Lab Communications Project 8
Signal Sources and Spectral Analysis

1 Introduction

In this lab the spectral content of a measured time-series signal will be determined. Therefore various methods for spectral analysis or spectral estimation will be studied.
Spectral analysis was first examined by Schuster in 1898 for detecting cyclic behavior in time-series. The word “spectrum” is derived from the latin word “specter”: “I’m being observed”. Newton used the word “spectrum” to describe the decomposition of white light into various colors.
There are many applications of spectral analysis. In mechanical engineering the wear and tear of the mechanical parts like balls and bearings can be analyzed using measured signals. In speech signal processing, speech analysis and many speech enhancement techniques are based on the estimated speech spectrum. In radar and sonar signal processing the location of the sources are present in the spectral contents of the signal received. Also it is used in various medical examinations, like the electroencephalogram (EEG) signals measured from patients which are spectrally analyzed to check for any symptoms.

2 Spectral Estimation

Intuitively, a spectrum of a signal can be thought of pieces of different frequencies arranged on a frequency scale. In order to determine the power of the spectral content, the signal can be passed through a set of bandpass filters of desired bandwidth $\Delta(e^{j\Omega})$ and divided by the bandwidth to obtain an estimated power of the spectral estimate.
Let $y(n)$ be a *deterministic* discrete-time signal sequence with finite energy

$$\sum_{n=-\infty}^{+\infty} |y(n)|^2 < \infty$$

then the *Energy Spectral Density* $S(e^{j\Omega})$ of $y(n)$ is given by

$$S(e^{j\Omega}) = |Y(e^{j\Omega})|^2 \tag{1}$$

where $Y(e^{j\Omega})$ is given by

$$Y(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} y(n)e^{-j\Omega n}. \tag{2}$$

Measured real-world signals are mostly random signals which are one of the many realizations of the random process from which these signals arise. The previous definition shown in Eq. (2) can’t be applied for measured signals, even though they don’t change after measurement, because they don't hold finite energy. Random signals are best described by their statistical properties like average power, second moment, etc. The spectral estimates of such signals are called *Power Spectral Densities* (PSD) given by

$$\phi(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} r(k)e^{-j\Omega k} \tag{3}$$

where $r(k)$ is the autocovariance of the random signal $y(n)$ defined as

$$r(k) = E\{y(n)\, y^*(n-k)\}. \tag{4}$$

A second definition of the PSD, which is equivalent to Eq. (3), is given by

$$S(e^{j\Omega}) = \lim_{N\to\infty} E\left\{ \frac{1}{N} \left| \sum_{n=0}^{N-1} y(n)e^{-j\Omega n} \right|^2 \right\}. \tag{5}$$

2.1 Non-Parametric Estimation

Non-parametric methods for estimating spectra are based on the definitions introduced before. These methods introduced here are generic ways of estimating spectra and are not dependent on any models or parameters, hence the name non-parametric.

2.1.1 Periodogram Method

By modifying Eq. (5) the measured spectra of a random signal can be obtained. This method is called Periodogram. The Periodogram is computed by

$$\hat{\phi}_{\mathrm{p}}(e^{j\Omega}) = \frac{1}{N} \left| \sum_{n=0}^{N-1} y(n)e^{-j\Omega n} \right|^2 . \tag{6}$$

Here the limit and the expectation have been omitted.

2.1.2 Corellogram Method

The correlation based computation of the PSD is called as Corellogram. From Eq. (3) the Correlogram can be computed by

$$\hat{\phi}_{\mathrm{c}}(e^{j\Omega}) = \sum_{k=-(N-1)}^{N-1} \hat{r}(k)e^{-j\Omega k}. \tag{7}$$

An estimate of the covariance can be obtained by

$$\hat{r}(k) = \frac{1}{N-k} \sum_{n=k+1}^{N} y(n)y^*(n-k) \qquad 0 \leq k \leq N-1. \tag{8}$$

2.1.3 Blackman-Tuckey Method

To reduce high statistical variability a modified version of the Correllogram was developed, called Blackman-Tuckey estimator. In this method a window $w(k)$ is applied to the estimated covariance. The window consits of non-zero values within the window length M and zeros outside. The window is an even function i.e. $w(-k) = w(k)$ and $M < N$ has to be fulfilled. It can be applied to the Periodogram by convolving $W(e^{j\Omega})$, the DTFT of the window, with $\phi_{\mathrm{p}}(e^{j\Omega})$.
The Blackman-Tuckey estimator is obtained by

$$\hat{\phi}_{\mathrm{BT}}(e^{j\Omega}) = \sum_{k=-(M-1)}^{M-1} w(k)\hat{r}(k)e^{-j\Omega k}. \tag{9}$$

and

$$\hat{\phi}_{\mathrm{BT}}(e^{j\Omega}) = \hat{\phi}_{\mathrm{p}}(e^{j\Omega}) * W(e^{j\Omega}). \tag{10}$$

Different windows such as the Rectangular, Bartlett, Hann, Hamming can be used corresponding to different requirements.

