Adaptive Filters – Processing Structures

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Contents of the Lecture

Today:

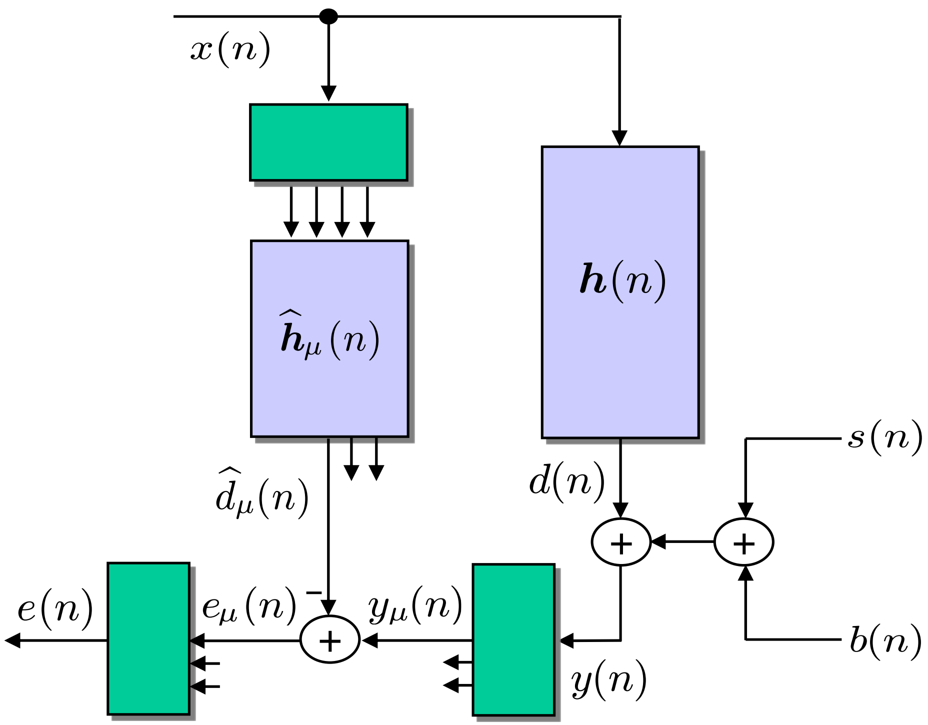
- Introduction and Motivation
- Adaptive Filters Operating in Subbands
- Filter Design for Prototype Lowpass Filters
- Examples

Problem and Objective

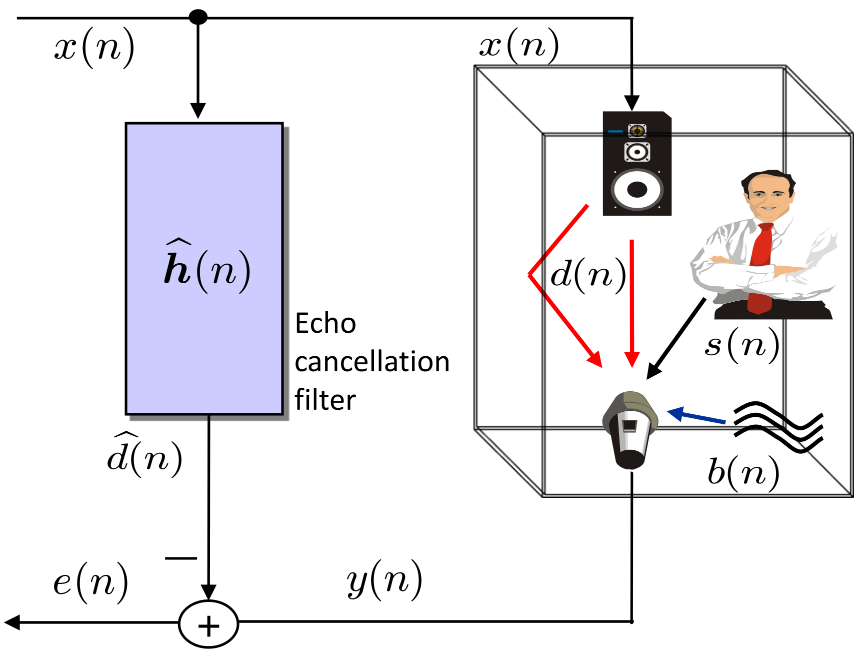
Objective:

Reduction of the computational complexity with nearly the same convergence properties by at least 75 %. If an additional delay is necessary, it should be lower than 30 ms.

Ansatz:



Application example:



Boundary Conditions

Boundary conditions:

The loudspeaker-enclosure-microphone system is modeled by an adaptive Filter with 4000 coefficients at a sampling rate of $f_\mathrm{s} =$8000 Hz. The filter should be adapted using the NLMS algorithm.

Computational complexity:

A convolution and an adaptation with 4000 elements has to be performed 8000 times per second. Assuming that a multiplication and an addition can be performed in one cycle on the target hardware, about

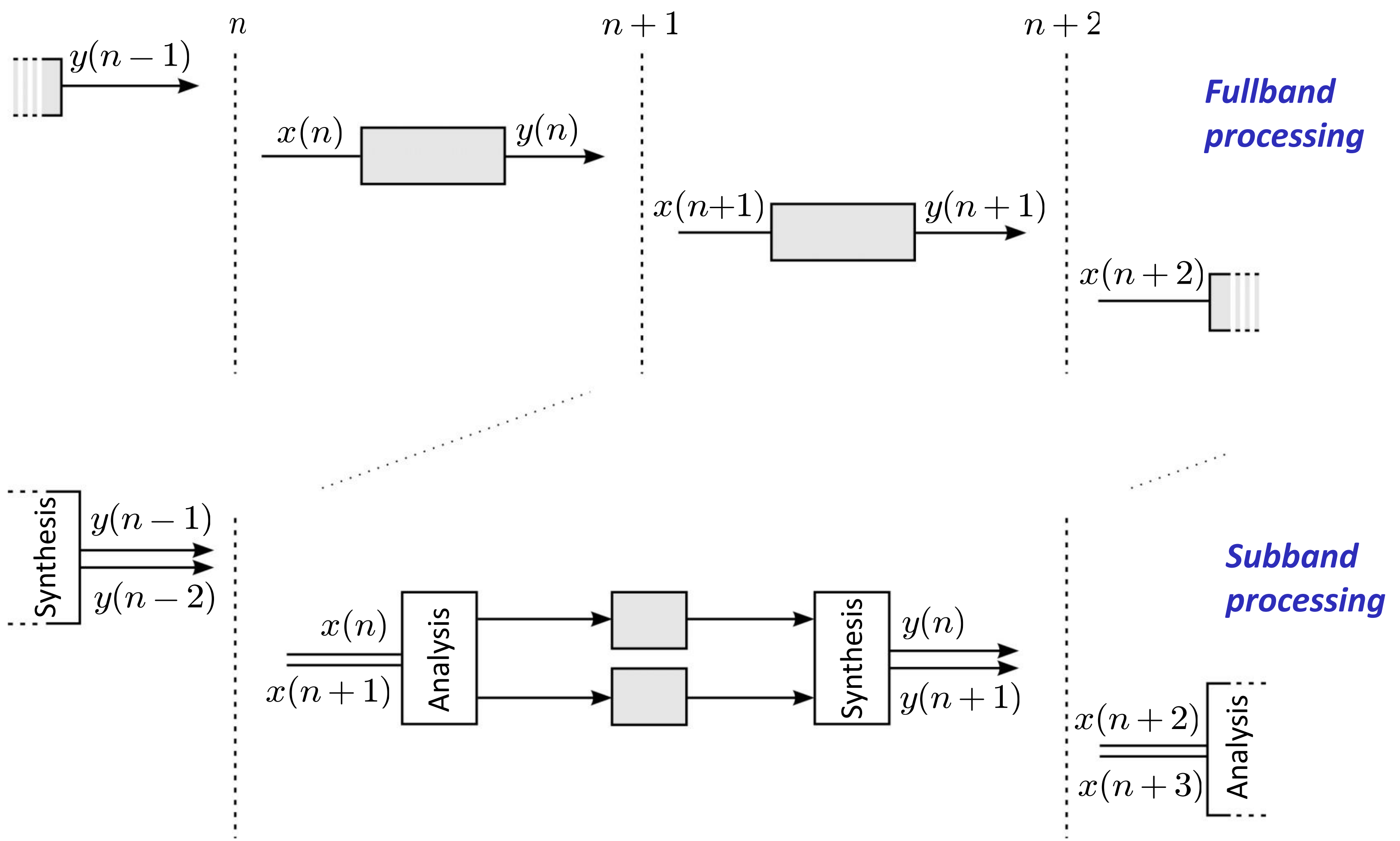
$$\frac{2 \cdot 8000 \cdot 4000}{1000000} = 64 \text{ million instructions per second (MIPS) are required.}$$

Pros and cons:

- \+ Rather simple and efficient algorithmic realization, only very little program memory is required.
- \+ A very good temporal resolution (for control purposes) is achieved.
- – Very high computational complexity.
- – Frequency selective control is not possible.

Application Example – Echo Cancellation

Ansatz – Part 1



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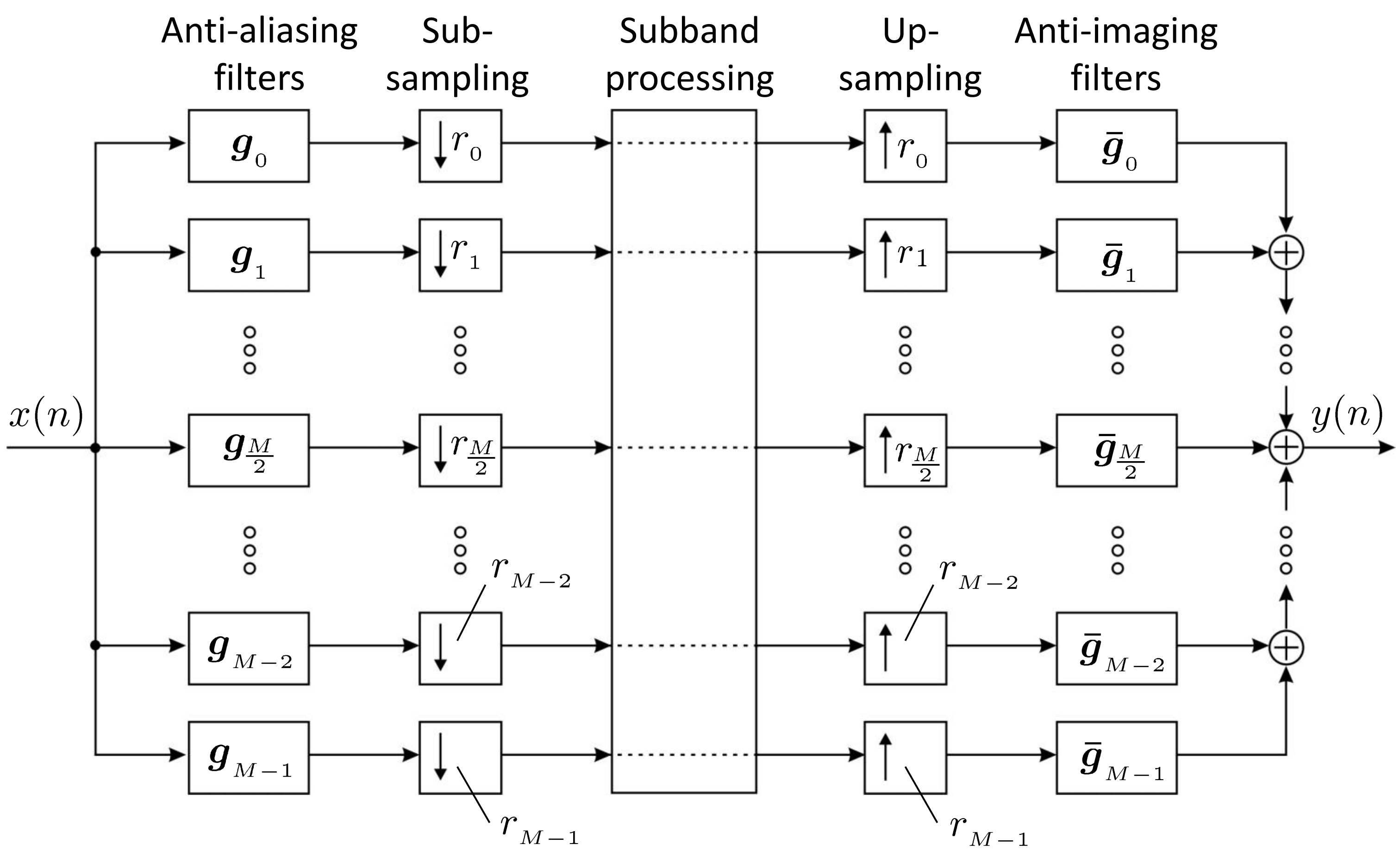
Ansatz – Part 2



