Adaptive Filters – Wiener Filter

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Today

Contents of the Lecture:

- ❑ Introduction and motivation
- ❑ Principle of orthogonality
- ❑ Time-domain solution
- ❑ Frequency-domain solution
- ❑ Application example: noise suppression

History and Assumptions

Filter design by means of minimizing the squared error (according to Gauß)

Independent development

1941: A. Kolmogoroff: *Interpolation und Extrapolation von stationären zufälligen Folgen*, Izv. Akad. Nauk SSSR Ser. Mat. 5, pp. 3 – 14, 1941 (in Russian)

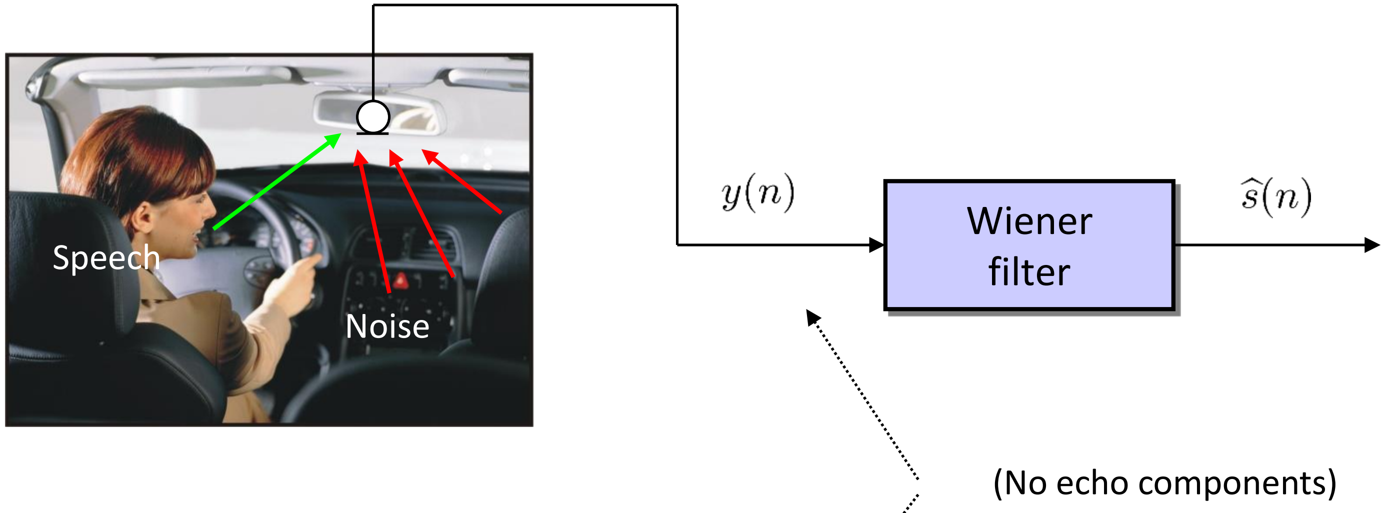
1942: N. Wiener: *The Extrapolation, Interpolation, and Smoothing of Stationary Time Series with Engineering Applications*, J. Wiley, New York, USA, 1949 (originally published in 1942 as MIT Radiation Laboratory Report)

Assumptions / design criteria:

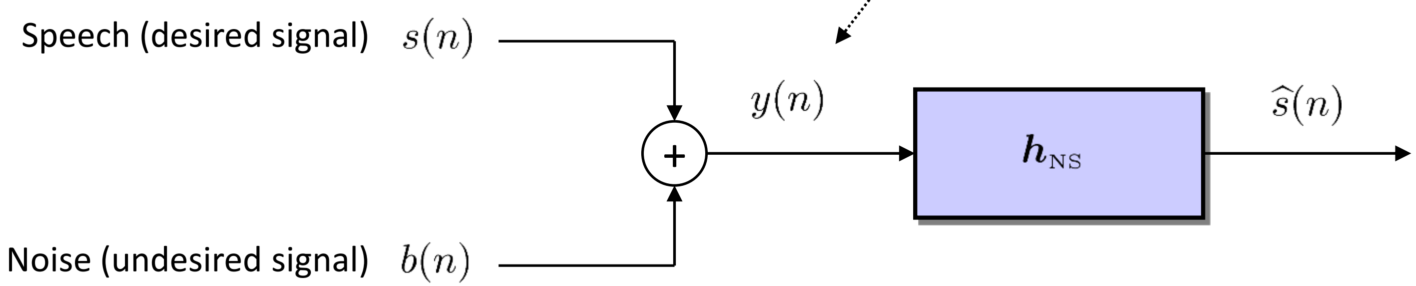
- Design of a filter that separates a desired signal optimally from additive noise
- Both signals are described as stationary random processes
- Knowledge about the statistical properties up to second order is necessary

Noise Suppression

Application example:



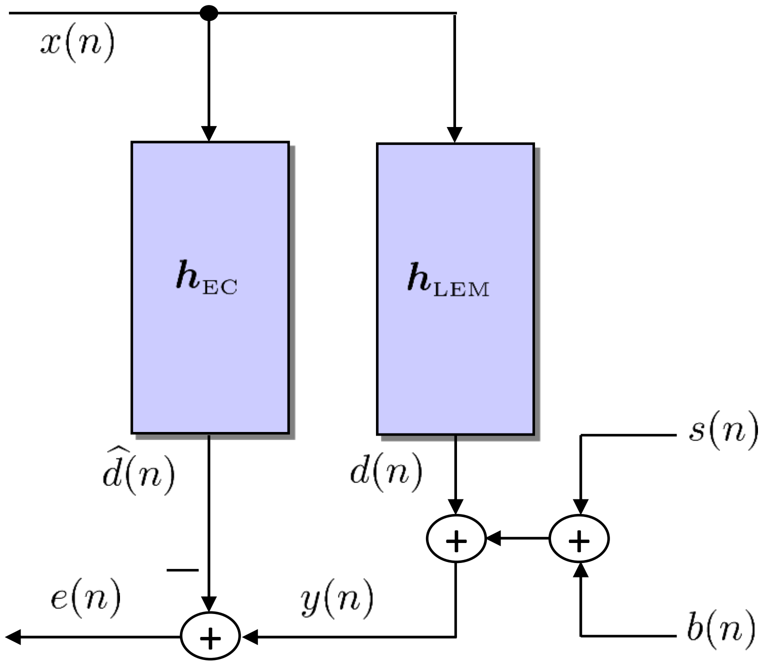
Model:



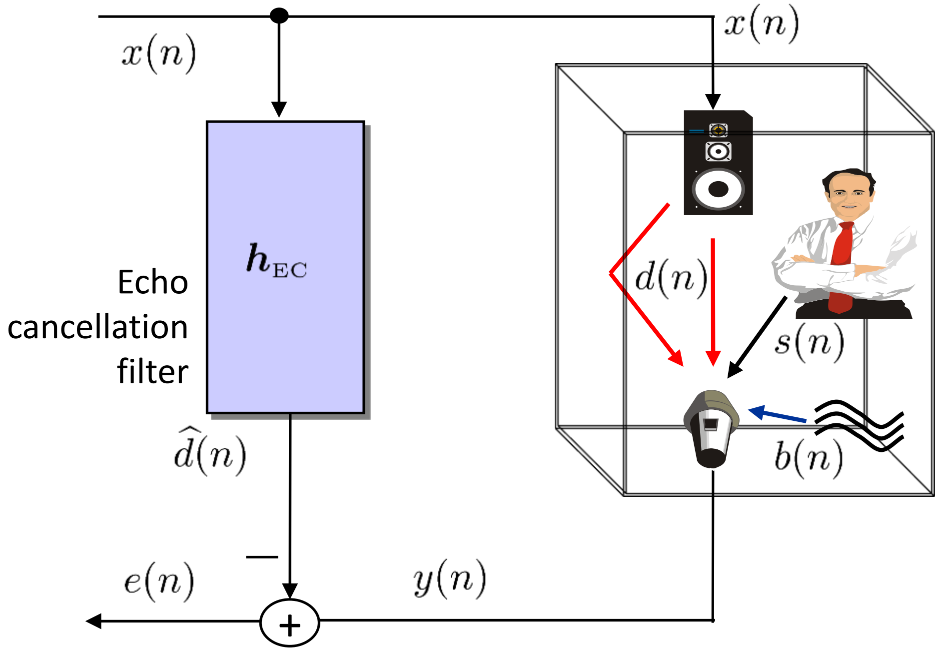
The Wiener solution if often applied in a "block-based fashion".

Echo Cancellation

Model:

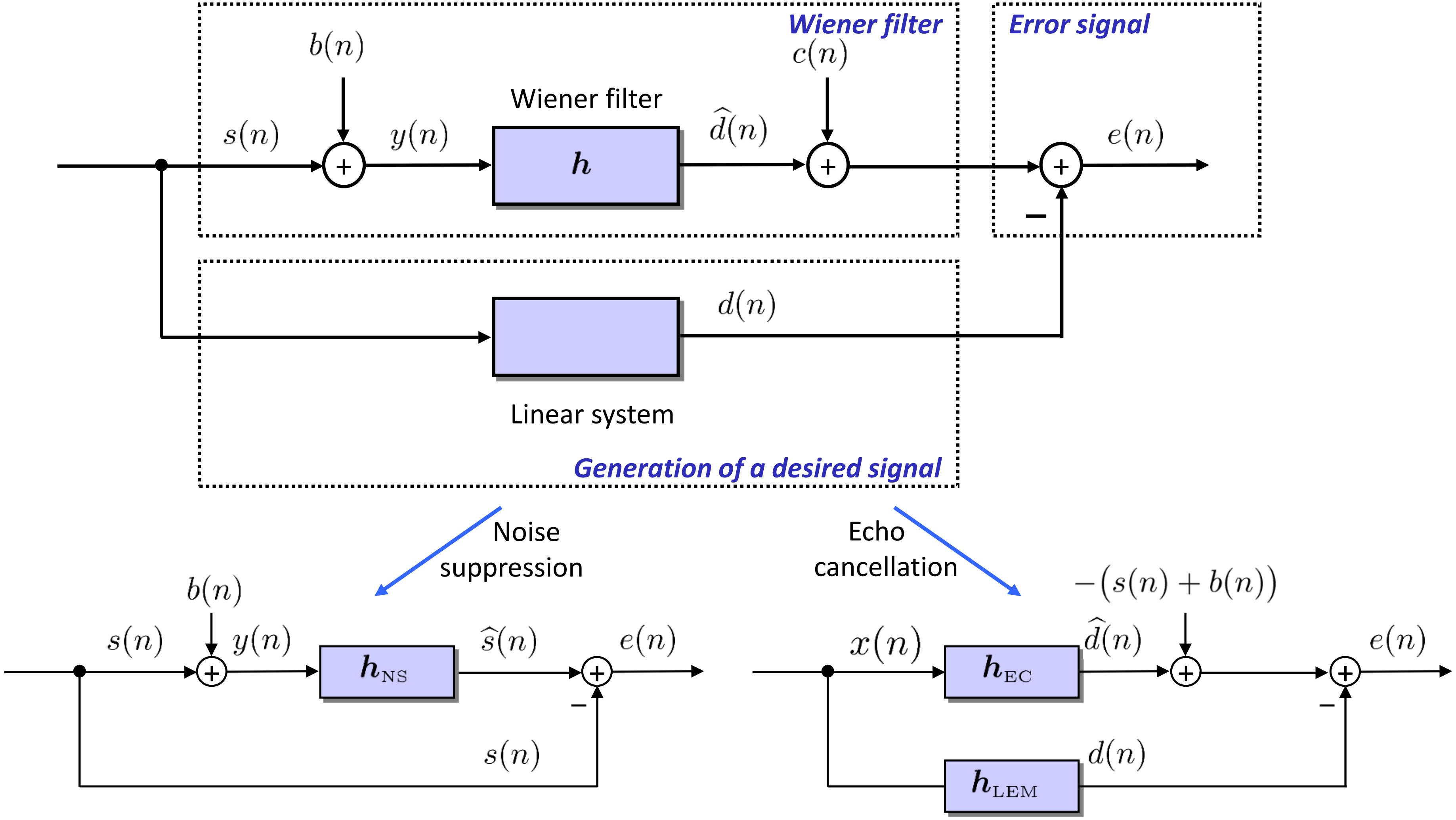


Application example:



The echo cancellation filter has to converge in an iterative manner (new = old + correction) towards the Wiener solution.