2.1.4 Bartlett Method

One simple idea to reduce the variance in the estimated PSDs is to compute multiple PSDs from the given data length and average all of them. This is achieved by dividing the data into smaller segments of length M according to

$$y_i(n) = y(n + (i-1)M) \text{ for } i = 1, ..., L \tag{11}$$

so that $N = L \cdot M$ where N is the lenght of the measured signal. The Periodogram is then computed according to

$$\hat{\phi}_{i,\mathrm{p}}(e^{j\Omega}, i) = \frac{1}{M}\left|\sum_{n=0}^{M-1} y_i(n)e^{-j\Omega n}\right|^2 \text{ for } i = 1, ..., L \tag{12}$$

and the Bartlett estimate is obtained according to

$$\hat{\phi}_{\mathrm{B}}(e^{j\Omega}) = \frac{1}{L}\sum_{i=1}^{L} \hat{\phi}_{i,\mathrm{p}}(e^{j\Omega}, i). \tag{13}$$

2.2 Parametric Estimation

A second approach for estimating the spectra of measured random signals is to compare the signal with a model. The parameters estimated in the model are used for the estimation of the spectra. When the estimated parameters are close to the real parameters, the estimated spectra are also more accurate. These methods are in general classified as parametric estimations.
In parametric modeling the measured signal $y(n)$ is considered as a result from a white noise input to a system with transfer function $H(z)$. The white noise process is described by a covariance of $\sigma_{\mathrm{wn}}^2\delta(k)$ and a PSD σ_{wn}^2. The transfer function $H(z)$ is defined as

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_q z^{-q}}{a_0 + a_1 z^{-1} + \cdots + a_p z^{-p}}. \tag{14}$$

- **Autoregressive process (AR)** when $q = 0$. The filter is all-pole, recursive.
- **Moving Average process (MA)** when $p = 0$. The filter is all-zero, non-recursive.
- **ARMA process** when both $p, q > 0$. The filter is pole-zero, recursive.

The parameters that need to be estimated are the a_i for AR processes, b_j for MA processes and both a_i, b_j for ARMA processes. The PSD of $y(n)$ is then given by

$$\phi(e^{j\Omega}) = |H(e^{j\Omega})|^2 \phi_{\mathrm{wn}}(e^{j\Omega}) = \left| \frac{b_0 + b_1 z^{-1} + \cdots + b_q z^{-q}}{a_0 + a_1 z^{-1} + \cdots + a_p z^{-p}} \right|^2 \sigma_{\mathrm{wn}}^2. \tag{15}$$

The parametric estimation is widely used in the analysis of speech signals. A simple all-pole filter can be assumed to describe the behavior of the vocal tract. Speech signals can be roughly divided into voiced and unvoiced segments. Unvoiced segments are those which are random noise alike and voiced segments are those which are passed through a glottal pulse filter $G(z)$. The vocal tract filter can be modeled as

$$H(e^{j\Omega}) = \frac{1}{1 + \sum\limits_{i=1}^{p} a_i e^{-j\Omega i}} \tag{16}$$

where p is the number of poles.

3 Lab Preparation

(a) Describe spectrum estimation in a few words .

(b) What is the difference between *Energy Spectral Density* and *Power Spectral Density*?

(c) What is Non-Parametric estimation?

(d) What is Parametric estimation?

(e) List out the properties of speech signals.

4 Lab Execution

4.1 Periodogram and Corellogram

1. Realize a function called `periodg1` which computes an estimate of the PSD based on Eq. (6).
2. Load data from p1.mat. Use the created function to estimate the PSD of the data.
3. Realize a function called `correlg1` which computes an estimate of the PSD based on Eq. (7).
4. Now use this function to compute the PSD of the above data.
5. Plot the two computed PSDs.
6. Repeat the above steps for p2.mat and p3.mat.
7. Plot all the estimations (using `subplot`).

4.2 Blackmann-Tuckey and Bartlett Estimation

1. Realize a function called `blackman1` which computes an estimate of the PSD based on Eq. (10). You could use `periodg1` as a template.
2. Load data from p1.mat. Use the created function to estimate the PSD of the data.
3. Change the window from rectangular to Barlett, Hann and Hamming.
4. Come up with a data sequence which would return the spectrum of the window used as PSD.
5. Plot the above spectrum for all windows stated above.
6. Realize a function called `bartlett1` which computes an estimate of the PSD based on Eq. (13).
7. Load data from p1.mat. Use the created function above to estimate the PSD of the data.
8. Try the computations for p2.mat and p3.mat.
9. Plot all the estimations (using `subplot`).

4.3 Speech Signal Analysis

1. Load the data s1.mat and s2.mat.
2. Model the vocal tract with an AR process of order 10 using the `aryule` function.
3. Compute and plot the PSDs for both data sets.
4. Try other values of the AR process order and plot the corresponding PSDs.
5. Based on the plots, is it possible to determine voiced or unvoiced speech segments?