Computational Complexity

Boundary conditions:

Design of a subband system with the following parameters:

- $M = 16$ subbands (with equal bandwidth)
- $r = 12$ (same subsampling rate for all subbands)
- $L_{\text{op}} = 3$ (average complexity ratio of complex and real operations)

Computational complexity:

- $\frac{2 \cdot 8000 \cdot 4000}{1000000} = 64$ MIPS (fullband)
- $\frac{\frac{M}{2} \cdot L_{\text{op}} \cdot 2 \cdot 8000/r \cdot 4000/r}{1000000} \approx \frac{8 \cdot 3 \cdot 2 \cdot 667 \cdot 333}{1000000} \approx 10.7$ MIPS (subband)

Reduction of about 83 %

Books

Basic text:

- E. Hänsler / G. Schmidt: *Acoustic Echo and Noise Control – Chapter 9 (Echo Cancellation),* Wiley, 2004
- E. Hänsler / G. Schmidt: *Acoustic Echo and Noise Control – Appendix B (Filterbank Design),* Wiley, 2004

Further details:

- P. P. Vaidyanathan: *Multirate* Systems *and Filter Banks,* Prentice-Hall, 1993
- R. E. Crochiere, L. R. Rabiner: *Multirate Digital Signal Processing,* Prentice-Hall, 1983
- H. G. Göckler, A. Groth: *Multiratensysteme,* Schlembach-Verlag, 2003/2004 (in German)

Repetition …

Known elements:

- Addition (signal + signal)
- Multiplication (signal * constant)
- Multiplication (signal * exponential series)
- Delay

$$x(n) + y(n) \quad \circ\!\!-\!\!\bullet \quad X\left(e^{j\Omega}\right) + Y\left(e^{j\Omega}\right)$$

$$K \cdot x(n) \quad \circ\!\!-\!\!\bullet \quad K \cdot X\left(e^{j\Omega}\right)$$

$$x(n) \cdot e^{j\Omega_0 n} \quad \circ\!\!-\!\!\bullet \quad X\left(e^{j(\Omega-\Omega_0)}\right)$$

$$x(n-k) \quad \circ\!\!-\!\!\bullet \quad X\left(e^{j\Omega}\right) \cdot e^{-j\Omega k}$$

Further necessary elements:

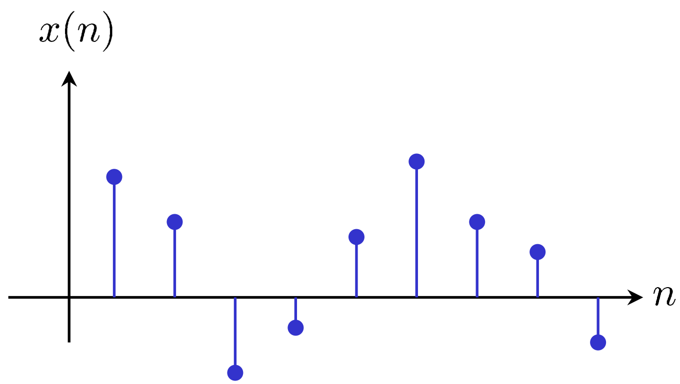
- Subsampling
- Upsampling

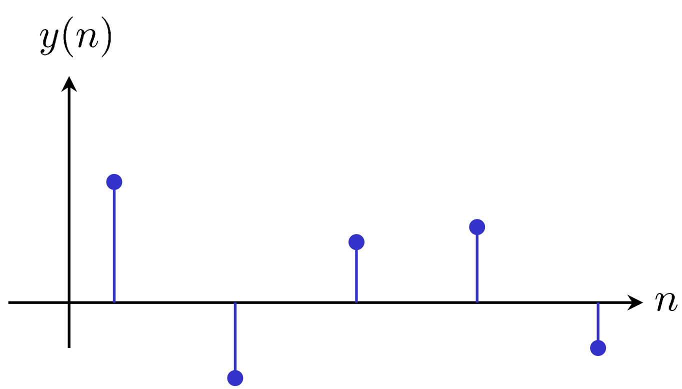
Up- and Downsampling – Part 1

(Derivation on the blackboard)

Up- and Downsampling – Part 2

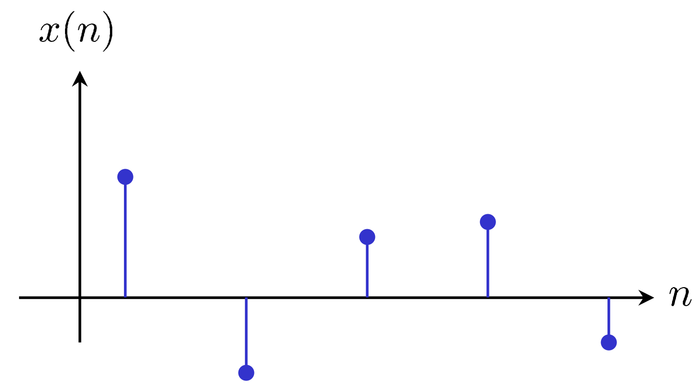
Downsampling (r = 2)

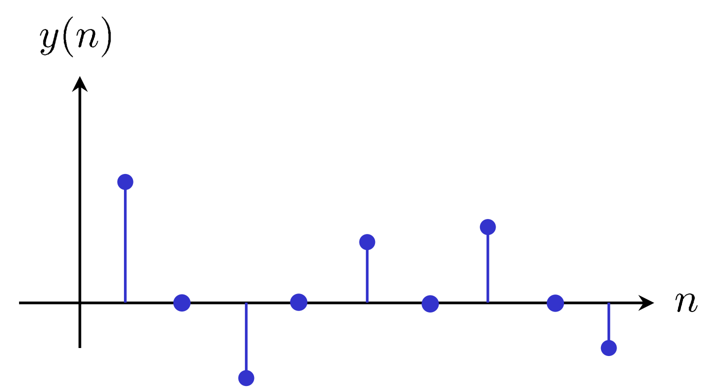




$$y(n) = x(n \cdot r)$$

Upsampling (r = 2)

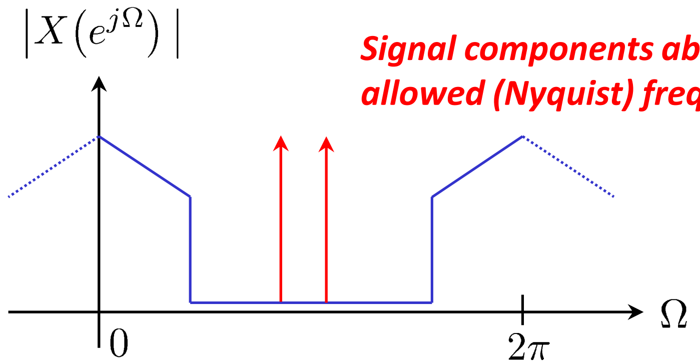


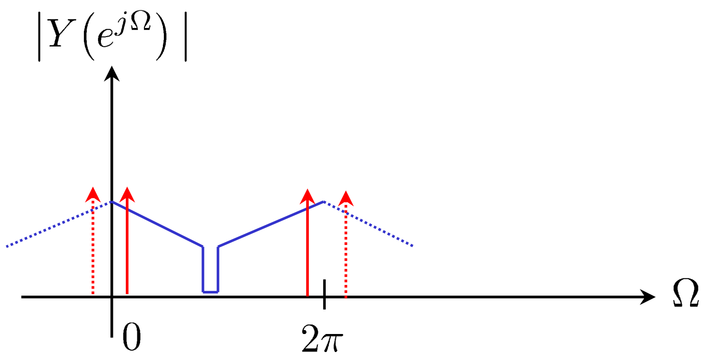


$$y(n) = \begin{cases} x\left(\frac{n}{r}\right), & \text{if } n \bmod r \equiv 0\,, \\ 0\,, & \text{else}\,. \end{cases}$$

Up- and Downsampling – Part 3

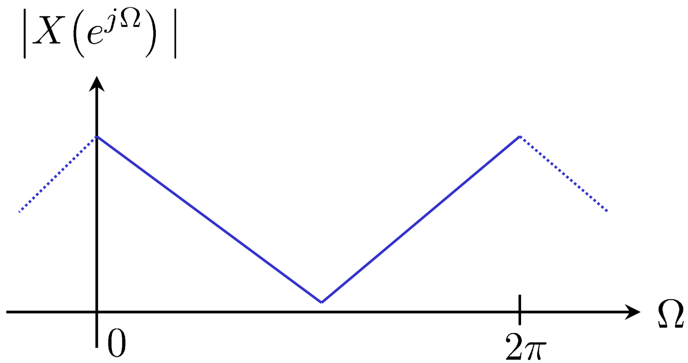
Downsampling (r = 2)

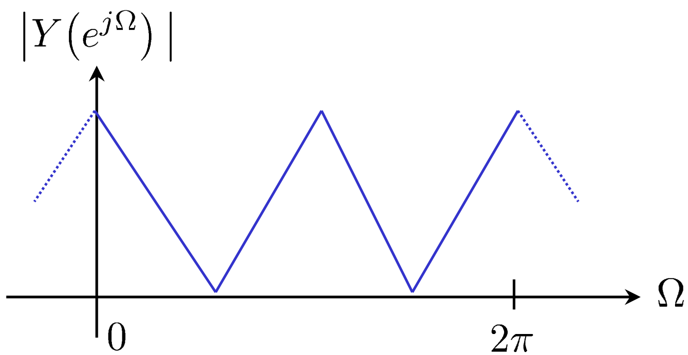




$$Y\left(e^{j\Omega}\right) = \frac{1}{r}\sum_{m=0}^{r-1} X\left(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\right)$$

Upsampling (r = 2)





$$Y\left(e^{j\Omega}\right) = X\left(e^{j\Omega r}\right)$$

How We Will Proceed …

Problem
(Reducing the complexity of adaptive filters)

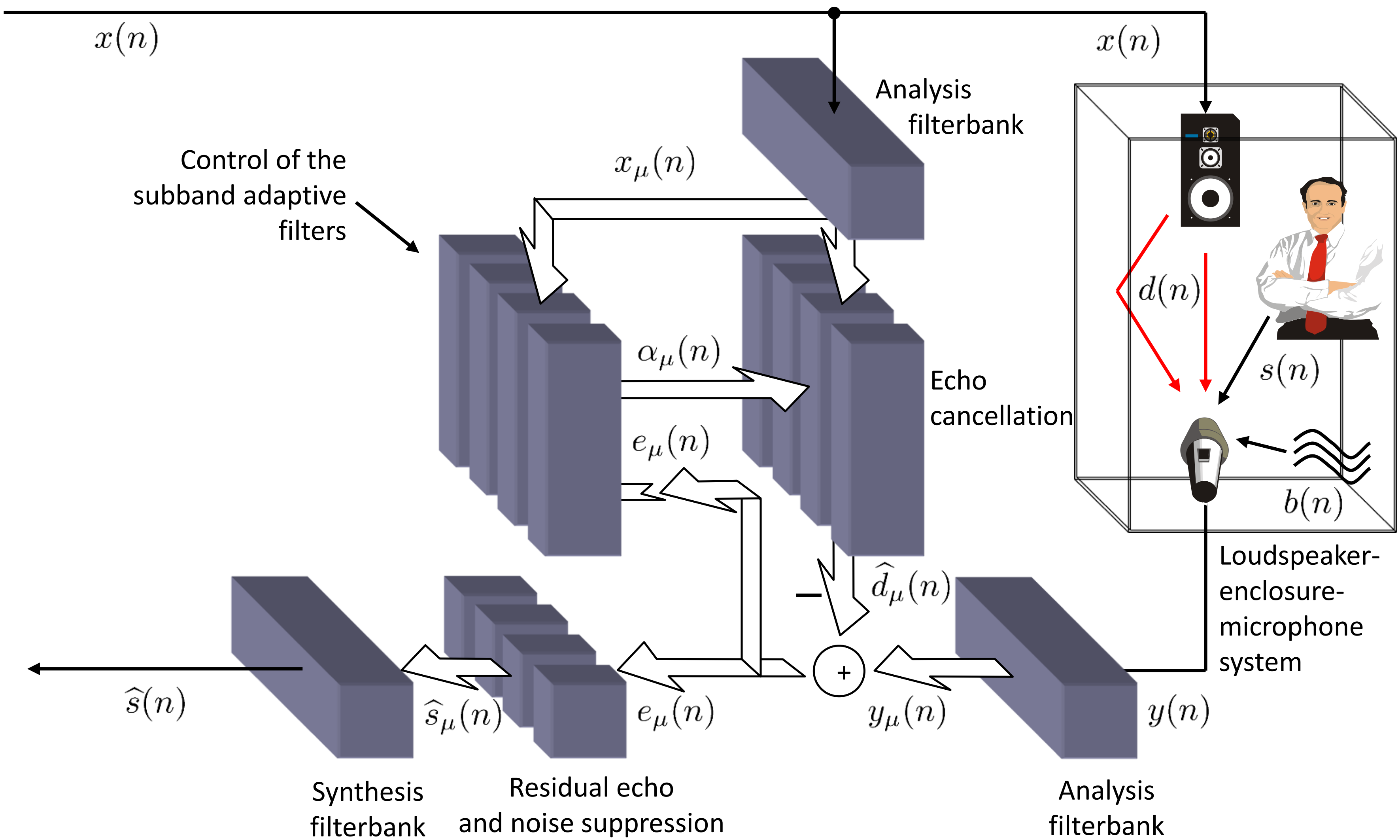
Efficient Subband Structures
(by inserting a few restrictions)