Noise Reduction and System Identification



Books

Main text:

- E. Hänsler / G. Schmidt: *Acoustic Echo and Noise Control – Chapter 5 (Wiener Filter),* Wiley, 2004

Additional texts:

- E. Hänsler: *Statistische Signale: Grundlagen und Anwendungen – Chapter 8 (Optimalfilter nach Wiener und Kolmogoroff),* Springer, 2001 (in German)
- M. S.Hayes: *Statistical Digital Signal Processing and Modeling – Chapter 7 (Wiener Filtering),* Wiley, 1996
- S. Haykin: *Adaptive Filter Theory – Chapter 2 (Wiener Filters),* Prentice Hall, 2002

Noise suppression:

- U. Heute: *Noise Suppression*, in E. Hänsler, G. Schmidt (eds.), Topics in Acoustic Echo and Noise Control, Springer, 2006

Derivation

Derivation during the lecture …

Principle of Orthogonality

A Deterministic Example

Derivation during the lecture …

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Time-Domain Solution

Derivation during the lecture …

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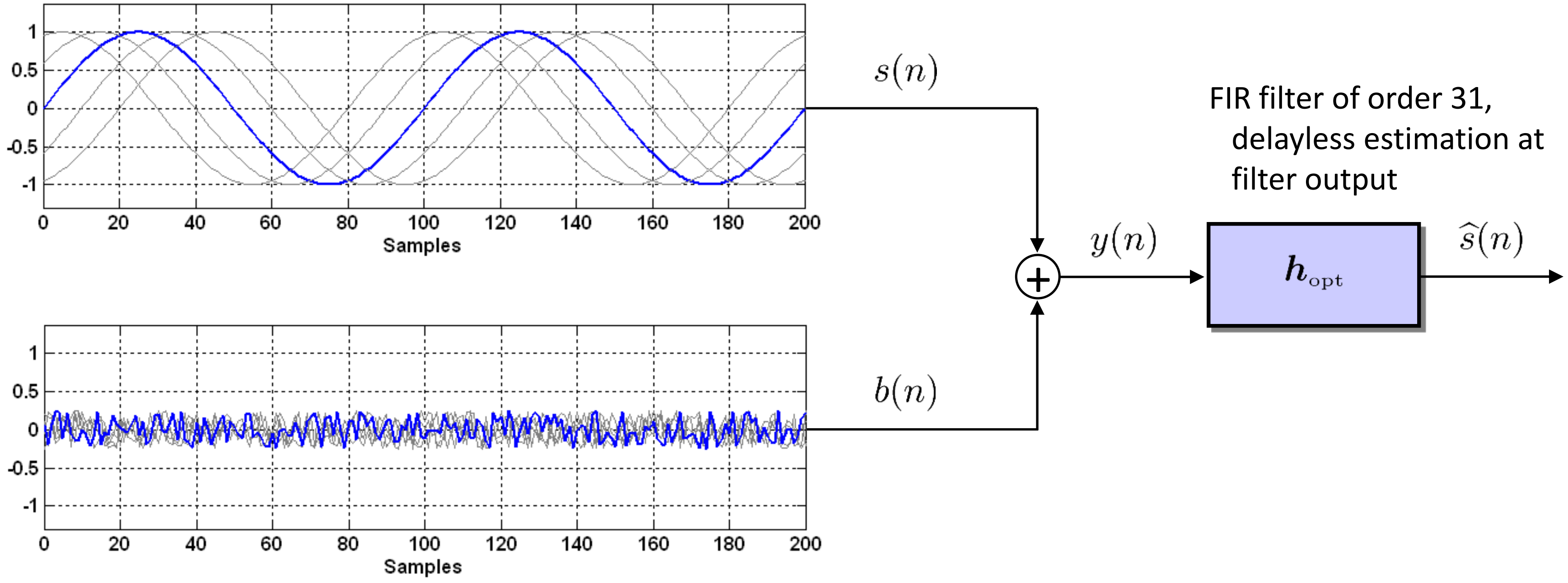
Example – Part 1

Desired signal: Sine wave with known frequency but with unknown phase, not correlated with noise



FIR filter of order 31, delayless estimation at filter output

Noise: White noise with zero mean, not correlated with desired signal

Example – Part 2

Wiener solution:

$$\boldsymbol{h}_{\text{opt}} = \boldsymbol{R}_{yy}^{-1}\,\boldsymbol{r}_{yd}(0)$$

Desired signal and noise are not correlated and have zero mean:

$$\mathrm{E}\big\{s(n)\cdot b(n+l)\big\} = \mathrm{E}\big\{s(n)\big\}\cdot\mathrm{E}\big\{b(n+l)\big\} = m_s\cdot m_b$$

$$\mathrm{E}\big\{s(n)\big\} = \mathrm{E}\big\{b(n)\big\} = 0$$

$$\Longrightarrow \quad \mathrm{E}\big\{s(n)\cdot b(n+l)\big\} = 0$$

Simplification according to the assumptions above:

$$\boldsymbol{R}_{yy} = \boldsymbol{R}_{ss} + \boldsymbol{R}_{bb}$$

$$\boldsymbol{r}_{yd}(0) = \boldsymbol{r}_{ys}(0) = \boldsymbol{r}_{ss}(0) + \underbrace{\boldsymbol{r}_{bs}(0)}_{0} = \boldsymbol{r}_{ss}(0)$$

Wiener solution (modified):

$$\boldsymbol{h}_{\text{opt}} = \big(\boldsymbol{R}_{ss} + \boldsymbol{R}_{bb}\big)^{-1}\,\boldsymbol{r}_{ss}(0)$$

