ADSP - Exercise 4: Peer Oral-Examination Cards

Instructions

Work in pairs. The examiner chooses two follow-ups and keeps the notes out of view.

- Round 1, 3 min: Student A examines student B using card A.
- Round 2, 3 min: Change roles; student B examines student A using card B.
- Feedback, 2 min: Use the checklist.

Card A - Linear filtering in blocks

Main question

How can a long signal be filtered with DFT blocks so that the result is the exact linear convolution? Derive the DFT-length condition for Overlap-Add and explain how Overlap-Save obtains valid outputs.

Follow-up questions

1. For $M_1 = 4$, $M_2 = 3$, where does local output index 5 appear when $M = 4$? Can the global addition repair this?
2. Why is the Overlap-Add shift νM_1 rather than νM?
3. For $M = 6$, $M_2 = 3$ with the smallest overlap, which Overlap-Save outputs are discarded and where does the second retained segment go?
4. Every block is cut off abruptly. Why is the assembled output nevertheless exact?
5. When would you prefer Overlap-Save, and when Overlap-Add?

Notes for the examiner

The input decomposition and linearity give

$$v(n) = \sum_{\nu} v_\nu(n - \nu M_1), \qquad y(n) = \sum_{\nu} (v_\nu * h)(n - \nu M_1).$$

The last local linear-convolution index is $M_1 + M_2 - 2$, so $M \geq M_1 + M_2 - 1$ prevents folding; neighbouring results overlap by $M_2 - 1$ samples and are added. Overlap-Save: output r needs inputs $r - \kappa$, $0 \leq \kappa < M_2$, which lie inside the block if $r \geq M_2 - 1$; keep $M_1 = M - M_2 + 1$ outputs and save $M_2 - 1$ input samples.

- With $M = 4$, local index 5 folds to 1. The values are mixed inside the block; adding blocks cannot separate them.
- The shift follows the position of the input block; padding to M only appends zeros.
- At $M = 6$: discard local 0-1, keep 2-5; the second segment occupies global 4-7. A discarded value may happen to match the reference.
- The rectangular blocks $w(n - \nu M_1)$ add up to one, and filtering is linear.
- Same transforms for equal M and M_1; OLS needs no output additions and suits streaming; OLA suits input that already arrives in separate blocks.

Useful prompt: "Which two contributions arrive at global index 4?" Expected: $7 + 2 = 9$.

Card B - Leakage and resolution

Main question

Why is the DFT spectrum of an observed cosine generally not a pair of lines? Derive the spectrum of the windowed sequence and explain what the window length and the window shape determine.

Follow-up questions

1. Why does $\cos(1.0 \cdot 2\pi n/L)$ with $M = L = 50$ give only two nonzero DFT values, but $\cos(1.2 \cdot 2\pi n/L)$ does not?
2. Does zero-padding remove leakage? What does it change?
3. State the criterion of slide 29 for $\Omega_0 = 0.2\pi$, $\Omega_1 = 0.22\pi$. Which L is needed with the rectangle, which with Hann?
4. A tone 40 dB weaker lies 0.13π away. Which window would you choose, and why?
5. Why is a longer observation not always better?

Notes for the examiner

Finite observation means $\hat{v}(n) = v(n)w(n)$. Inserting the inverse DTFT gives

$$\hat{V}(e^{j\Omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} V(e^{j\theta})W(e^{j(\Omega-\theta)})\,d\theta, \qquad \hat{V}(e^{j\Omega}) = \tfrac{1}{2}\left[W(e^{j(\Omega-\Omega_0)}) + W(e^{j(\Omega+\Omega_0)})\right]$$

for a cosine (slide 19). The DFT samples this spectrum at $\Omega_\mu = 2\pi\mu/M$. Rectangle: first zero at $\pm 2\pi/L$; the length sets the main-lobe width, the shape sets the sidelobes.

- With $M = L$ and an integer number of periods, all other samples fall on zeros of W; the periodic continuation has no jump. For $c = 1.2$ it jumps, and all values are nonzero.
- Zero-padding samples the same $\hat{V}(e^{j\Omega})$ more densely; leakage stays.
- $|\Omega_1 - \Omega_0| = 0.02\pi$: rectangle $L > 100$, Hann about $L > 200$. At $L = 100$ two displaced maxima arise from cancellation; they do not satisfy the criterion.
- Weak tone far outside the main lobes: Hann, because the rectangle's sidelobes cover it.
- Stationarity over $T_0 = LT$, delay and effort (slides 18, 27).

Useful prompt: "What is the spectrum of an infinitely long cosine, and what does multiplication by the window do to it?"

Peer feedback checklist

Criterion	Secure	Partly	Unclear
Derives the DFT length from the linear support length	☐	☐	☐
Explains shift and addition (OLA) and the valid suffix (OLS)	☐	☐	☐
Explains why cutting blocks does not disturb filtering	☐	☐	☐
Attributes leakage to the finite observation, not to the DFT	☐	☐	☐
Distinguishes zero-padding from a longer observation	☐	☐	☐
Relates window length and shape to resolution and sidelobes	☐	☐	☐

Give one positive observation and one concrete suggestion for improvement.