Design of so-called „prototype lowpass filters“

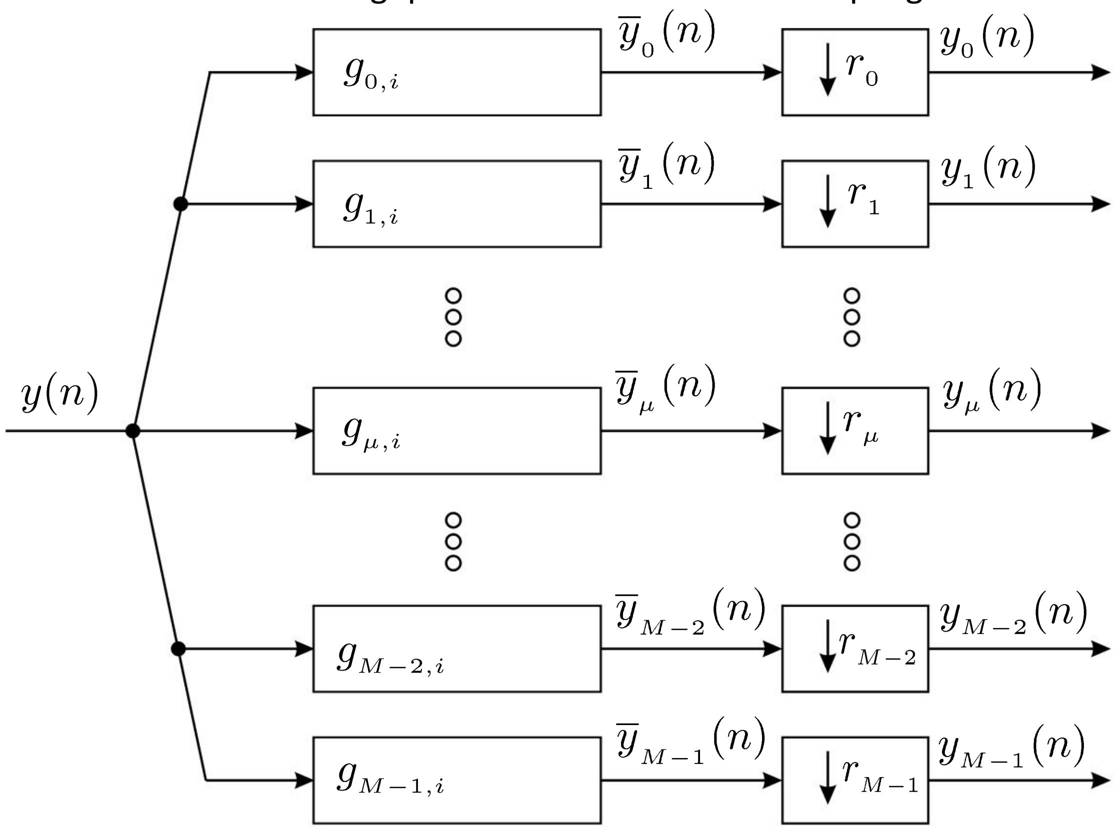
Verification
(Complexity reduction and adaptation performance)

Solution
(Complexity reduction but also an additional delay)

Basic Structure



Basic Structure of the Analysis Filterbank



Restrictions That Lead to Efficient Implementations

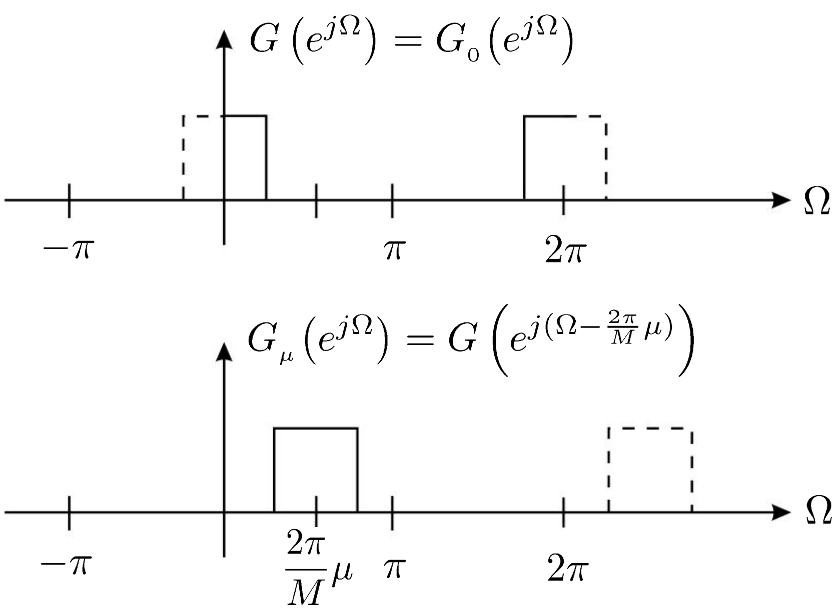
The same subsampling rate and the same prototype filter in all channels:

- Using the same subsampling rate for all channels/subbands:

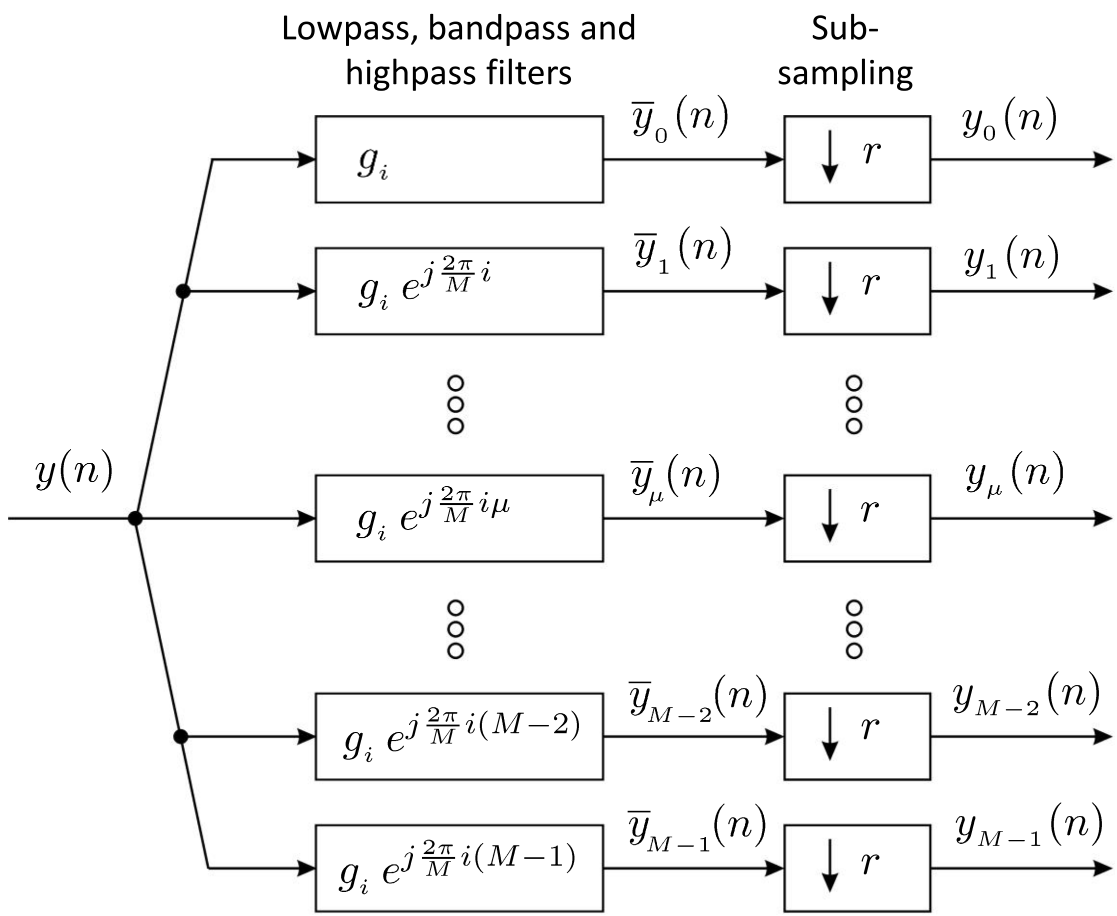
$$\boxed{r_\mu = r}$$

- Realizing the bandpass and the highpass filters as frequency shifted version of a lowpass filter:

$$\boxed{g_{\mu,i} = g_i \, e^{j\frac{2\pi}{M}i\mu}}$$



Structure of the Analysis Filterbank (with Restrictions)



Analysis of the Filterbank Structure – Part 1

Signal after subsampling:

$$y_\mu(n) = \overline{y}_\mu(r\,n)$$

Assuming a causal prototype lowpass filter:

$$g_i = 0, \quad \text{if } i < 0$$

Signal after anti-aliasing filtering:

$$\overline{y}_\mu(n) = \sum_{i=0}^{\infty} y(n-i)\, g_i\, e^{j\frac{2\pi}{M}i\mu}$$

Inserting results in:

$$\begin{aligned} y_\mu(n) &= \overline{y}_\mu(r\,n) \\ &= \sum_{i=0}^{\infty} y(rn-i)\, g_i\, e^{j\frac{2\pi}{M}i\mu} \end{aligned}$$

Analysis of the Filterbank Structure – Part 2

Previous result:

$$y_\mu(n) \;=\; \sum_{i=0}^{\infty} y(rn-i)\, g_i\, e^{j\frac{2\pi}{M}i\mu}$$

Splitting the summation index:

$$i \;=\; lM+\nu \qquad \text{with } l \in \{0,\,1,\,2,\,...\} \text{ and } \nu \in \{0,\,...,\,M-1\}$$

Inserting results in:

$$\begin{aligned} y_\mu(n) &= \sum_{l=0}^{\infty}\sum_{\nu=0}^{M-1} y(rn-lM-\nu)\, g_{lM+\nu}\, e^{j\frac{2\pi}{M}(lM+\nu)\mu} \\ &= \sum_{\nu=0}^{M-1}\sum_{l=0}^{\infty} y(rn-lM-\nu)\, g_{lM+\nu}\, e^{j\frac{2\pi}{M}(lM+\nu)\mu} \\ &= \sum_{\nu=0}^{M-1} e^{j\frac{2\pi}{M}\nu\mu} \sum_{l=0}^{\infty} y(rn-lM-\nu)\, g_{lM+\nu}\, \underbrace{e^{j\frac{2\pi}{M}lM\mu}}_{=1} \\ &= \sum_{\nu=0}^{M-1} e^{j\frac{2\pi}{M}\nu\mu} \sum_{l=0}^{\infty} y(rn-lM-\nu)\, g_{lM+\nu} \end{aligned}$$

Exchanging the order of the sums

Resolving the exponential term

Simplification

Analysis of the Filterbank Structure – Part 3

Previous result:

$$y_\mu(n) = \sum_{\nu=0}^{M-1} e^{j\frac{2\pi}{M}\nu\mu} \sum_{l=0}^{\infty} y(rn - lM - \nu)\, g_{lM+\nu}$$

Specialties:

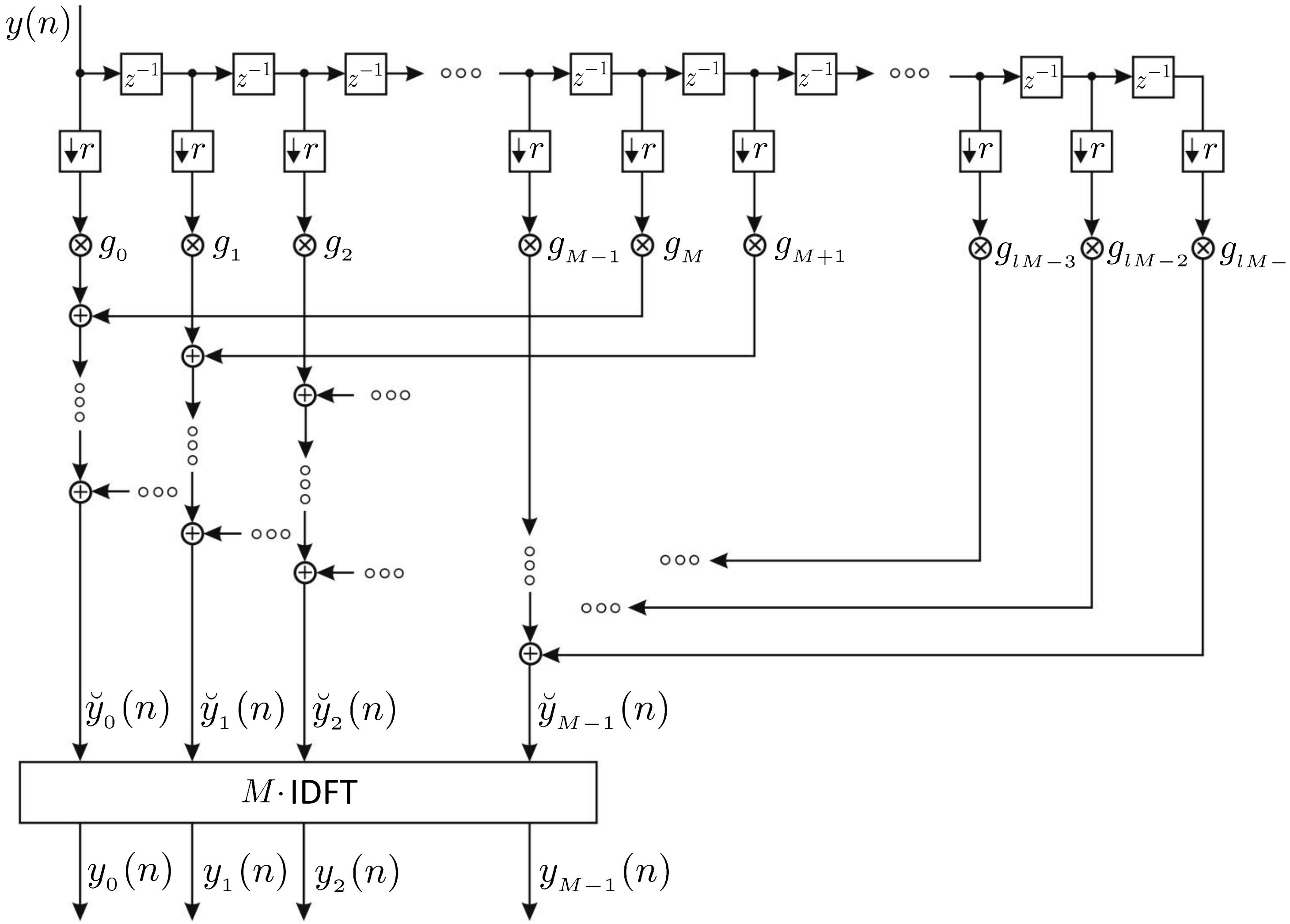
$$\breve{y}_\nu(n) = \sum_{l=0}^{\infty} y(rn - lM - \nu)\, g_{lM+\nu}$$

… does not depend on μ !

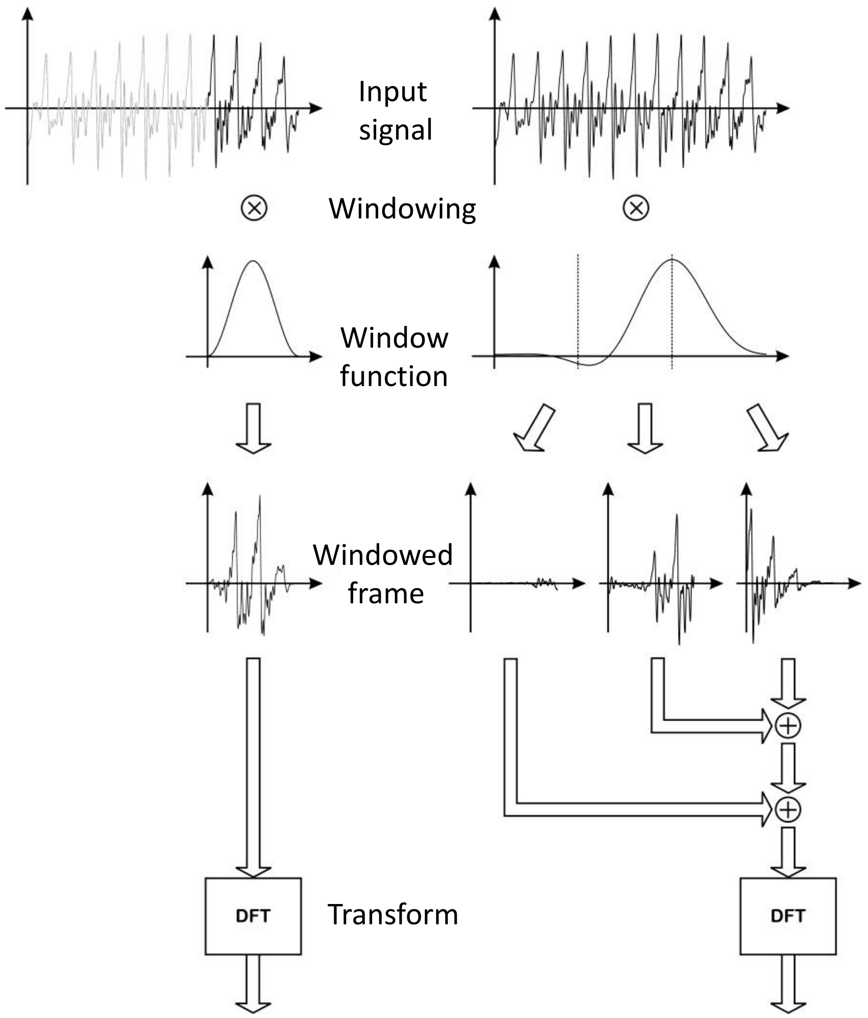
$$y_\mu(n) = \sum_{\nu=0}^{M-1} e^{j\frac{2\pi}{M}\nu\mu}\, \breve{y}_\nu(n)$$

… is a weighted inverse DFT and can be realized efficiently as an inverse FFT!

DFT-Modulated Polyphase Analysis Filterbank

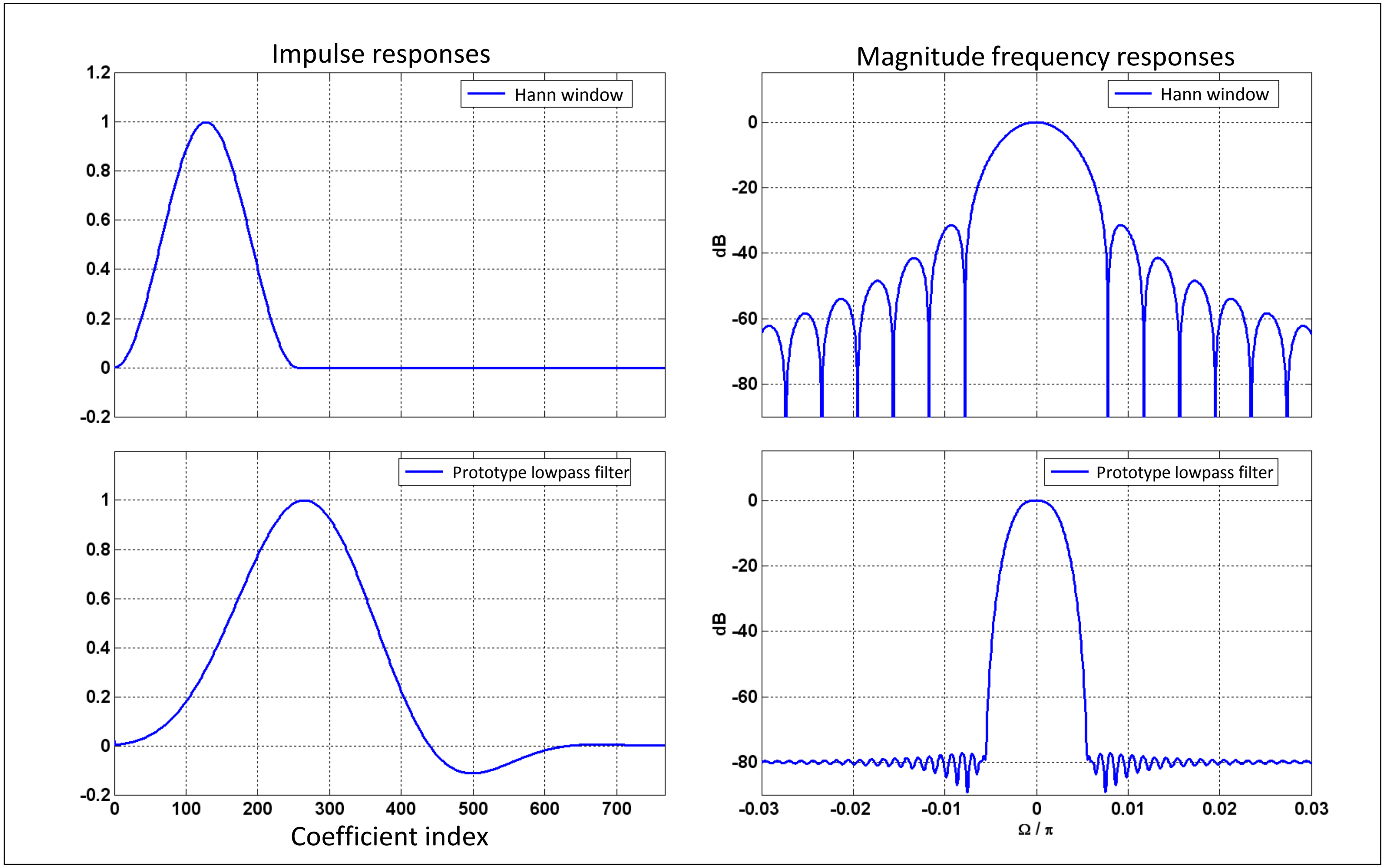


Filterbank Versus DFT – Part 1



Filterbanks allow for lengths of window functions larger than the DFT size. This can lead to improved frequency resolution.

Filterbank Versus DFT – Part 2



Both filters were designed for a DFT order of 256.

Comparison of the Basic and the Efficient Structure

Direct implementation:

Filterlength of the (FIR-) lowpass filter

We have to compute

- $r \cdot M$ (real-complex) convolutions with N coefficients

per input frame.