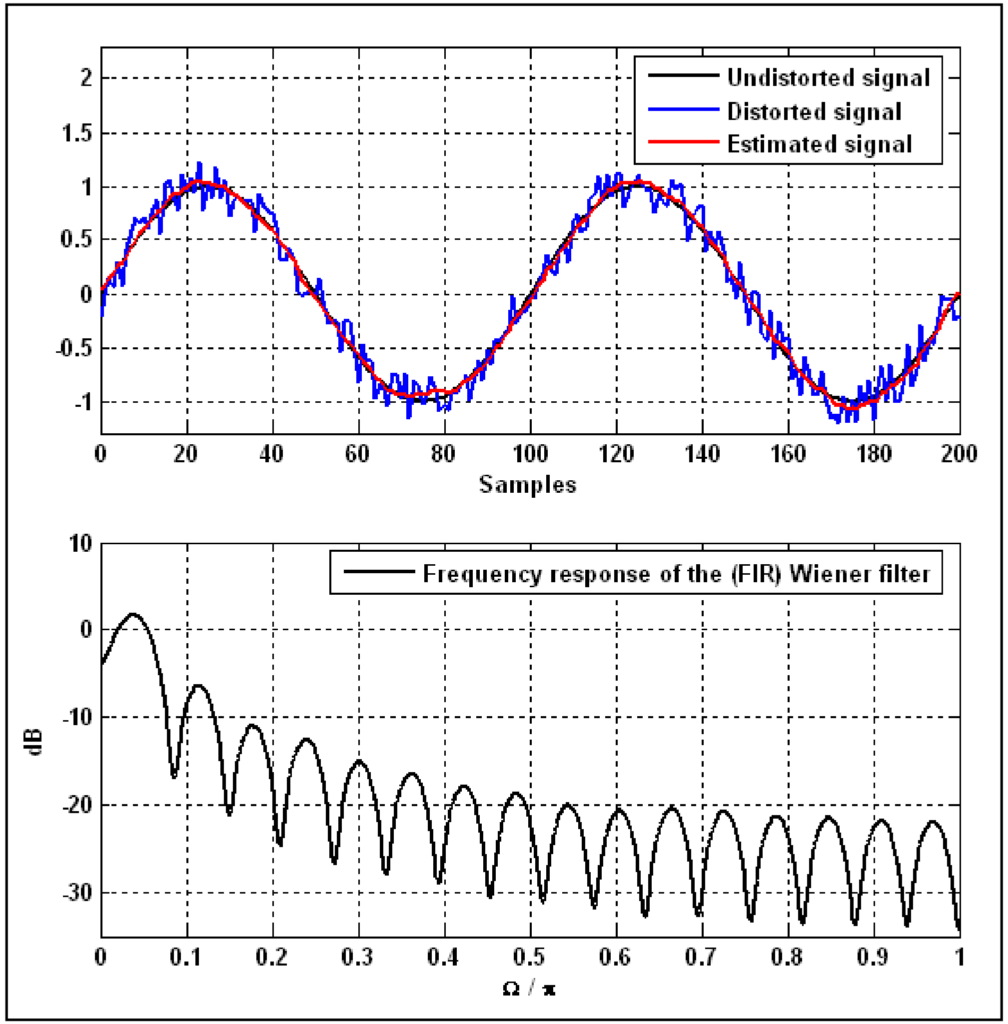
Example – Part 3

Input signals:

Excitation: sine wave
Noise: white noise

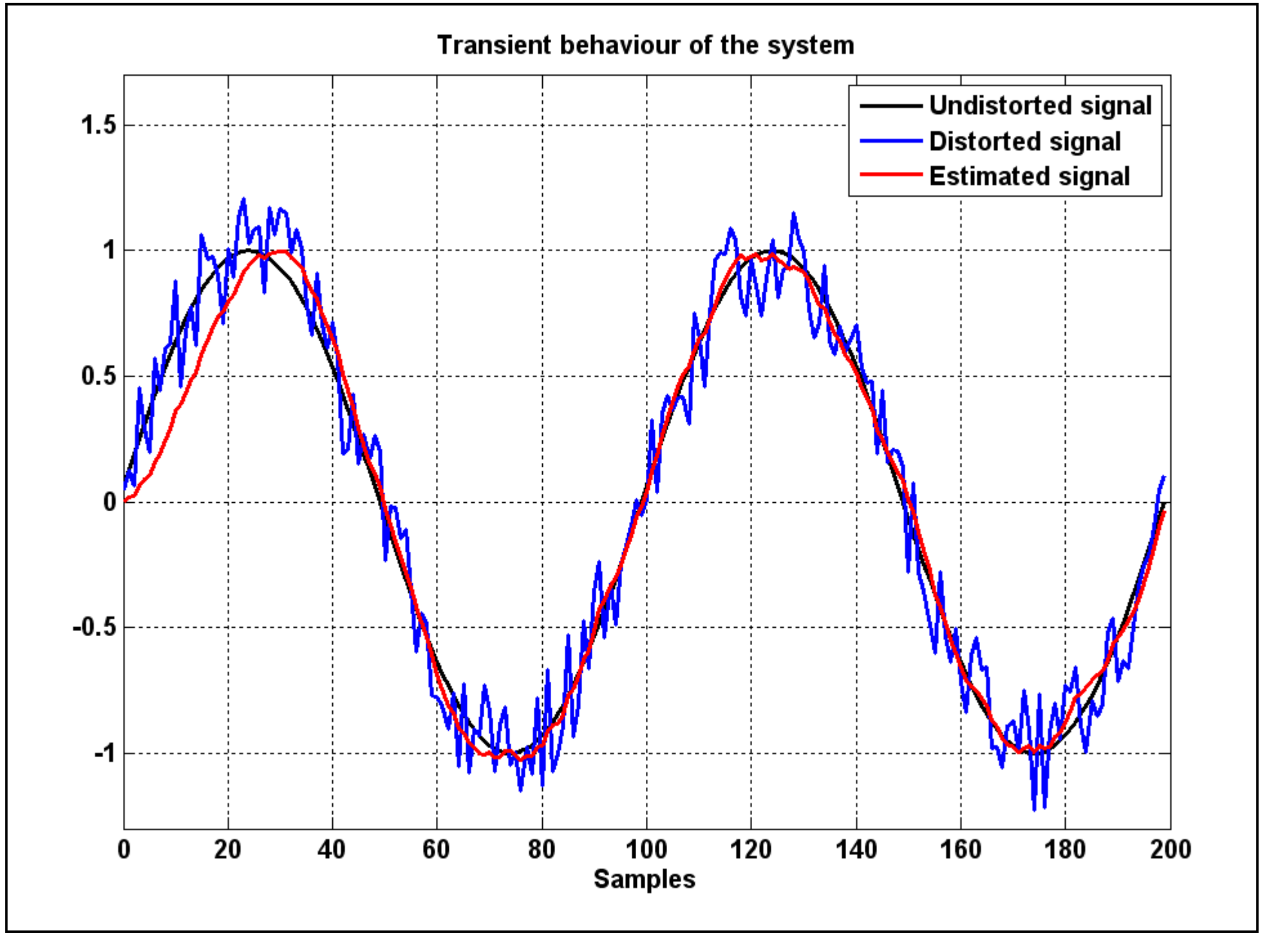
Assumptions:

- Knowledge of the mean values and of the autocorrelation functions of the desired and of the undesired signal
- Desired signal and noise are not correlated
- Desired signal and noise have zero mean
- 32 FIR coefficients should be used by the filter



Example – Part 4

- After a short initialization time the noise suppression performs well (and does not introduce a delay!)



