ADSP - Ex 2

A sinusoid signal $v(n) = 5\sin(\frac{\omega_0}{\omega_s} \cdot n)$ with $f_0 = 5$ Hz and $f_s = 10$ kHz has to be quantized ($v_q = Q[v(n)]$) with a midtreat quantizer. The range of the signal is ±5 V and the word length of the quantizer 4 bits. The quantizer at digital full scale.

(a) How many quantization levels L does the quantizer have? What is the value of Δ?

$V(n) = 5 \cdot \sin\left(\frac{\omega_0}{\omega_s} \cdot n\right)$, $\omega_0 = 2\pi \cdot f_0 = 2\pi \, 5\,Hz$

$\omega_s = 2\pi \cdot f_s = 2\pi \cdot 10\,kHz$

discrete in time: n, continuous in value: $V(n) \in [-5V, +5V)$

↓ quantize values

$V_q = Q[V(n)]$

each value represented by a 4-bit word

0000
0001
⋮
1110
1111

} Quantization levels

16 different combinations

a.) $L = 2^b = 2^4 = 16$

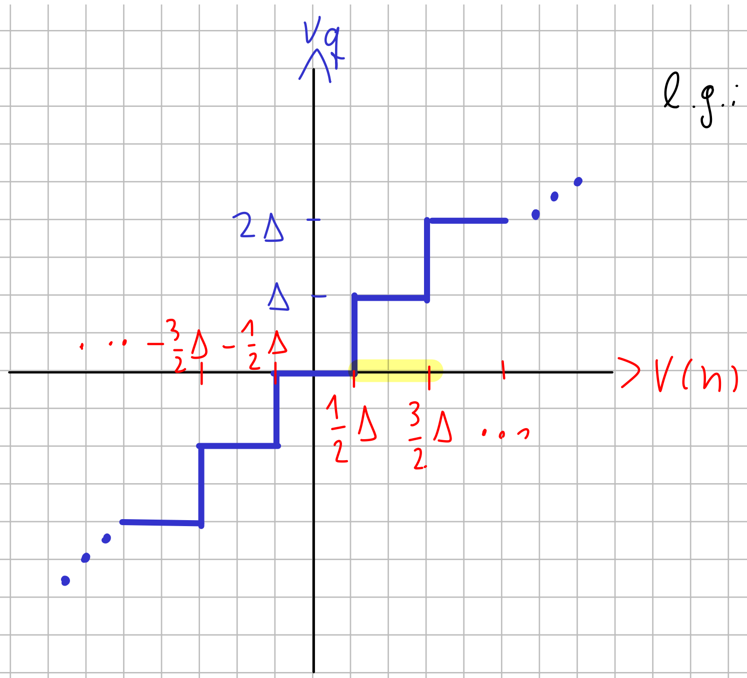
Quantization step Δ: Difference in value between two quantization levels

Quantization range R: $V(n) \in [-5V, +5V] \rightsquigarrow R = 5V - (-5V) = 10V$

↳ $\Delta = R/L$

$= 10V/16 = 0{,}625\,V$

(b) Sketch the input-output characteristic of the quantizer. How different is a midtreat quantizer to a midrise quantizer.



e.g.: $v(n) = 0{,}5V \in \left[\frac{1}{2}\Delta, \frac{3}{2}\Delta\right]$ — mapped to Δ

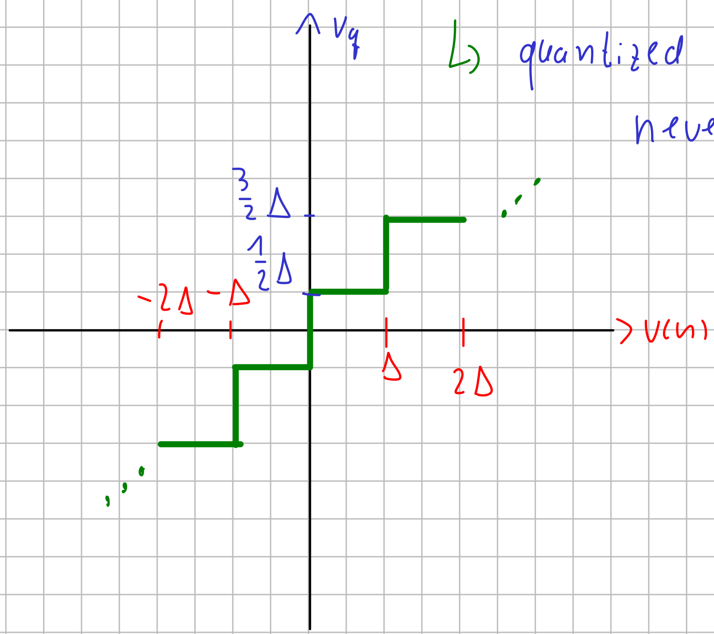
$\downarrow$

$v_q(n) = \Delta = 0{,}625\,V$

midtread: zero as quantization level

mid-rise: zero as decision level

↳ quantized signal never equal to zero!