2 r M N operations
= 49152 operations

Example for M=16, r=12, N=128

Efficient implementation:

We have to compute

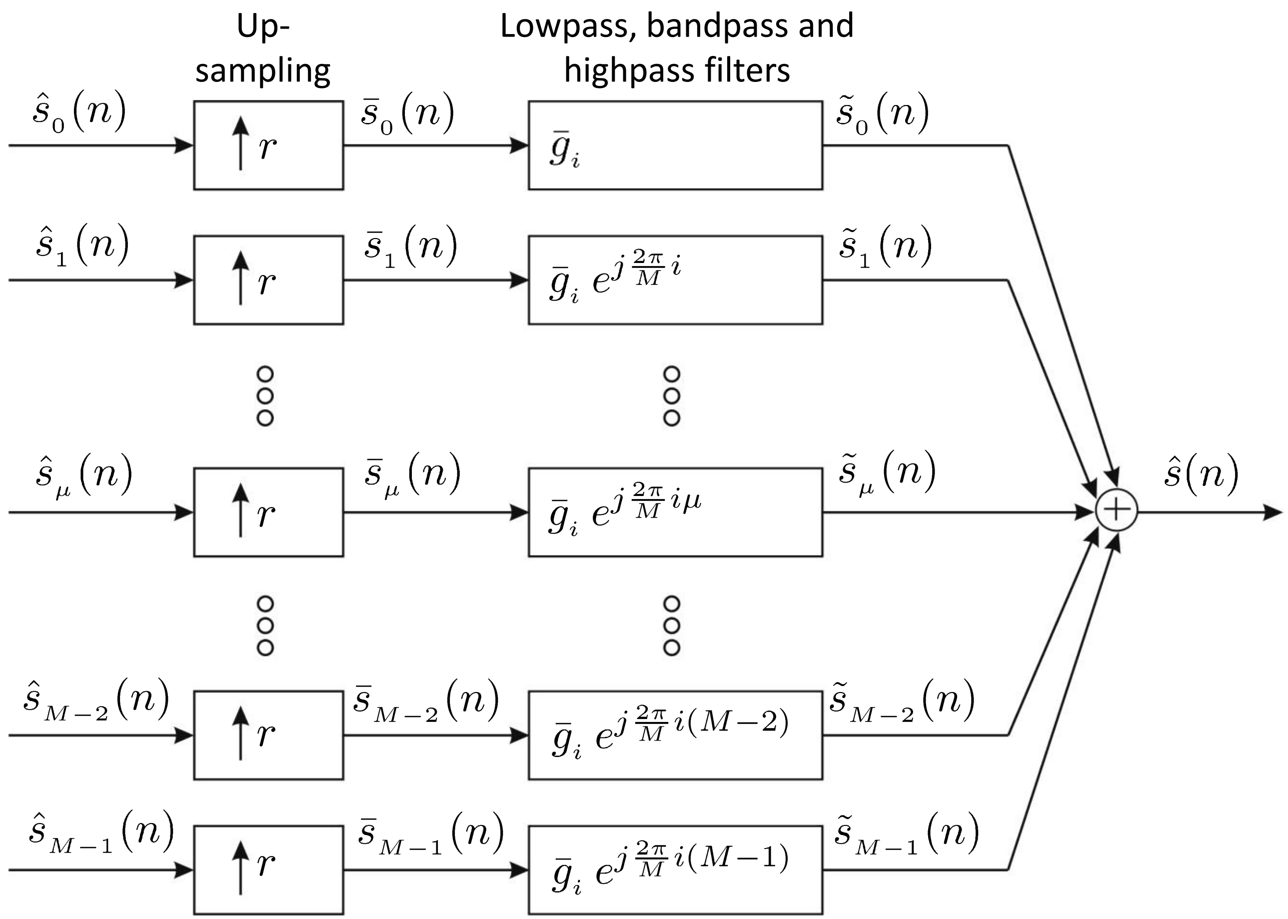
- 1 (real-real) convolution with N coefficients and
- 1 IFFT of order M

per input frame.

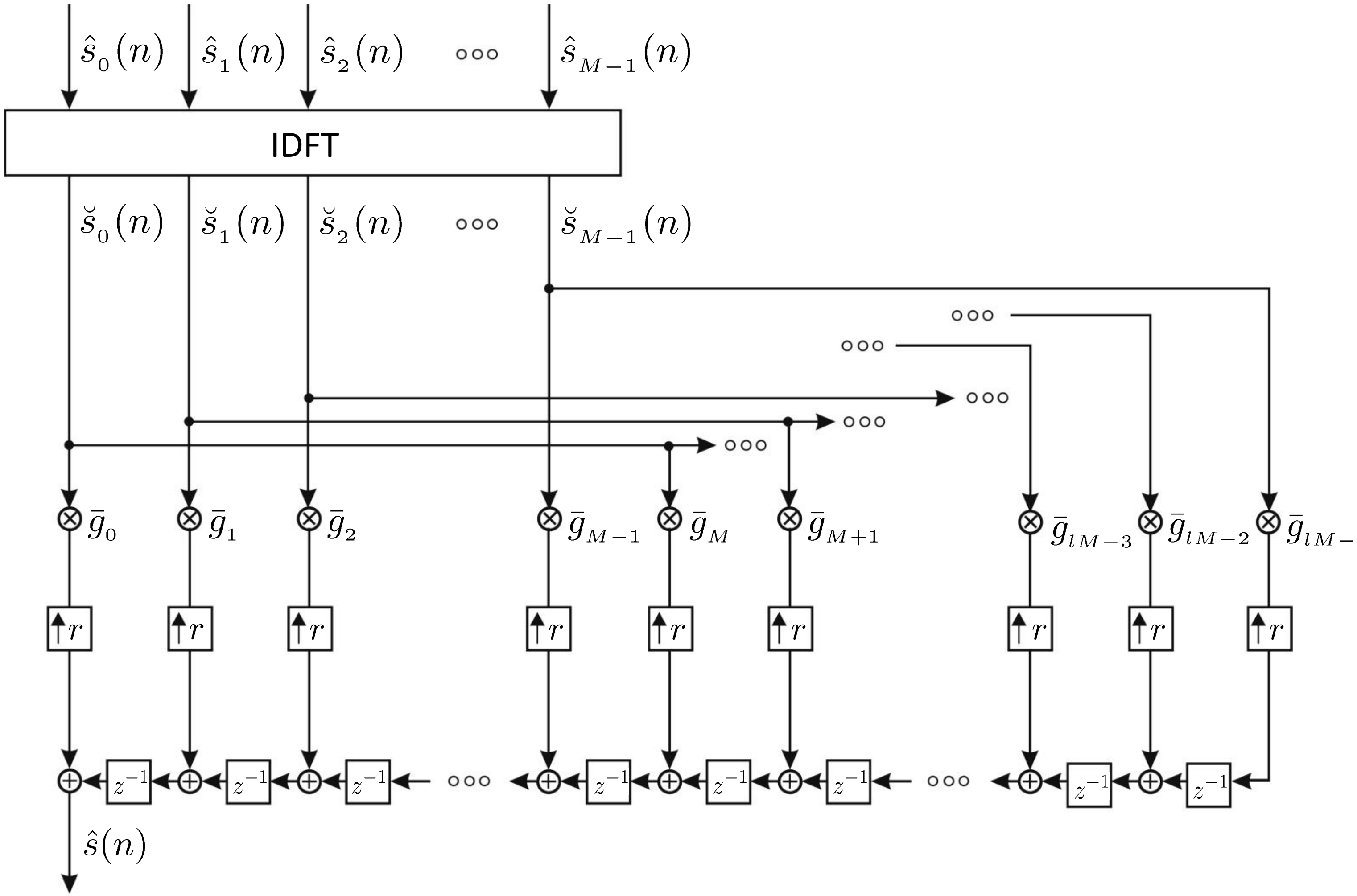
N + M $\log_2 M$ operations
= 192 operations

Synthesis Filterbanks – Part 1

A comparable structure can be derived for the synthesis filterbank (details e.g. in E. Hänsler, G. Schmidt: Acoustic Echo and Noise Control, Wiley, 2004)



Synthesis Filterbanks – Part 2



What we have done so far …

Problem
(Reducing the complexity of adaptive filters)

Efficient subband structures
(by inserting a few restrictions)

Design of so-called „prototype lowpass filters“

Verification
(Complexity reduction and adaptation performance)

Solution
(Complexity reduction but also an additional delay)

The Following Steps …

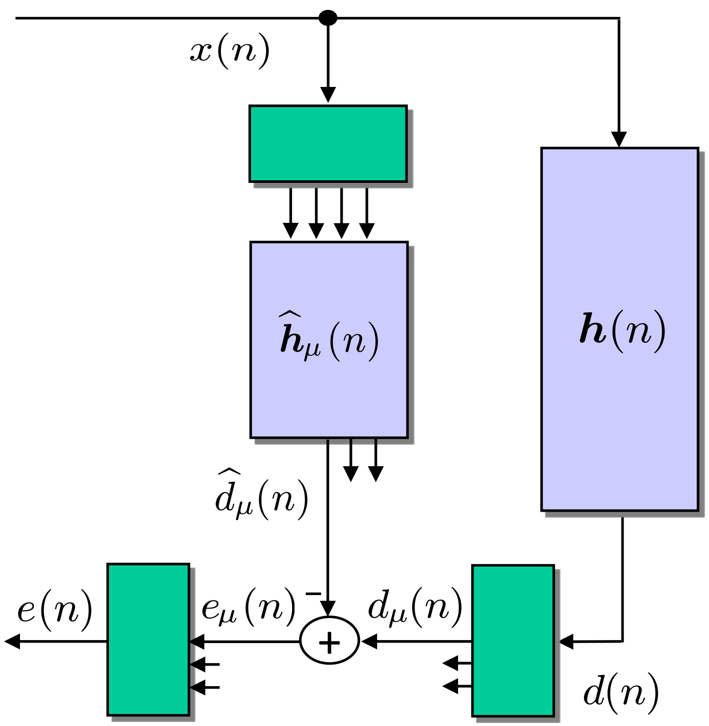
Our steps up to now:

- Derivation of an efficient analysis structure using the filter g_i.
- Derivation of an efficient synthesis structure using the filter $\bar{g}_i$.

What is still missing:

- Derivation of the requirements on the entire analysis-synthesis structure.
  - The entire system, consisting of the analysis and the synthesis part, should not insert more than 20 … 40 ms delay. For a sampling rate of 8 kHz this means less than 160 to 320 samples delay.
  - The magnitude frequency response should not deviate by more than 0.5 dB from the desired 0 dB value.
  - The group delay should not fluctuate more than 0.5 samples.
  - The subsampling rate should be chosen as large as possible in order to reduce the computational complexity as much as possible.
  - The aliasing components should be kept as small as possible in order to allow fast convergence and a good steady-state performance of the adaptive subband filters. The aliasing components should be about 40 dB smaller than the desired signal components.
- Design of the filters g_i and $\bar{g}_i$.

Aliasing Components



Echo spectrum (broadband):

$$D\big(e^{j\Omega}\big) = X\left(e^{j\Omega}\right) H\left(e^{j\Omega}\right)$$

Echo spectrum (subband):

$$\begin{aligned} D_\mu\big(e^{j\Omega}\big) &= \frac{1}{r}\sum_{m=0}^{r-1} D\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big)\cdot G_\mu\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big) \\ &= \frac{1}{r}\sum_{m=0}^{r-1} X\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big)\cdot \boxed{H\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big)} \\ &\qquad \cdot G_\mu\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big) \end{aligned}$$

Estimated echo spectrum (subband):

$$\begin{aligned} \widehat{D}_\mu\big(e^{j\Omega}\big) &= \frac{1}{r}\sum_{m=0}^{r-1} X\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big)\cdot G_\mu\Big(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\Big) \\ &\qquad \cdot \boxed{\widehat{H}_\mu\big(e^{j\Omega}\big)} \end{aligned}$$

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Restrictions for the Synthesis Filter – Part 1

Analysis and synthesis filters have the same requirements for the magnitude frequency response. The phase (and thus also the group delay) can be chosen differently. We will make the following ansatz:

$$\boxed{\bar{g}_i = g_{N-i}}$$

Frequency response of the analysis filter:

$$G\left(e^{j\Omega}\right) = \sum_{i=0}^{N-1} g_i\, e^{-j\Omega i}$$

For the synthesis filter we obtain:

$$\begin{aligned}\bar{G}\left(e^{j\Omega}\right) &= \sum_{i=-\infty}^{\infty} \bar{g}_i\, e^{-j\Omega i} = \sum_{i=-\infty}^{\infty} g_{N-i}\, e^{-j\Omega i} \\ &= \sum_{i=-\infty}^{\infty} g_i\, e^{-j\Omega(N-i)} = e^{-j\Omega N} \sum_{i=0}^{N-1} g_i\, e^{j\Omega i} \\ &= e^{-j\Omega N}\, G^*\left(e^{j\Omega}\right)\end{aligned}$$

The synthesis filter has the same magnitude response but a different phase response!

