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A Generic Acoustic and System-Level Modeling Framework for Multistatic SONAR Systems

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Abstract

Multistatic SONAR networks offer great potential for underwater surveillance applications but pose a complex analytical challenge due to their acoustic and systemic interactions. This work presents a generic modeling framework that accounts for acoustic, systemic, and network-related aspects of multistatic configurations. In addition to modeling acoustic propagation conditions, node-dependent parameters such as directional sensitivity and mechanical constraints of individual systems are incorporated. Unlike many previous studies that assume homogeneous propagation conditions, the framework also allows the consideration of littoral, i.e., spatially heterogeneous, environments in the system evaluation. The goal is to establish an extensible foundation for the development of future optimization and evaluation methods for multistatic SONAR networks.

Introduction

The need for large-scale and reliable underwater coverage has increased significantly in recent years. The operational objective in underwater surveillance is to determine the position and type of a participant within a defined area at any given time. A variety of SONAR configurations are available for this purpose, each exhibiting distinct characteristics. The simultaneous operation of such systems and the corresponding information fusion form a heterogeneous multistatic SONAR network (hMSN) [1].

While the optimization of individual systems remains relevant, the composition and coordination of such networks are of increasing importance. Consequently, a variety of modeling approaches for hMSNs have been proposed in the literature.

However, many existing approaches rely on simplifying assumptions. In particular, homogeneous propagation conditions and identical system parameters across all nodes are commonly assumed. Spatially varying environmental conditions, node-specific properties such as source level, directivity, or detection behavior, as well as their interactions, are often only implicitly or partially considered. In addition, reverberation is frequently simplified or neglected, although it can significantly influence detection performance, especially in shallow and littoral environments.

As a result, existing models are often limited in their ability to realistically evaluate heterogeneous system configurations. In particular, the combined impact of node-specific properties and spatially varying reverberation on system-level performance remains insufficiently addressed.

Against this background, this work presents an extended modeling framework for heterogeneous multistatic SONAR networks that complements existing approaches by incorporating a spatially resolved acoustic modeling perspective. In particular, spatially varying propagation conditions, node-specific system parameters, and bottom reverberation are explicitly considered. This enables a more realistic evaluation of network performance and provides a basis for future optimization and coordination strategies.

The main contributions of this work are:

- A spatially resolved acoustic modeling framework for heterogeneous multistatic SONAR networks.
- Explicit modeling of node-dependent system characteristics, including directional effects.
- Integration of a bistatic, cell-based bottom reverberation model.
- Analysis of the impact of reverberation and node heterogeneity on detection performance.

Modeling of Performance

The proposed modeling framework enables a spatially resolved evaluation of the performance of a heterogeneous multistatic SONAR network. The modeling is based on a multi-stage processing chain. The starting point is the parameterization of the network, which includes both the geometric configuration defined by the positions and orientations of the individual systems, as well as node-specific properties such as source level, directivity, and signal parameters (e.g., pulse and ping duration). In addition, environmental information such as bathymetry, coastline geometry, and physical water properties (e.g., temperature and salinity) is taken into account.

Based on these inputs, the acoustic signal propagation between transmitter, target position, and receiver is modeled, including transmission loss, noise, and reverberation effects. This results in a detection-related metric for each position within the area of interest, describing the local system performance.

The acoustic evaluation is based on the active bistatic SONAR equation [1]. For the general case of a separate transmitter $s \in [0, N_{\mathrm{Tx}} - 1]$ and receiver $r \in [0, N_{\mathrm{Rx}} - 1]$, the signal-to-noise ratio at a target position $\boldsymbol{p}$ is given by

$$\begin{aligned} SNR^{(s,r)}(\boldsymbol{p}) = SL^{(s)}&(\boldsymbol{u}_{\mathrm{Tx}}^{(s)}) \\ &- TL_{\mathrm{Tx}}^{(s)}(\boldsymbol{p}) - TL_{\mathrm{Rx}}^{(r)}(\boldsymbol{p}) \\ &+ TS^{(s,r)}(\boldsymbol{u}_{\mathrm{Tgt}}) \\ &- IL^{(r)}(\boldsymbol{p}) + DI^{(r)}(\boldsymbol{u}_{\mathrm{Rx}}^{(r)}). \end{aligned}$$

