ADSP - Ex. 10

Problem 15 (Multirate Digital Signal Processing)

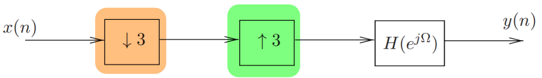
Consider the system shown in the figure. For each of the following input signals $x(n)$, indicate whether the output $y(n) = x(n)$.

(a) $x(n) = cos(\pi n/4)$

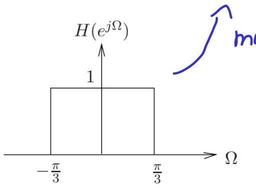
(b) $x(n) = cos(\pi n/2)$

(c) $x(n) = (\frac{\sin(\pi n/8)}{\pi n})^2$

upsampling with $M=3$



downsampling with a factor of $M=3$



mult with anti-imaging LP

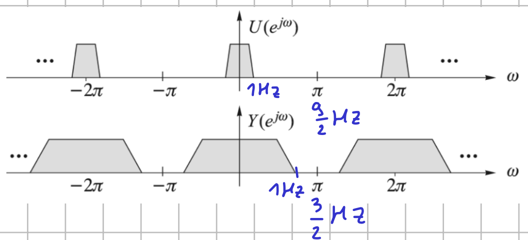
Downsampling: Re-sample a signal $x(n)$ that has a sampling rate of f_s with a new sample rate of $\frac{f_s}{M}$ } Downsampling factor

! Spectra repititions at multiples of $\frac{f_s}{M}$

$$Y(e^{j\Omega'}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\Omega'-k2\pi)/M}).$$

↳ Aliasing if sampling theorem is violated

↳ Anti-imaging filter (LP) before DS to avoid aliasing } decimation



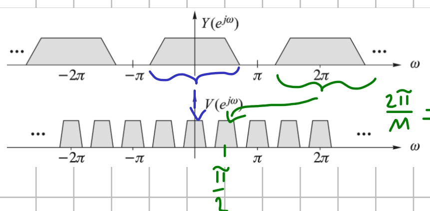
e.g.: $f_s = 9\,Hz$, $M = 3 \rightsquigarrow \frac{f_s}{M} = 3\,Hz$

} spectrum looks stretched after DS as f_s is lowered & π represents $\frac{f_s}{2}$

! only visual in Ω-domain

Upsampling: Insert zeros in-between sample values

↳ scaling of freq. axis by M

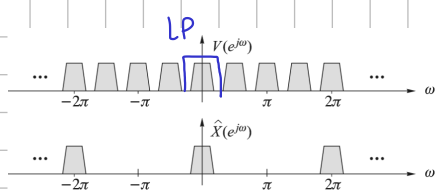


e.g.: $M=4$

$\frac{2\pi}{M} = \frac{2\pi}{4} = \frac{\pi}{2}$

} compression of freq. axis

Anti-imaging & reconstruction:



LP: rect in freq. ⇝ mult with si in time ⇝ convolve with

↳ interpolate samples

Problem 15 **(Multirate Digital Signal Processing)**

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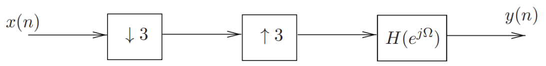
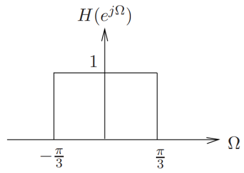
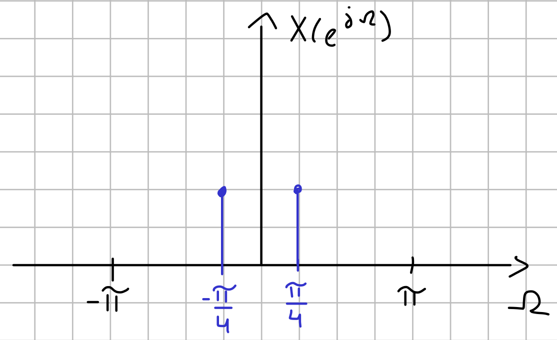
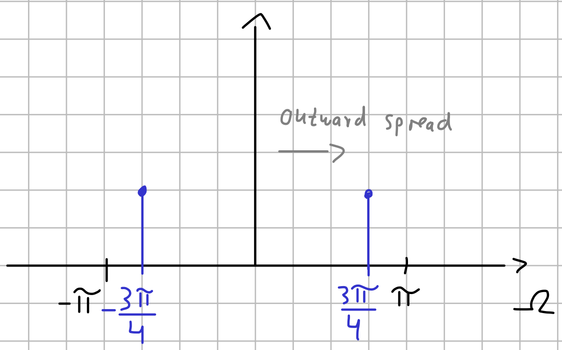




a.) $x(n) = \cos\left(n \cdot \frac{\pi}{4}\right)$

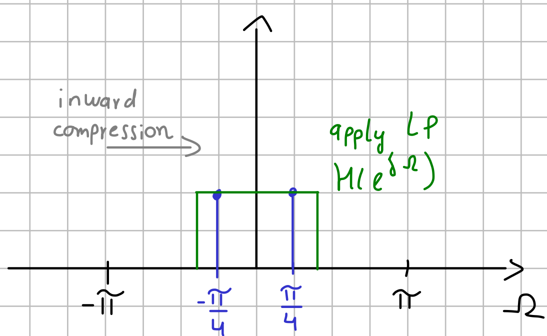


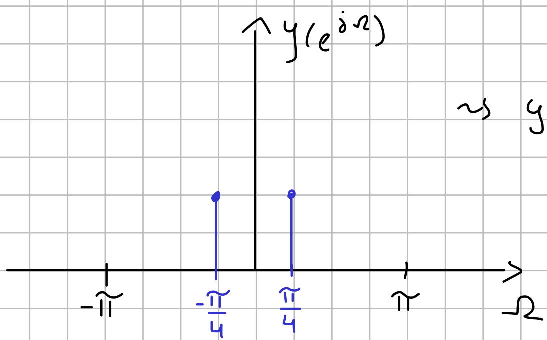
DS with $M=3$:



No aliasing from DS as max freq. $\frac{3\pi}{4} \leq \underbrace{\pi}_{\hat{=}\ \frac{f_s}{2}}$

US with $M=3$:



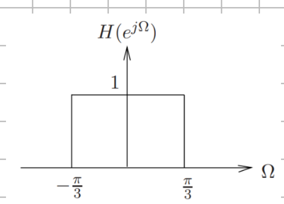


$\rightsquigarrow$ $y(n) = x(n)$

b.) $x(n) = \cos\left(n \cdot \frac{\pi}{2}\right)$

$\frac{\pi}{2} > \frac{\pi}{3}$ component gets filtered out by $H(e^{j\Omega})$

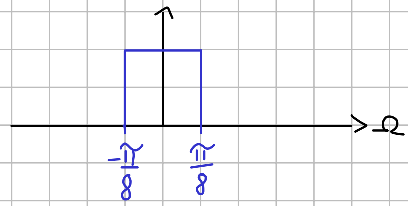
$y(n) \neq x(n)$



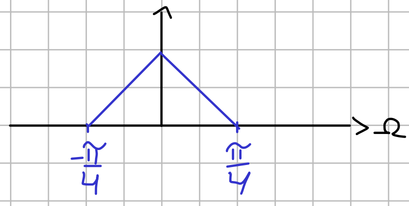
c.) $x(n) = \left(\frac{\sin(n\cdot\frac{\pi}{8})}{n\cdot\pi}\right)^2 = \frac{1}{8^2}\left(\frac{\sin(n\cdot\frac{\pi}{8})}{\underbrace{n\cdot\frac{\pi}{8}}_{\text{sinc}(n\cdot\frac{\pi}{8})}}\right)^2 = \frac{1}{64}\cdot \text{sinc}(n\cdot\frac{\pi}{8})^2$

convolution of spectra due to squaring

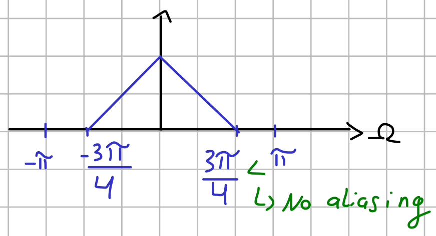
Spectrum:



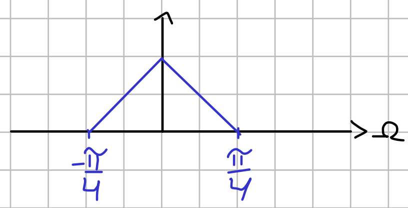
Spectrum:



after DS:



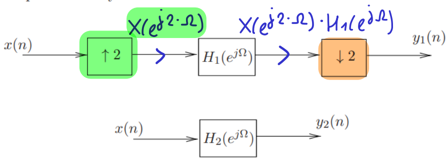
after US:



anti-imaging: $\underset{\text{max freq}}{\frac{\pi}{4}} < \frac{\pi}{3}$ ⇝ spectrum passes LP & $y(n) = x(n)$

Problem 16 (Multirate Digital Signal Processing)

Consider the systems shown in the figure. Suppose that $H_1(e^{j\Omega})$ is fixed and known. Find $H_2(e^{j\Omega})$, the frequency response of an LTI system, such that $y_2(n) = y_1(n)$, if the inputs to the systems are the same.



$y_1(e^{j\Omega})$ is $X(e^{j2\cdot\Omega})\cdot H_1(e^{j\Omega})$ downsampled by $M = 2$

↓

$$\boxed{Y(e^{j\Omega'}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\Omega' - k2\pi)/M}).}$$

$$y_1(e^{j\Omega}) = \frac{1}{\underset{M}{2}}\cdot\left[X(e^{j2\Omega/2})\cdot H_1(e^{j\Omega/2}) + X(e^{j2\cdot(\Omega-2\pi)/2})\cdot H_1(e^{j(\Omega-2\pi)/2})\right]$$

$$= \frac{1}{2}\cdot\left[X(e^{j\Omega})\cdot H_1(e^{j\Omega/2}) + X(e^{j(\Omega-2\pi)})\cdot H_1(e^{j(\frac{\Omega}{2}-\pi)})\right]$$

$$= X(e^{j\Omega})\cdot \underbrace{\frac{1}{2}\left[H_1(e^{j\Omega/2}) - H_1(e^{j(\frac{\Omega}{2}-\pi)})\right]}_{H_2(e^{j\Omega}),\ \text{freq. response of equivalent system}}$$

$$= X(e^{j\Omega})\cdot H_2(e^{j\Omega})$$