Restrictions for the Synthesis Filter – Part 2

Previous result:

$$\bar{g}_i = g_{N-i} \quad \circ\!\!-\!\!\bullet \quad \bar{G}\left(e^{j\Omega}\right) = e^{-j\Omega N}\, G^*\left(e^{j\Omega}\right) \longrightarrow \left|\bar{G}\left(e^{j\Omega}\right)\right| = \left|G\left(e^{j\Omega}\right)\right|$$

Connecting the analysis and synthesis filters:

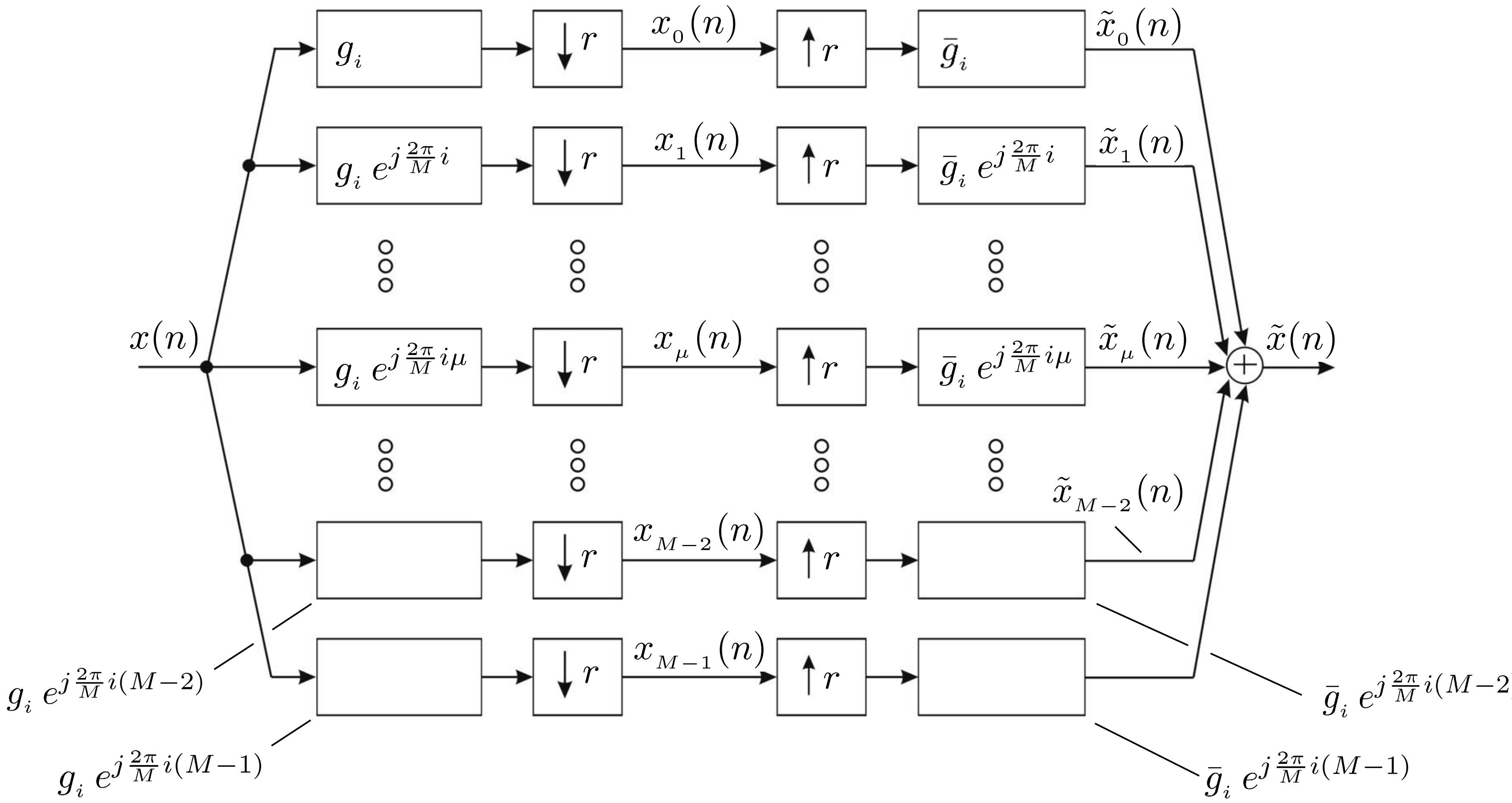
$$\begin{aligned} G_\mu\left(e^{j\Omega}\right) \cdot \bar{G}_\mu\left(e^{j\Omega}\right) &= G\Big(e^{j\left(\Omega - \frac{2\pi\mu}{M}\right)}\Big) \cdot \bar{G}\Big(e^{j\left(\Omega - \frac{2\pi\mu}{M}\right)}\Big) \\ &= G\Big(e^{j\left(\Omega - \frac{2\pi\mu}{M}\right)}\Big) \cdot e^{-j\left(\Omega - \frac{2\pi\mu}{M}\right)N} \cdot G^*\Big(e^{j\left(\Omega - \frac{2\pi\mu}{M}\right)}\Big) \\ &= \Big|G\Big(e^{j\left(\Omega - \frac{2\pi\mu}{M}\right)}\Big)\Big|^2 \cdot e^{-j\left(\Omega - \frac{2\pi\mu}{M}\right)N} \end{aligned}$$

If the filter length N is chosen as a multiple of the subband number M, we get:

$$G_\mu\left(e^{j\Omega}\right) \cdot \bar{G}_\mu\left(e^{j\Omega}\right) = \Big|G\Big(e^{j\left(\Omega - \frac{2\pi\mu}{M}\right)}\Big)\Big|^2 \cdot e^{-j\Omega N}$$

Linear phase filter (with constant group delay of N samples, independent of the subband index)

Analysis-Synthesis Structure – Part 1



Analysis-Synthesis Structure – Part 2

Output spectrum of one channel of the analysis filter bank (after subsampling):

$$X_\mu(e^{j\Omega}) = \frac{1}{r}\sum_{m=0}^{r-1} X\left(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{r}m\right)}\right) G\left(e^{j\left(\frac{\Omega}{r}-\frac{2\pi}{M}\mu-\frac{2\pi}{r}m\right)}\right)$$

1. convolution with a frequency shifted lowpass
2. subsampling

Output spectrum of a synthesis channel:

$$\begin{aligned}\widetilde{X}_\mu(e^{j\Omega}) &= X_\mu(e^{j\Omega r})\,\bar{G}_\mu(e^{j\Omega}) \\ &= \frac{1}{r}\left[\sum_{m=0}^{r-1} X\left(e^{j\left(\Omega-\frac{2\pi}{r}m\right)}\right) G\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu-\frac{2\pi}{r}m\right)}\right)\right]\bar{G}_\mu(e^{j\Omega}) \\ &= \frac{1}{r}\left[\sum_{m=0}^{r-1} X\left(e^{j\left(\Omega-\frac{2\pi}{r}m\right)}\right) G\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu-\frac{2\pi}{r}m\right)}\right)\right]\bar{G}\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right)\end{aligned}$$

1. upsampling
2. convolution with a frequency shifted lowpass

Analysis-Synthesis Structure – Part 3

Previous result:

$$\widetilde{X}_\mu\big(e^{j\Omega}\big) = \frac{1}{r}\left[\sum_{m=0}^{r-1} X\left(e^{j\left(\Omega-\frac{2\pi}{r}m\right)}\right) G\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu-\frac{2\pi}{r}m\right)}\right)\right] \bar{G}\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right)$$

Separation into a linear part and aliasing components:

$$\begin{aligned}\widetilde{X}_\mu\big(e^{j\Omega}\big) &= \frac{1}{r}\, X\big(e^{j\Omega}\big) \underbrace{G\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right) \bar{G}\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right)}_{\text{linear part}} \\ &\quad + \frac{1}{r}\sum_{m=1}^{r-1} X\left(e^{j\left(\Omega-\frac{2\pi}{r}m\right)}\right) \underbrace{G\left(e^{j\left(\Omega-\frac{2\pi}{r}m-\frac{2\pi}{M}\mu\right)}\right) \bar{G}\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right)}_{\text{aliasing components}}\end{aligned}$$

Neglecting the aliasing components:

$$\widetilde{X}_\mu\big(e^{j\Omega}\big) \approx \frac{1}{r}\, X\big(e^{j\Omega}\big)\, G\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right) \bar{G}\left(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\right)$$

Analysis-Synthesis Structure – Part 4

Previous result:

$$\widetilde{X}_\mu\big(e^{j\Omega}\big) \approx \frac{1}{r}\, X\big(e^{j\Omega}\big)\; G\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\; \bar{G}\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)$$

Synthesis of the broadband spectrum:

$$\begin{aligned}\widetilde{X}\big(e^{j\Omega}\big) &= \sum_{\mu=0}^{M-1} \widetilde{X}_\mu\big(e^{j\Omega}\big) \\ &\approx X\big(e^{j\Omega}\big)\; \frac{1}{r}\sum_{\mu=0}^{M-1} G\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\; \bar{G}\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\end{aligned}$$

Inserting the restrictions for the synthesis filter:

$$\begin{aligned}\widetilde{X}\big(e^{j\Omega}\big) &\approx X\big(e^{j\Omega}\big)\; \frac{1}{r}\sum_{\mu=0}^{M-1} \Big|G\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\Big|^2\; e^{-j\Omega N} \\ &= X\big(e^{j\Omega}\big)\; e^{-j\Omega N}\; \frac{1}{r}\sum_{\mu=0}^{M-1} \Big|G\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\Big|^2\end{aligned}$$

Analysis-Synthesis Structure – Part 5

Previous result:

$$\widetilde{X}\big(e^{j\Omega}\big) \approx X\big(e^{j\Omega}\big)\ e^{-j\Omega N}\ \frac{1}{r}\sum_{\mu=0}^{M-1}\left|G\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\right|^2$$

Requirements for the analysis-synthesis system:

$$\tilde{x}(n) = x(n-N) \quad \circ\!\!-\!\!\bullet \quad \widetilde{X}\big(e^{j\Omega}\big) = X\big(e^{j\Omega}\big)\ e^{-j\Omega N}$$

This results in:

$$1 = \frac{1}{r}\sum_{\mu=0}^{M-1}\left|G\Big(e^{j\left(\Omega-\frac{2\pi}{M}\mu\right)}\Big)\right|^2$$

Design approach for the filters

Transformation into the time domain:

$$\delta_{\mathrm{K},i} = \frac{1}{r}\,\psi_i\sum_{\mu=0}^{M-1} e^{j\frac{2\pi\mu i}{M}} \qquad \text{with} \qquad \psi_i = g_i * g_{-i} = \sum_{\kappa=-\infty}^{\infty} g_\kappa\, g_{\kappa-i}$$

Analysis-Synthesis Structure – Part 6

Previous result:

$$\delta_{\mathrm{K},i} = \frac{1}{r}\,\psi_i \sum_{\mu=0}^{M-1} e^{j\frac{2\pi\mu i}{M}}$$

Ansatz:

$$\psi_i = \psi_{\mathrm{B},i} \cdot \psi_{\mathrm{F},i}$$

with

$$\begin{aligned}\psi_{\mathrm{B},i} &= \frac{1}{M}\,\mathrm{sinc}\left(\frac{i\pi}{M}\right)\\ &= \begin{cases} \dfrac{1}{M} & : \ \text{for } i = 0,\\ \dfrac{1}{M}\,\dfrac{\sin\left(\frac{i\pi}{M}\right)}{\frac{i\pi}{M}} & : \ \text{else.}\end{cases}\end{aligned}$$

Analysis-Synthesis Structure – Part 7

Previous result:

$$\delta_{\mathrm{K},i} \;=\; \frac{1}{r}\,\psi_i \sum_{\mu=0}^{M-1} e^{j\frac{2\pi\mu i}{M}}$$

Finite geometrical series:

$$\sum_{\mu=0}^{M-1} a^{\mu} \;=\; \frac{1-a^M}{1-a}$$

Applied to our problem:

$$\sum_{\mu=0}^{M-1} e^{j\frac{2\pi\mu i}{M}} \;=\; \begin{cases} M, & \text{if } i \in \{0, \pm M, \pm 2M, ...\} \\ 0, & \text{else.} \end{cases}$$

Requirements for the autocorrelation function of the lowpass filter:

$$\psi_i \;=\; \begin{cases} \frac{r}{M} & : \; i = 0, \\ 0 & : \; i \in \{\pm M, \pm 2M, ...\}\,, \\ \text{arbitrary} & : \; \text{else.} \end{cases}$$

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Analysis-Synthesis Structure – Part 8

The first part of the autocorrelation function can be an ideal lowpass filter:

$$\psi_{\mathrm{B},i} = \frac{1}{M}\,\mathrm{sinc}\left(\frac{i\pi}{M}\right) \quad \circ\!\!-\!\!\bullet \quad \Psi_{\mathrm{B}}\big(e^{j\Omega}\big) = \begin{cases} 1 & : \; 0 \le |\Omega| < \dfrac{\pi}{M}, \\ 0 & : \; \dfrac{\pi}{M} \le |\Omega| \le \pi. \end{cases}$$