Here, $SL^{(s)}$ denotes the directional source level, TL_{Tx} and TL_{Rx} the transmission losses along the respective paths, TS the target strength, and $IL^{(r)}$ the interference level at the receiver, while $DI^{(r)}$ accounts for the directional reception characteristics.

The directional dependencies are described via the respective orientation vectors $\boldsymbol{u}_{\mathrm{Tx}}^{(s)}$, $\boldsymbol{u}_{\mathrm{Rx}}^{(r)}$, and the target orientation $\boldsymbol{u}_{\mathrm{Tgt}}$, allowing both transmitting and receiving characteristics to be represented.

This formulation enables the explicit consideration of node-specific properties as well as direction-dependent effects, thereby allowing heterogeneous system configurations to be modeled.

Direction-dependent Quantities

Source Level and Reception Characteristics

Both the source level and the reception characteristics are generally direction-dependent. For a given target position $\boldsymbol{p}$, these quantities are determined by the relative orientation between the system alignment and the target direction. The effective source level $SL^{(s)}$ and the reception directivity $DI^{(r)}$ can therefore be expressed as functions of the respective orientation vectors and the target position.

Target Strength

The target strength TS depends strongly on the relative geometry between transmitter, target, and receiver, and is particularly influenced by the bistatic incidence and scattering angles. Since the true target orientation is typically unknown, this introduces uncertainty into the modeling of TS.

In multistatic configurations, however, a larger diversity of bistatic angle combinations increases the likelihood of observing favorable scattering conditions. An explicit modeling of this effect is beyond the scope of this work and is left for future investigations.

Position-Dependent Quantities

Transmission Loss

The transmission loss is described using a simplified model consisting of geometric spreading and absorption:

$$\begin{aligned} TL_{\mathrm{Tx}}^{(s)}(\boldsymbol{p}) &= \beta \cdot 10\log_{10}\left(d_{\mathrm{Tx}}^{(s)}(\boldsymbol{p})\right) + \alpha\, d_{\mathrm{Tx}}^{(s)}(\boldsymbol{p}) \\ TL_{\mathrm{Rx}}^{(r)}(\boldsymbol{p}) &= \beta \cdot 10\log_{10}\left(d_{\mathrm{Rx}}^{(r)}(\boldsymbol{p})\right) + \alpha\, d_{\mathrm{Rx}}^{(r)}(\boldsymbol{p}). \end{aligned}$$

Here, $\beta \in [1,2]$ represents the spreading factor, corresponding to cylindrical or spherical propagation, and α denotes the absorption coefficient of the medium. For bistatic configurations, the total transmission loss is obtained by combining the transmitter–target and target–receiver paths.

Transmission loss often represents the dominant factor influencing detection performance. Many existing approaches therefore rely on surrogate models that directly map bistatic distance to detection probability. In the present work, such simplifications are avoided in order to explicitly account for additional position- and direction-dependent effects.

Noise and Reverberation

The interference term consists of ambient noise and reverberation and is combined in linear power:

$$IL^{(s,r)}(\boldsymbol{p}) = 10\log_{10}\left(10^{NL^{(r)}/10} + 10^{RL^{(s,r)}(\boldsymbol{p})/10}\right).$$

The ambient noise level is estimated using established models (e.g., [2]) based on the power spectral density $N_0(f)$:

$$NL^{(r)} = 10\log_{10}\left(\int_{f_\mathrm{l}}^{f_\mathrm{u}} N_0(f)\,\mathrm{d}f\right).$$

