Processing Structures

1 Questions

1. What are basic approaches for up- and downsampling a signal, respectively?
2. When looking at the filter bank principle, why are we allowed to violate the basic sampling theorem ($f_{\mathrm{max}} < f_{\mathrm{s}}/2$)?
3. What quantities describe the effect of window functions in the frequency domain? How are these quantities influenced when window functions are allowed to become longer than the DFT size and even asymmetric?
4. What is the advantage of designing a prototype lowpass filter?
5. Why is the inverse FFT (paradoxically) used in the efficient implementation of an analysis filterbank?

2 Answers

1. When upsampling with a postitive integer factor r, $r-1$ zero-valued samples are added between each sample of the source signal. When downsampling with r, only every r-th sample is retained, the $r-1$ samples in between are not set to zero but are omitted. To avoid aliasing (downsampling) and imaging (upsampling), anti-aliasing and anti-imaging filters are applied to the signal before downsampling and after upsampling, respectively.

2. We usually violate the theorem when downsampling one of the higher frequency bands, as the maximum frequency contained after the bandpass / highpass is higher than half of the new sampling frequency. We are allowed to violate the theorem since the processing chain consecutively includes the anti-imaging filters, which will get rid of the aliasing terms. This only works properly if the aliasing terms do not overlap with the actual frequency content, hence, if the effective bandwidth of the signal to be downsampled (and therefore the bandwidth of the anti-aliasing / imaging filters) satisfies the sampling theorem instead of the maximum frequency.

3. Window functions are characterized in the frequency domain by the main lobe (passband) width and by the side lobe (stopband) attenuation. When there is more freedom in the design of the function in the time-domain (length, symmetry,..) both main lobe width, which corresponds to the bandwidth, and side lobe attenuation, which influences e.g. the aliasing terms, can be decreased.

4. If all frequency bands are to have the same bandwidth, they can utilize the same filter (prototype) with band-individual frequency shift. While it is convenient to only have to design one instead of M filters, this approach also yields a significant complexity reduction during runtime (see polyphase implementation).

5. The frequency-shift of the prototype filter corresponds in the time-domain to the multiplication with exponential terms holding positive exponents. Due to the positive exponents, rewriting the equations leads to the application of the scaled IFFT instead of the FFT, which utilizes negative exponents.