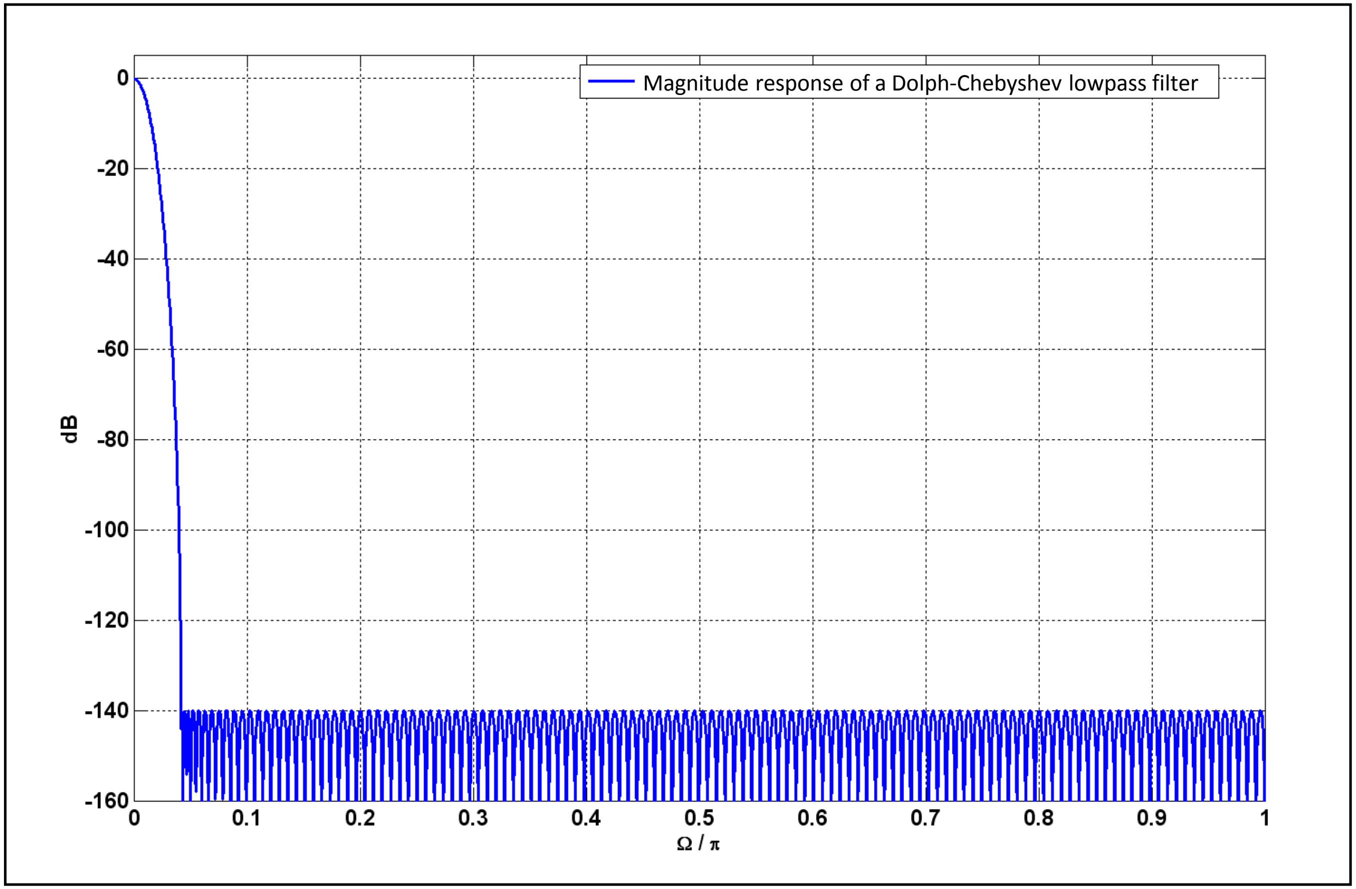
Ideal lowpass filter with the desired width for the passband.

The multiplication (in the time domain) with a window function (corresponds to a convolution in the frequency domain) will widen the passband of the resulting filter. For that reason the window function should be a lowpass filter with minimal passband width, e.g. a Dolph-Chebyshev window of length 2N-1:

$$\tilde{\Psi}_{\mathrm{F}}\big(e^{j\Omega}\big) = \begin{cases} K\;\cosh\left((2N-1)\,\mathrm{arccosh}\left(\dfrac{\cos\left(\frac{\Omega}{2}\right)}{\cos\left(\frac{\Omega_{\mathrm{g}}}{2}\right)}\right)\right) & : \; 0 \le |\Omega| < \Omega_{\mathrm{g}}\,, \\[2ex] K\;\cos\left((2N-1)\,\mathrm{arccos}\left(\dfrac{\cos\left(\frac{\Omega}{2}\right)}{\cos\left(\frac{\Omega_{\mathrm{g}}}{2}\right)}\right)\right) & : \; \Omega_{\mathrm{g}} \le |\Omega| \le \pi\,. \end{cases}$$

Analysis-Synthesis Structure – Part 9

Dolph-Chebyshev lowpass filter:



Since we are designing the magnitude square of the filter, we have to take care about sufficient stopband attenuation (twice as much as usual).

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Analysis-Synthesis Structure – Part 10

Previous result:

$$\psi_i = \psi_{\mathrm{B},i} \cdot \psi_{\mathrm{F},i} \qquad \text{with} \qquad \psi_i = g_i * g_{-i} = \sum_{\kappa=-\infty}^{\infty} g_\kappa \, g_{\kappa-i}$$

What is missing finally is the decomposition of the autocorrelation function in an impulse response of the analysis filter and an impulse response of the synthesis filter. The solution of this problem is not unique.

Transformation into the z-domain:

$$\psi_i = g_i * g_{-i} \quad \circ\!\!-\!\!\!-\!\!\bullet \quad \Psi(z) = G(z) \cdot G(z^{-1})$$

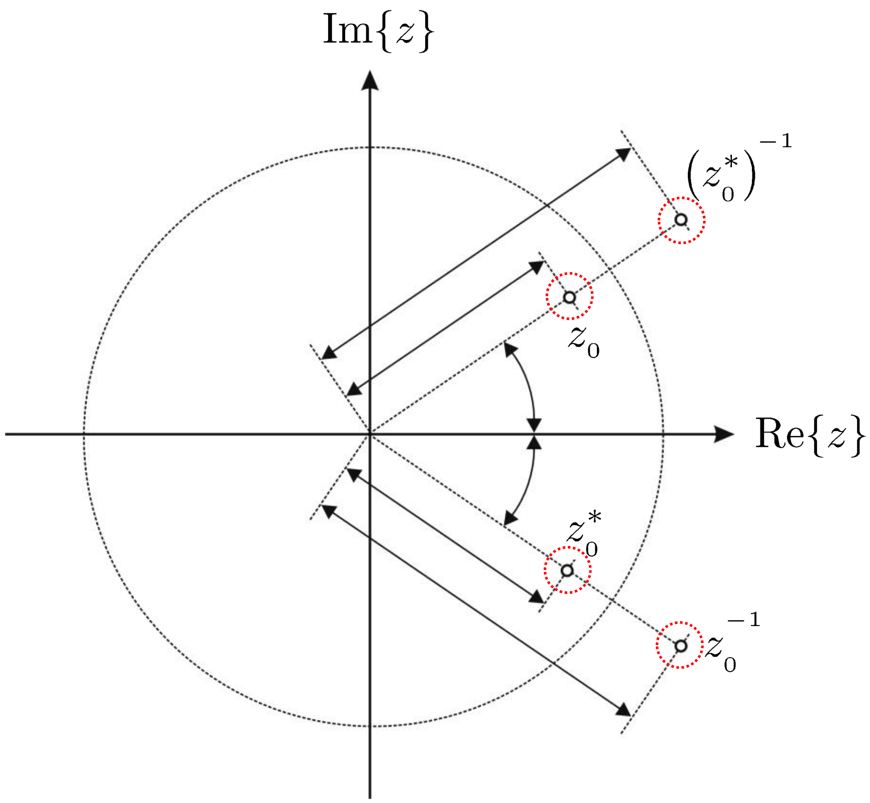
This means: for each pair of zeros of G(z), a pair with inverse magnitude belongs to $G(z^{-1})$.

Analysis-Synthesis Structure – Part 11

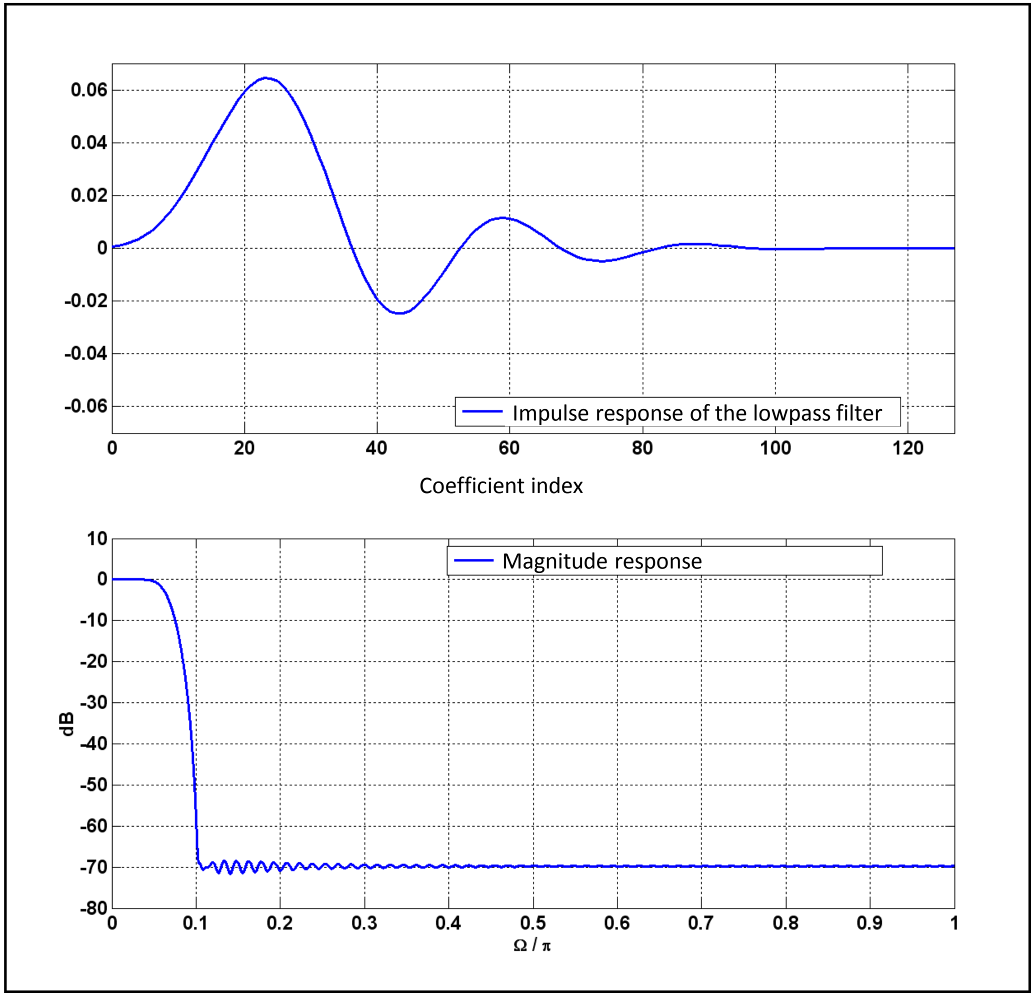
Splitting the pairs of zeroes [half for $G(z)$, half for $G(z^{-1})$]:

Since finding the zeros of high-order polynoms leads to numerical problems it is beneficial to use approaches that avoid the explicit computation of the zeros. These approaches split the polynom into two of lower order: one containing all zeros that are located within the unit circle, the other containing all outside the unit circle. This is called a minimum-phase/maximum-phase decomposition.

(Details in Hänsler/Schmidt: Acoustic Echo and Noise Control, Wiley, 2004).