In shallow water environments, reverberation often represents a dominant interference component. It is modeled as the spatial integration of contributions from the seafloor. The contribution associated with an infinitesimal surface element $\mathrm{d}A$ at position $\boldsymbol{p}$ is given by:

$$\begin{aligned} RL^{(s,r)}(\boldsymbol{p}) = SL^{(s)}(\boldsymbol{p}) &- TL_{\mathrm{Tx}}^{(s)}(\boldsymbol{p}) - TL_{\mathrm{Rx}}^{(r)}(\boldsymbol{p}) \\ &+ SS^{(s,r)}(\boldsymbol{p}) + 10\log_{10}(\mathrm{d}A). \end{aligned}$$

Here, $SS^{(s,r)}(\boldsymbol{p})$ denotes the bistatic scattering strength per unit area (in dB re $1\,\mathrm{m}^2$), and $\mathrm{d}A$ represents the corresponding surface element.

The scattering strength is modeled as a function of the incidence and scattering angles:

$$SS^{(s,r)}(\boldsymbol{p}) = 10\log_{10}\left(\sigma_{\mathrm{bottom}}(\theta_i, \theta_s, \phi)\right).$$

Using a simplified Lambert model, this yields [3]:

$$\sigma_{\mathrm{bottom}}(\theta_i, \theta_s, \phi) = \mu \cdot \cos(\theta_i) \cdot \cos(\theta_s) \cdot f(\phi).$$

Only seafloor elements that are visible from both transmitter and receiver are considered, i.e., contributions

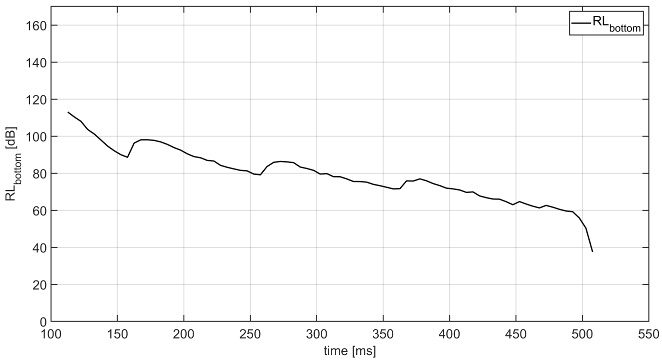


Figure 1: Reverberation level over time for a fixed transmitter–receiver configuration on ripple-structured bathymetry.

are restricted to positions with line-of-sight (LOS) along both propagation paths. The effective reverberation level (in dB re $1\,\mu$Pa) is obtained by integrating over all positions whose echoes arrive within the time interval $[t, t+\tau_{\text{pulse}})$:

$$RL^{(s,r)}_{\text{bottom}}(t) = 10\log_{10}\left(\sum_{\mathcal{P}(t)} 10^{RL^{(s,r)}(\boldsymbol{p})/10}\,\mathrm{d}\boldsymbol{p}\right)$$

with

$$\mathcal{P}(t) = \left\{\boldsymbol{p}\;\middle|\; t \le \tau^{(s,r)}(\boldsymbol{p}) < t + \tau_{\text{pulse}}\right\},$$

where τ_{pulse} denotes the transmit pulse duration and $\tau^{(s,r)}(\boldsymbol{p})$ is the bistatic propagation delay associated with position $\boldsymbol{p}$.

Different bathymetric configurations have been considered. For demonstration purposes, the following example (Fig. 1 and Fig. 2) is based on a seafloor with ripple-like structures.

During temporal aggregation, it must be considered that the area of interest may only partially cover the bistatic isochrones associated with a given time interval. As a result, not all contributing seafloor regions are included, which would otherwise lead to systematically underestimated reverberation levels if not properly compensated. To isolate the impact of node placement on the reverberation level, a reference scenario is considered in which the transmitter–receiver configuration is rotated around a fixed target location. The resulting reverberation level is evaluated as a function of the rotation angle, while all other parameters remain unchanged, ensuring that variations are solely due to geometric configuration.

