ADSP - Exercise 3: Peer Oral-Examination Cards

Instructions

Work in pairs. The examiner may use the notes but should not show them during the answer.

- Round 1, 3 min: Student A examines student B using card A.
- Round 2, 3 min: Change roles; student B examines student A using card B.
- Feedback, 2 min: Use the checklist.

Card A - Circular versus linear convolution

Main question

Explain why multiplication of two M-point DFTs followed by an IDFT generally produces circular rather than linear convolution.

Follow-up questions

1. What happens to a linear-convolution sample whose index is $n \geq M$?
2. Derive the minimum DFT length for lengths M_1 and M_2, once from the indices and once from sampling $Y(e^{j\Omega})$.
3. Is a power-of-two DFT length mathematically necessary?
4. Is the DFT result incorrect when M is too short for linear convolution?

Choose two follow-up questions.

Notes for the examiner

Expected key ideas:

- The DFT represents length-M periodic sequences.
- Frequency-domain multiplication corresponds to M-point circular convolution.
- Linear indices are reduced modulo M; colliding samples are added.
- The last possible linear-convolution index is $M_1 + M_2 - 2$.
- Therefore $M \geq M_1 + M_2 - 1$ prevents time-domain aliasing.
- Spectral view (slide 9): $\tilde{V}_M(\mu)\tilde{H}_M(\mu)$ are M samples of $Y(e^{j\Omega})$; the IDFT of M spectral samples periodizes $y_{\text{lin}}(n)$ with period M. The product is correct; too few samples cause the overlap.
- A power of two is an implementation convenience, not the mathematical condition.

Useful prompt:

> Where does the sample at linear index M appear in an M-point circular result?

Card B - DFT as samples of the DTFT

Main question

Explain the relationship between the DFT of a finite sequence and its DTFT.

Follow-up questions

1. At which frequencies does an M-point DFT sample the DTFT?
2. What changes when zeros are appended before computing the DFT?
3. Does zero-padding create new spectral information or improve intrinsic frequency resolution?
4. Why can a plotted spectrum look smoother after zero-padding?
5. A 16-sample sinusoid with exactly two periods gives a 16-point DFT with only two nonzero samples. Is its spectrum an ideal line spectrum?

Choose two follow-up questions.

Notes for the examiner

Expected key ideas:

$$\tilde{V}_M(\mu) = V(e^{j\Omega})\big|_{\Omega=2\pi\mu/M}, \qquad M \geq M_1.$$

- The DTFT is continuous in Ω and 2π-periodic.
- The DFT gives M equally spaced samples of that function.
- Zero-padding increases the number of sampling locations.
- The original nonzero sequence and its DTFT remain unchanged.
- A denser grid interpolates the display but does not add observations or narrow features set by the finite record.
- Coherent case: the 16-point grid hits the two main-lobe peaks and otherwise only zeros of the DTFT. The DTFT still has finite-width main lobes and side lobes; zero-padding makes them visible.

Useful prompt:

> Did appending zeros add any new measured signal samples?

Peer feedback checklist

Criterion	Secure	Partly secure	Not yet clear
DFT and IDFT conventions are stated consistently.	☐	☐	☐
Circular and linear convolution are distinguished.	☐	☐	☐
Wrap-around is explained using modulo-M indexing.	☐	☐	☐
$M \geq M_1 + M_2 - 1$ is derived, not merely quoted.	☐	☐	☐
DFT samples are connected correctly to the DTFT.	☐	☐	☐
Zero-padding is not confused with new information.	☐	☐	☐

Give one positive observation and one concrete suggestion for improvement.