Analysis-Synthesis Structure – Part 12



Result after minimum phase splitting of the zeros

What we have done so far …

Problem
(Reducing the complexity of adaptive filters)

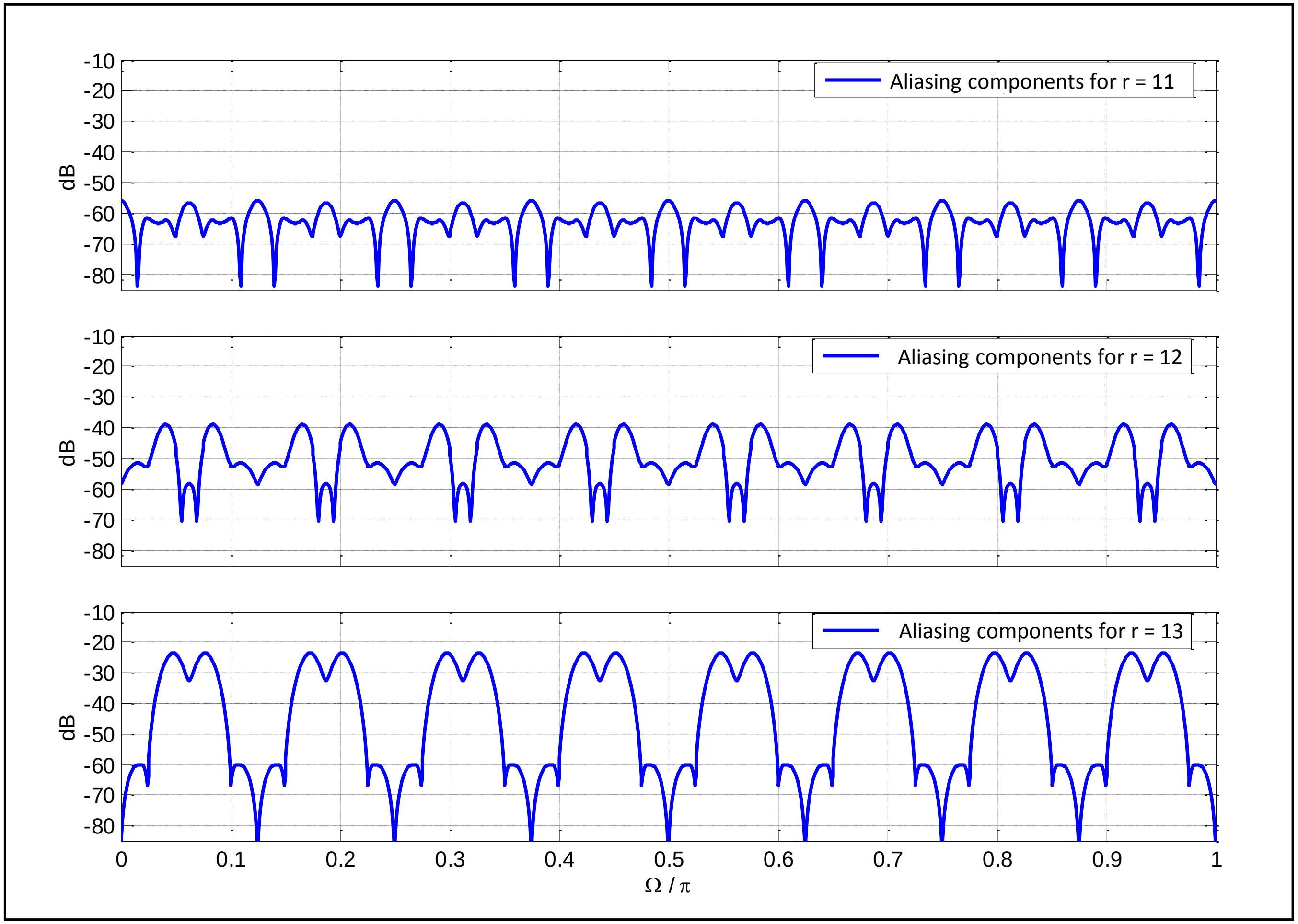
Efficient subband structures
(by inserting a few restrictions)

Design of so-called „prototype lowpass filters“

Verification
(Complexity reduction and adaptation performance)

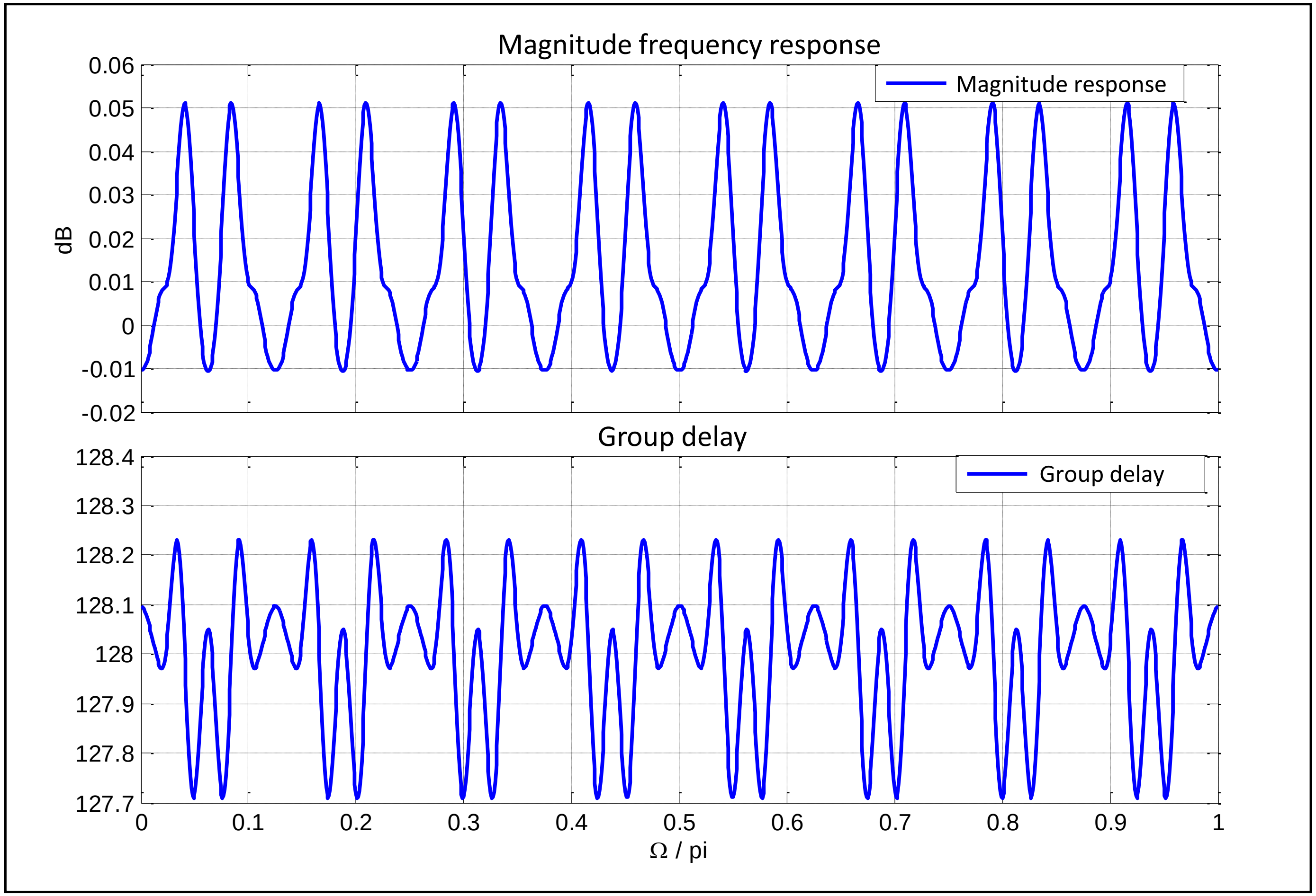
Solution
(Complexity reduction but also an additional delay)

Verification - Aliasing



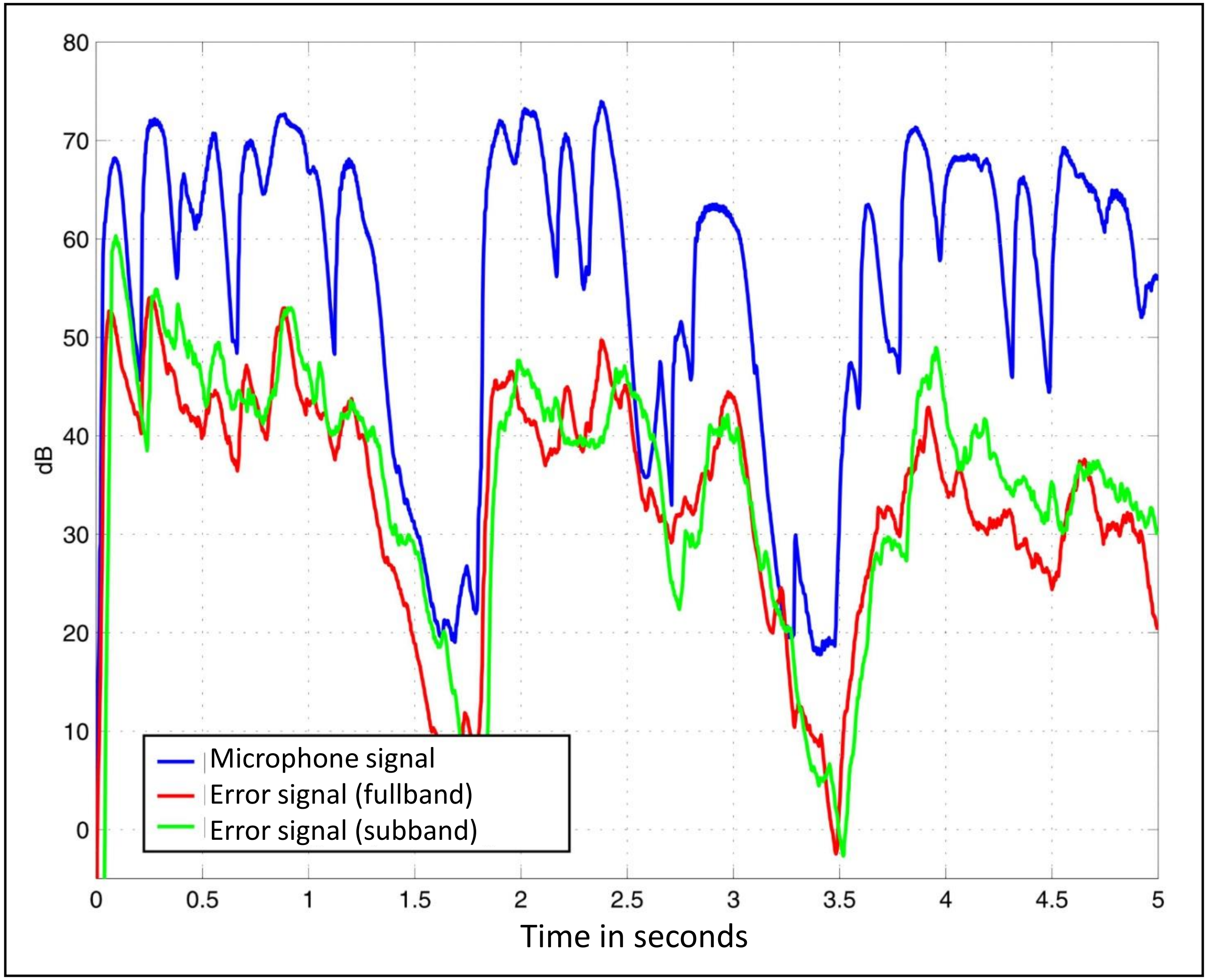
Analysis of the aliasing components for different subsampling rates (excitation with a Kronecker impulse at n = 0)

Verification – Magnitude and Phase Response (Group Delay)



Analysis of the magnitude frequency response and of the group delay for r = 12 (excitation with a Kronecker impulse at n = 0)

Verification – Convergence Analysis



Comparison of two convergence runs (excitation: speech, no local distortion, all step sizes = 1, fixed first-order prediction error filter for improving the speed of convergence in the fullband structure)

What we have done so far …

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Results – Part 1

Boundary conditions:

Design of a subband system with the following parameters:

- $M = 16$ subbands (with equal bandwidth)
- $r = 12$ (same subsampling rate for all subbands)
- $L_{\mathrm{op}} = 3$ (average complexity ration of complex and real operations)
- $N = 128$ (length of the prototype lowpass filter)

Computational complexity:

- $\dfrac{2 \cdot 8000 \cdot 4000}{1000000} = 64$ MIPS (fullband)
- $$\frac{\frac{M}{2} \cdot L_{\mathrm{op}} \cdot 2 \cdot 8000/r \cdot 4000/r + 3 \cdot M/r \log_2 M + 3 \cdot N/r}{1000000}$$

$$\approx \frac{8 \cdot 3 \cdot 2 \cdot 667 \cdot 333 + 3 \cdot 16/12 \cdot 4 + 3 \cdot 128/12}{1000000}$$

≈ 10.7 MIPS (subband)

Reduction of about 83 %

Results – Part 2

Results:

- Reduction of the computational complexity down to about 17 % of the starting value.
- A delay of about 16 ms was necessary to achieve this (before it has been zero).
- The filterbanks were designed such that the aliasing components are about 40 dB lower compared to the desired signal components. This leads to a maximum echo reduction of about 40 dB.
- The distortions of the magnitude frequency response of the entire filterbank systems were below 0.1 dB.
- The distortions of the group delay of the entire filterbank were below 0.5 samples.

The design objectives were met!

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Summary and Outlook

This week:

- Introduction and Motivation
- Adaptive Filters Operating in Subbands
- Filter Design for Prototype Lowpass Filters
- Examples

Next weeks:

- Applications of Linear Prediction