The results show that the reverberation level strongly depends on the geometric configuration. This indicates that reverberation has a direct impact on the optimal parameterization of the network and should therefore be explicitly considered in acoustic modeling.

System-level Fusion

For each transmitter–receiver pair (s, r) and each position $\boldsymbol{p}$, the signal-to-noise ratio $SNR^{(s,r)}(\boldsymbol{p})$ can be determined. Based on this, the signal excess (SE) is defined as the difference between the resulting SNR and a system-specific detection threshold $DT^{(r)}$:

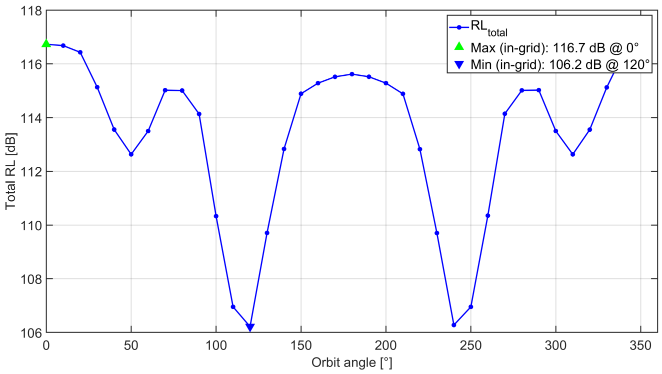


Figure 2: Reverberation level as a function of the rotation angle of the transmitter–receiver configuration around a fixed target position.

$$SE^{(s,r)}(\boldsymbol{p}) = SNR^{(s,r)}(\boldsymbol{p}) - DT^{(r)}.$$

The signal excess serves as the input to the detection model, allowing the mapping to a detection probability. Different detection models can be incorporated to represent various sensor types and signal processing approaches. For example, conventional coherent detectors as well as methods based on direction-of-arrival (DOA) estimation can be consistently integrated.

The detection probability is expressed as:

$$P^{(s,r)}_{D}(\boldsymbol{p}) = f^{(s,r)}_{D}\left(SE^{(s,r)}(\boldsymbol{p})\right).$$

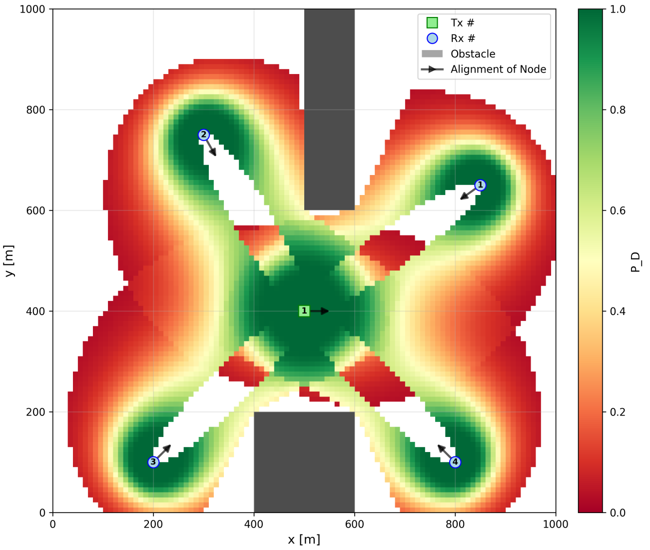
This formulation enables a consistent coupling between acoustic modeling and detection behavior.

$$P_{\mathrm{D}}(\boldsymbol{p}) = 1 - \prod_{(s,r)}\left(1 - P^{(s,r)}_{\mathrm{D}}(\boldsymbol{p})\right)$$