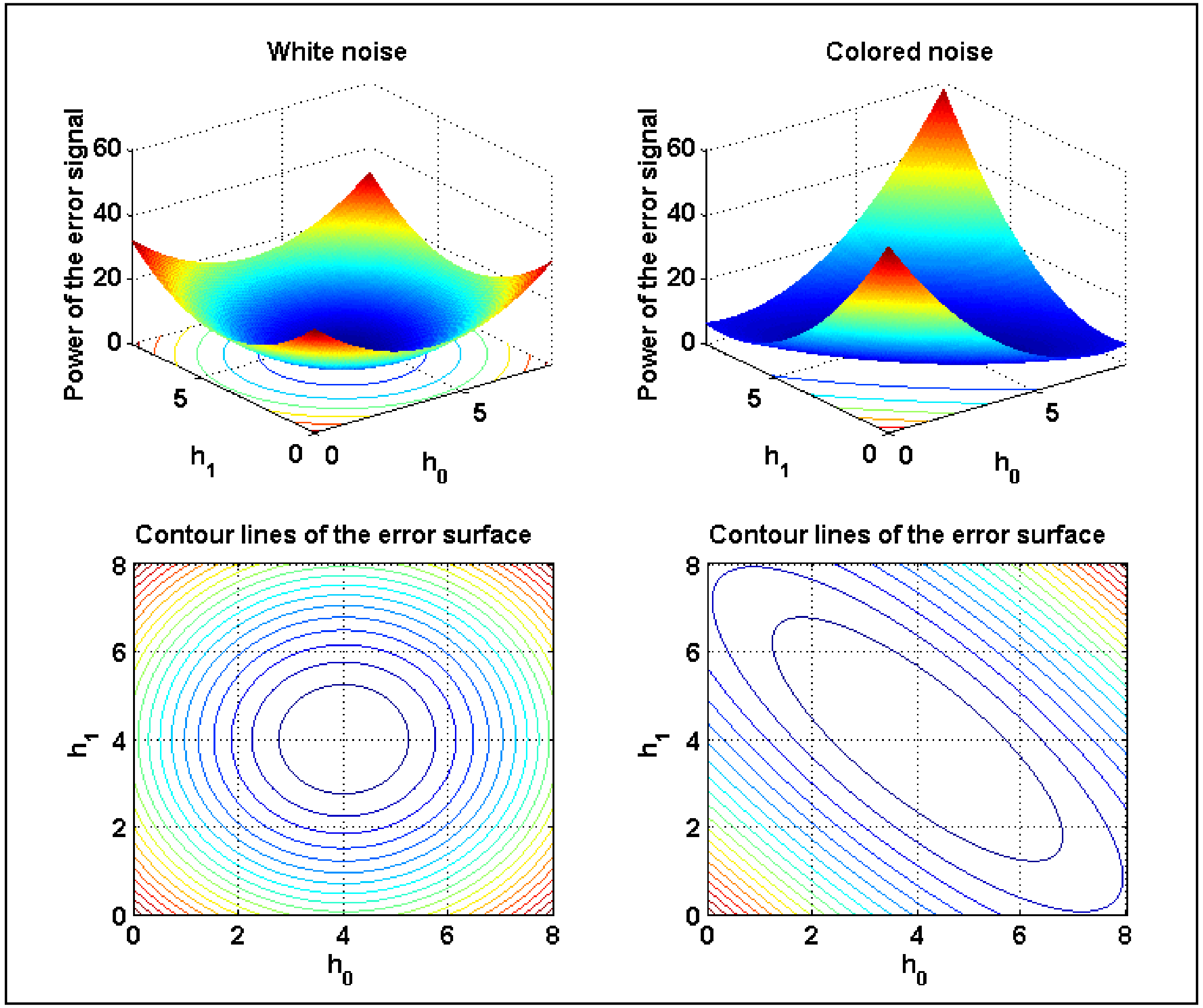
Derivation – Part 1

Derivation during the lecture …

Derivation – Part 2

Error surface for:

- $\boldsymbol{R}_{yy} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\boldsymbol{R}_{yy} = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$



Properties:

- Unique minimum (no local minima)
- Error surface depends on the correlation properties of the input signal

Frequency-Domain Solution

Derivation

Derivation during the lecture …

Noise Suppression – Part 1

Frequency-domain Wiener solution (non-causal):

$$H_{\text{opt}}\left(e^{j\Omega}\right) = \frac{S_{yd}(\Omega)}{S_{yy}(\Omega)}$$

Desired signal = speech signal:

$$d(n) = s(n) \qquad \Longrightarrow \qquad H_{\text{opt}}\left(e^{j\Omega}\right) = \frac{S_{ys}(\Omega)}{S_{yy}(\Omega)}$$

Desired signal and noise are orthogonal:

$$\mathrm{E}\big\{s(n_1)\,b(n_2)\big\} = 0, \;\text{for all } n_1 \text{ and } n_2$$

$$\begin{aligned}
\Longrightarrow \qquad H_{\text{opt}}\left(e^{j\Omega}\right) &= \frac{S_{ys}(\Omega)}{S_{yy}(\Omega)} = \frac{S_{ss}(\Omega)+S_{bs}(\Omega)}{S_{yy}(\Omega)} \\
&= \frac{S_{ss}(\Omega)}{S_{yy}(\Omega)} = \frac{S_{yy}(\Omega)-S_{bb}(\Omega)}{S_{yy}(\Omega)} \\
&= 1 - \frac{S_{bb}(\Omega)}{S_{yy}(\Omega)}
\end{aligned}$$

Noise Suppression – Part 2

Frequency-domain solution:

$$H_{\mathrm{opt}}(e^{j\Omega}) \quad = \quad 1 - \frac{S_{bb}(\Omega)}{S_{yy}(\Omega)}$$

Approximation using short-term estimations:

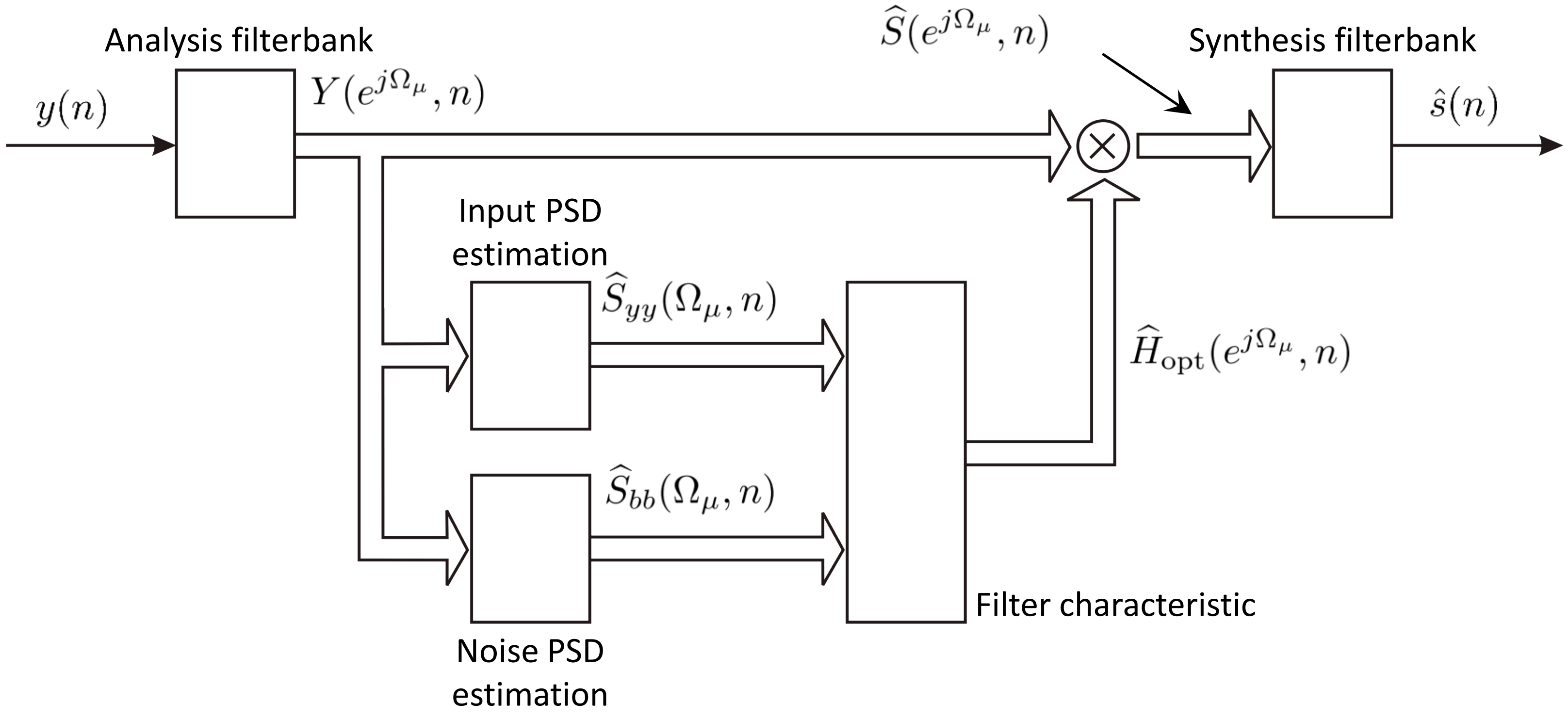
$$\widehat{H}_{\mathrm{opt}}(e^{j\Omega}, n) \quad = \quad \max\left\{0,\, 1 - \frac{\widehat{S}_{bb}(\Omega, n)}{\widehat{S}_{yy}(\Omega, n)}\right\}$$

Practical approaches:

- Realization using a filterbank system (time-variant attenuation of subband signals)
- Analysis filters with length of about 15 to 100 ms
- Frame-based processing with frame shifts between 1 and 20 ms
- The basic Wiener characteristic is usually „enriched“ with several extensions (overestimation, limitation of the attenuation, etc.)

Noise Suppression – Part 3

Processing structure:



PSD = power spectral density

Noise Suppression – Part 4

Power spectral density estimation for the input signal:

$$\widehat{S}_{yy}(\Omega_\mu, n) = \left|Y(e^{j\Omega_\mu}, n)\right|^2$$

Power spectral density estimation for the noise:

Estimation schemes using voice activity detection(VAD)

Tracking of minima of short-term power estimations

Noise Suppression – Part 5

Schemes with voice activity detection:

$$\widehat{S}_{bb}(\Omega_\mu, n) = \begin{cases} \beta\, \widehat{S}_{bb}(\Omega_\mu, n-1) + (1-\beta)\, \widehat{S}_{yy}(\Omega_\mu, n), & \text{during speech pauses,} \\ \widehat{S}_{bb}(\Omega_\mu, n-1), & \text{else.} \end{cases}$$

Tracking of minima of the short-term power:

$$\overline{S_{yy}(\Omega_\mu, n)} = \beta\, \overline{S_{yy}(\Omega_\mu, n-1)} + (1-\beta)\, \widehat{S}_{yy}(\Omega_\mu, n)$$

Constant slightly larger than 1

Bias correction

$$\widehat{S}_{bb}(\Omega_\mu, n) = K \begin{cases} \max\left\{S_{\min},\, \widehat{S}_{bb}(\Omega_\mu, n-1)\right\} \Delta_{\text{inc}}, \\ \qquad \text{if } \overline{S_{yy}(\Omega_\mu, n)} > \widehat{S}_{bb}(\Omega_\mu, n-1), \\ \max\left\{S_{\min},\, \widehat{S}_{bb}(\Omega_\mu, n-1)\right\} \Delta_{\text{dec}}, \\ \qquad \text{else.} \end{cases}$$

Constant slightly smaller than1

Noise Suppression – Part 6

Problem:

- The short-term power of the input signal usually fluctuates faster than the noise estimate – also during speech pauses. As a result the filter characteristic opens and closes in a randomized manner, with results in tonal residual noise (so-called musical noise).