(c) For time index $n = 1250$ calculate the quantized value $v_q(n)$, the quantization error $e_q(n)$ and represent $v_q(n)$ using bipolar code (sign and magnitude representation).

$$v(n=1250) = 5\sin\left(\frac{\omega_0}{\omega_s}\cdot 1250\right) = 2{,}925$$

$$v_q(n=1250) = Q[v(n=1250)]$$
$$= Q[2{,}925]$$
$$= 3{,}125$$

$2{,}925/\Delta = 4{,}68$

$[4{,}5\cdot\Delta, 5{,}5\cdot\Delta]$

$\downarrow$

$5\cdot\Delta = 3{,}125$

$e_q(n) = V(n) - V_q(n)$

↳ $e_q(1250) = -0{,}2$

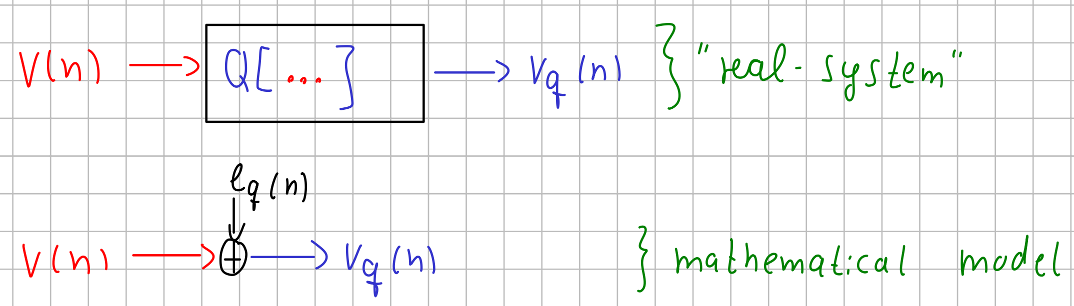
bipolar code (sign-magnitude)

Number	Positive reference	Negative reference	Sign and magnitude	Two's complement	Offset binary	One's complement
+7	+7/8	-7/8	0111	0111	1111	0111
+6	+3/4	-3/4	0110	0110	1110	0110
+5	+5/8	-5/8	0101	0101	1101	0101
+4	+1/2	-1/2	0100	0100	1100	0100
+3	+3/8	-3/8	0011	0011	1011	0011
+2	+1/4	-1/4	0010	0010	1010	0010
+1	+1/8	-1/8	0001	0001	1001	0001
+0	0+	0-	0000	0000	1000	0000
-0	0-	0+	1000	(0000)	(1000)	1111
-1	-1/8	+1/8	1001	1111	0111	1110
-2	-1/4	+1/4	1010	1110	0110	1101
-3	-3/8	+3/8	1011	1101	0101	1100
-4	-1/1	+1/1	1100	1100	0100	1011
-5	-5/8	+5/8	1101	1011	0011	1010
-6	-3/4	+3/4	1110	1010	0010	1001
-7	-7/8	+7/8	1111	1001	0001	1000
-8	-1	+1		1000	0000	

$V_q(n) = 5 \cdot \Delta$ ⇝ 0101

The quantization error over time can be modeled as a noise that is added to the input signal.

(d) Sketch the real system and the mathematical model of the system with the added quantization noise.



(e) Calculate the power P_n of the quantization noise.

$$P_n = \sigma_{e_q}^2 = \int_{-\Delta/2}^{\Delta/2} e^2 f_e(e)\, de = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2\, de = \frac{\Delta^2}{12}$$

↳ $P_n = \frac{\Delta^2}{12} = \frac{0{,}625^2}{12} = 0{,}03255$

(f) Determine the SNR in dB and in linear scale.

$$SNR/\text{dB} = 10\log_{10}\left(\frac{P_v}{P_n}\right)$$

$$SNR/\text{dB} = 10\log_{10}\left(\frac{\sigma_v^2}{\sigma_{e_q}^2}\right) = 10\log_{10}\left(\frac{12\,2^{2b}\,\sigma_v^2}{R^2}\right) = 6.02\,b + 10.8 - 20\log_{10}\left(\frac{R}{\sigma_v}\right)$$

σ_v : root-mean-square of $v(n) = \frac{R}{2\sqrt{2}} \rightsquigarrow \frac{10}{2\sqrt{2}} = 3{,}5355$

$$SNR/dB = 6{,}02 \cdot 4 + 10{,}8 - 20\log_{10}\left(\frac{10}{3{,}5355}\right)$$
$$= 25{,}849\,dB$$

$$SNR = 10^{\frac{SNR/dB}{10}} = 384{,}503$$

The signal's amplitude is changed to ±1 V, while the range R of the quantizer remains the same as before.

(g) How is SNR affected with this change?

↳ new $\sigma_v = \frac{2}{2\sqrt{2}} = 0{,}707$

↳ $SNR/dB = 11{,}87\,dB$

↳ SNR decrease if quantizer range is not re-adjusted to value range!

(h) What word length has to be chosen to achieve an SNR > 45 dB?

Increasing bits ⇝ Decreased q. step Δ ⇝ finer quantization

↳ SNR ↑

1V : $45dB < 6{,}02 \cdot b + 10{,}8 - 20\log\left(\frac{10}{0{,}707}\right)$

⋮

$b \geq$ round$(9{,}503) = 10$

5V : $b \geq 8$

} less additional bits required if R matches input range