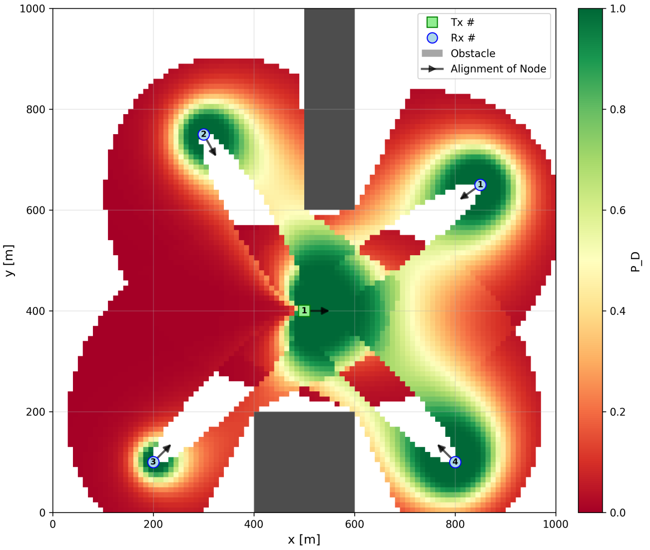
$$\overline{P_{\mathrm{D}}} = \frac{1}{|\mathcal{P}|}\sum_{\boldsymbol{p}\in\mathcal{P}} P_{\mathrm{D}}(\boldsymbol{p})$$

The cumulative detection probability $P_{\mathrm{D}}(\boldsymbol{p})$ represents the probability that a target at position $\boldsymbol{p}$ is detected by at least one transmitter–receiver pair, assuming statistically independent detections [1]. To obtain a system-level performance measure, the cumulative detection probability is averaged over all discrete grid positions within the area of interest. The resulting mean detection probability $\overline{P_{\mathrm{D}}}$ therefore reflects the overall detection capability of the network. Fig. 3 shows a comparison of detection maps with and without node-specific parameterization and directional modeling.

The comparison clearly shows that neglecting node-specific characteristics and directional effects leads to



(a) Homogeneous network without directional characteristics resulting in $\overline{P_\mathrm{D}} = 30\,\%$.



(b) With node-specific orientation resulting in $\overline{P_\mathrm{D}} = 23\,\%$.

Figure 3: Comparison of detection maps for homogeneous and heterogeneous system configurations.

simplified and potentially misleading performance patterns. Incorporating these aspects results in a more differentiated and realistic representation of the detection performance.

Modeling Assumptions and Limitations

The presented modeling approach is based on a set of simplifying assumptions that are chosen deliberately in order to focus on the dominant effects relevant for system-level evaluation.

In particular, the reverberation model is restricted to first-order bottom scattering using a diffuse Lambert-type formulation. Surface reverberation, volume scattering, and higher-order multipath propagation are not explicitly considered. While these effects can contribute to the overall interference level, especially in complex environments, they are omitted here to isolate the influence of bathymetry and geometric system configuration on reverberation.

This simplification allows for a computationally efficient and spatially resolved evaluation of large-scale network configurations, while still capturing the primary dependency of reverberation on transmitter–receiver geometry.

Furthermore, the transmission loss is modeled using a simplified spreading and absorption model, and target strength is treated as a generalized parameter without explicit aspect-dependent modeling.

Despite these simplifications, the results indicate that the proposed model captures the dominant trends relevant for system-level analysis. The extension towards more advanced propagation and scattering models, including surface interactions and ray-based approaches, represents a natural direction for future work.

Summary and Outlook

In this work, an extended modeling framework for heterogeneous multistatic SONAR networks was presented, enabling a spatially resolved evaluation of system performance. The approach integrates node-specific system parameters, spatially varying propagation conditions, and a bistatic, cell-based reverberation model within a unified modeling chain.

The results demonstrate that both reverberation and geometric configuration have a significant influence on the resulting detection performance. In particular, neglecting reverberation effects can lead to misleading performance estimates and suboptimal system configurations.

The proposed framework provides a flexible basis for the evaluation of heterogeneous system configurations and allows the consistent integration of different detection models.

Future work will focus on extending the reverberation model towards more complex propagation effects, including surface interactions and higher-order multipath, as well as on incorporating optimization strategies for network configuration.

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