Simple solution:

- By inserting a fixed overestimation

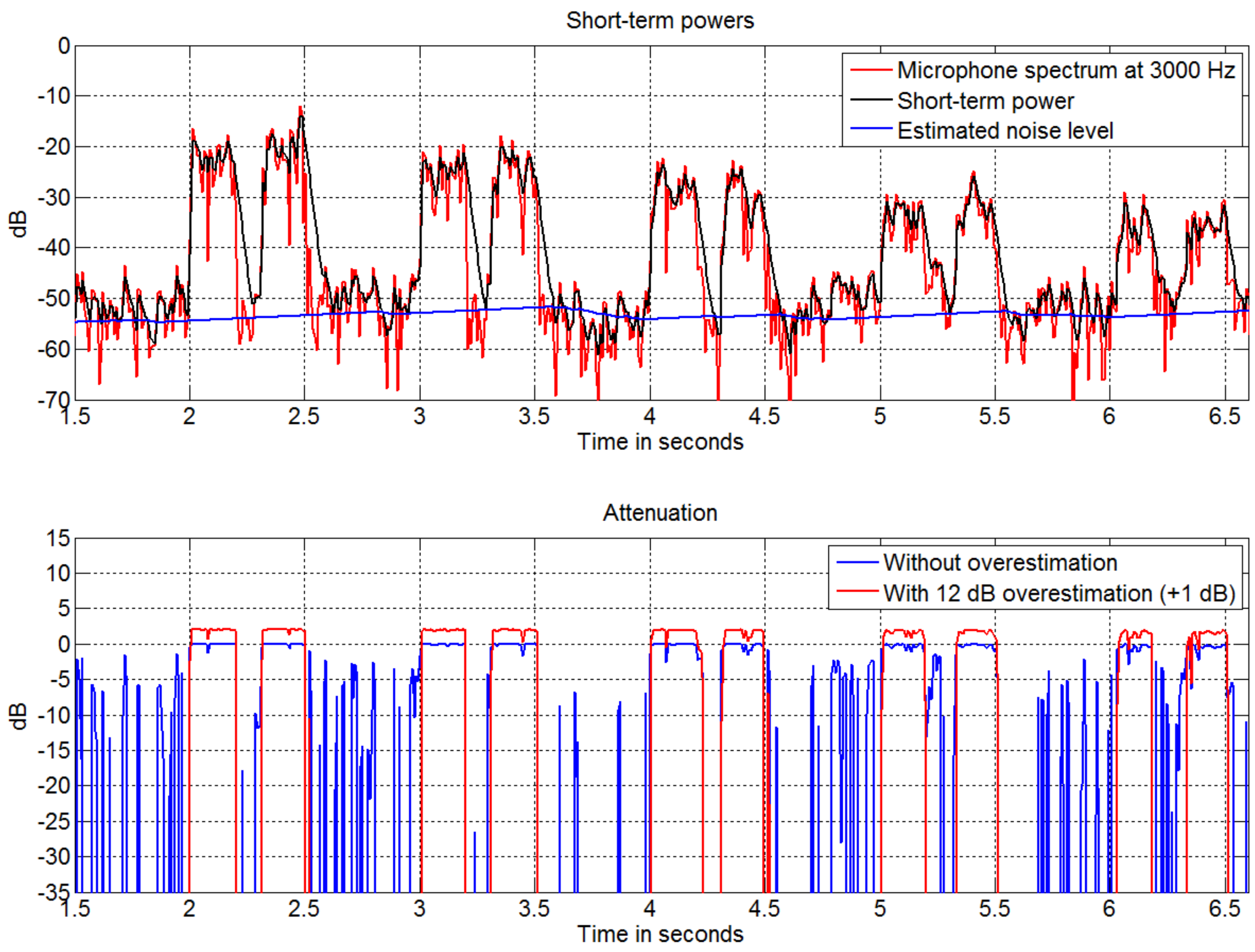
$$\widehat{S}_{bb}(\Omega_\mu, n) \longrightarrow K_{\text{over}}\, \widehat{S}_{bb}(\Omega_\mu, n)$$

the randomized opening of the filter can be avoided. This comes, however, with a more aggressive attenuation characteristic that attenuates also parts of the speech signal.

Enhanced solutions:

- More enhanced solutions will be presented in the lecture “Speech and Audio Processing – Audio Effects and Recognition” (offered next term by the “Digital Signal Processing and System Theory” team).

Noise Suppression – Part 7



: Microphone signal

: Output without overestimation

: Output with 12 dB overestimation

Noise Suppression – Part 8

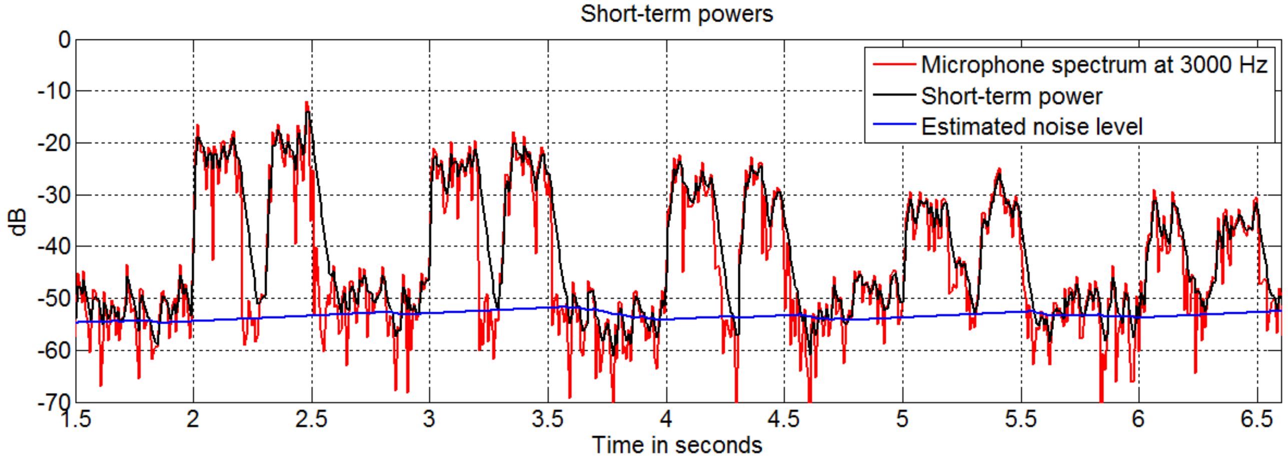
Limiting the maximum attenuation:

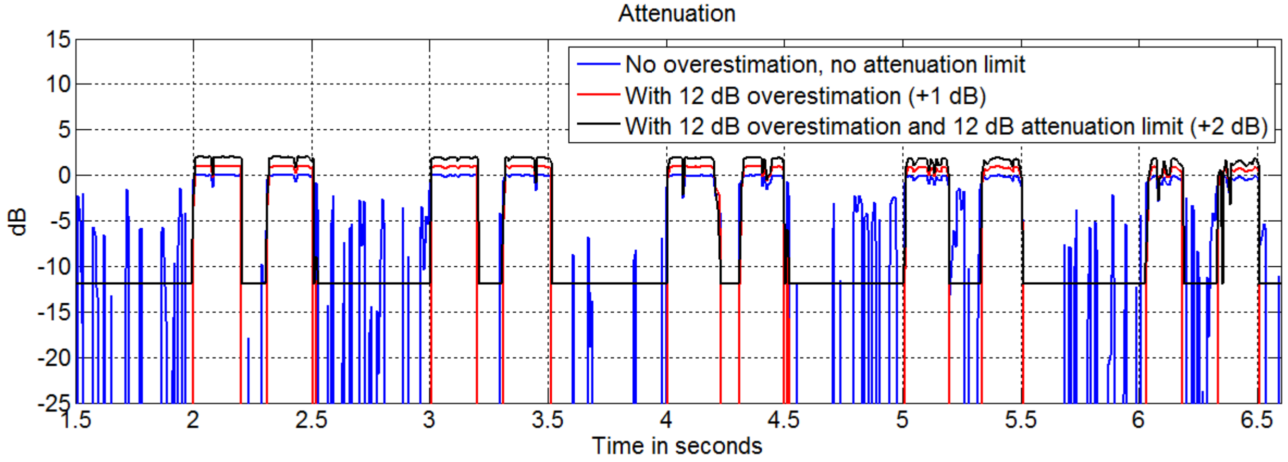
- For several application the original shape of the noise should be preserved (the noise should only be attenuated but not completely removed). This can be achieved by inserting a maximum attenuation:

$$H_{\min}(e^{j\Omega_\mu}, n) \quad = \quad H_{\min}.$$

- In addition, this attenuation limits can be varied slowly over time (slightly more attenuation during speech pauses, less attenuation during speech activity).

Noise Suppression – Part 9



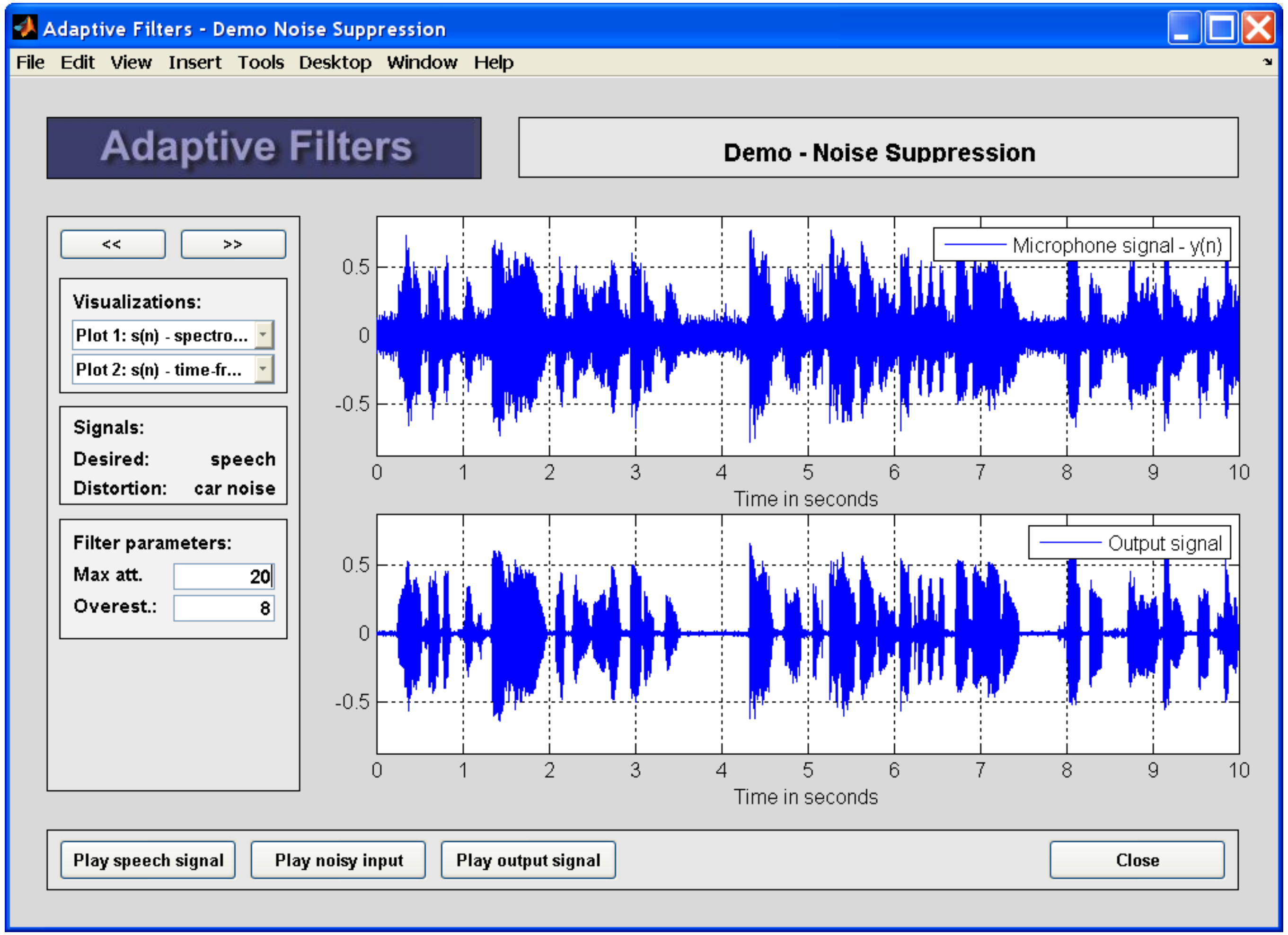


: Microphone signal

: Output without attenuation limit

: Output with attenuation limit

Noise Suppression – Part 10



Summary and Outlook

This week:

- Introduction and motivation
- Principle of orthogonality
- Time-domain solution
- Frequency-domain solution
- Application example: noise suppression

Next week:

- Linear Prediction