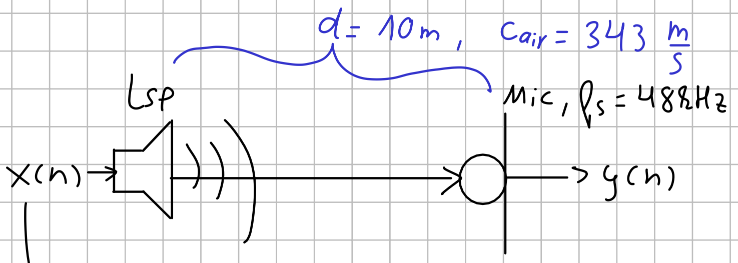
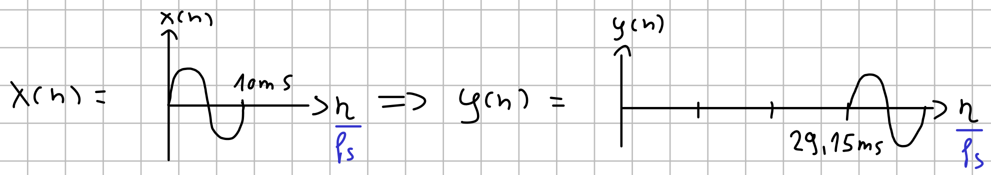


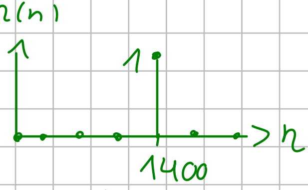
Propagation delay: $\Delta t = \frac{d}{c_{air}} = \frac{10\,m}{343\,\frac{m}{s}} = 0{,}02915\,s = 29{,}15\,ms$

↳ Samples?: $\Delta\tau = \Delta t \cdot f_s = 0{,}02915 \cdot 48000 = 1399{,}41 \approx 1400$

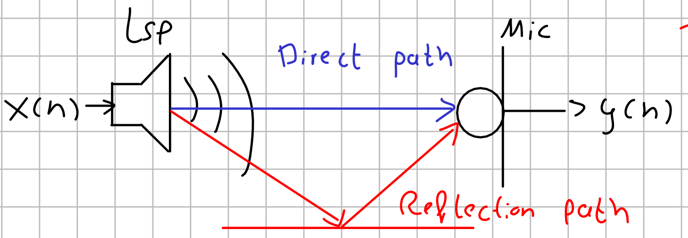


$$x(n) \longrightarrow \boxed{z^{-1400}} \longrightarrow y(n) = x(n-1400)$$

(Finite) Impulse response: $h(n) =$



Determine & analyze for e.g. distance estimation

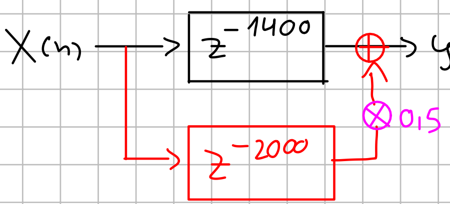


Multipath propagation

Reflecting wall

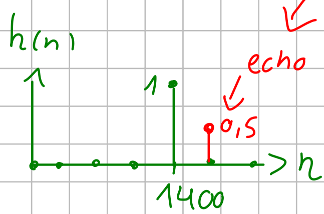
↳ Damping factor $0{,}5 \rightsquigarrow -6\,dB$

↳ e.g.: $14{,}29\,m \rightsquigarrow 2000$ samples delay



$$y(n) = x(n-1400) + 0{,}5 \cdot x(n-2000)$$

(Finite) Impulse response: $h(n) =$

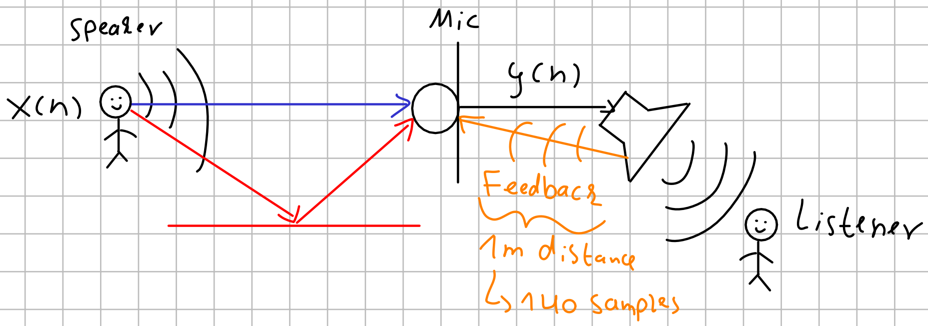


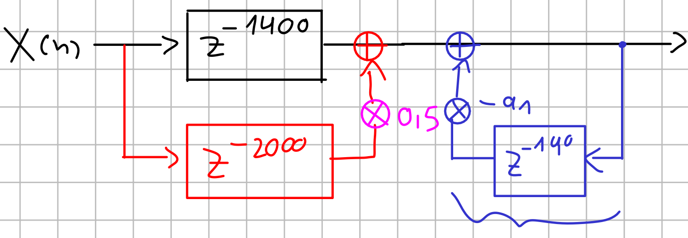
Acoustic characteristics of propagation path (e.g. room)

↳ Revert influence of room? $Y(\mu) = X(\mu) \cdot H(\mu)$

↳ Filter with inverse response $\rightsquigarrow X(\mu) = \frac{Y(\mu)}{H(\mu)} \xrightarrow{IDFT} x(n)$

Recover original input sequence



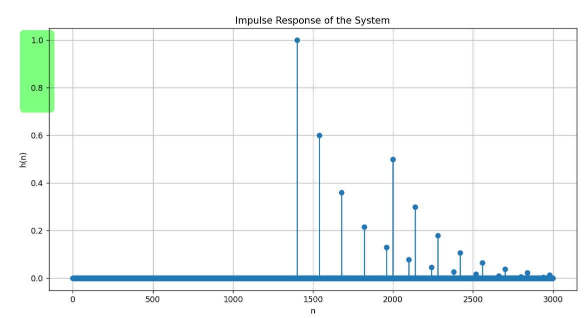


$y(n) = x(n-1400) + 0{,}5 \cdot x(n-2000)$

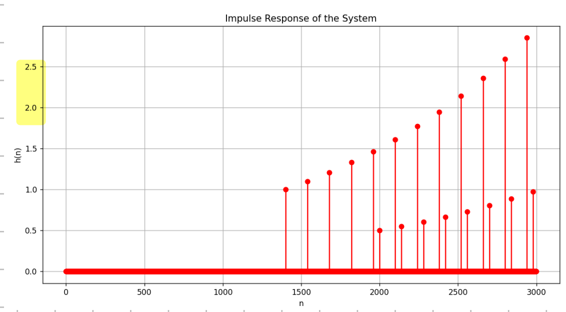
$-a_1 \cdot y(n-140)$

Quiet Lsp ~> e.g. $-a_1 = 0{,}6$

Loud Lsp: e.g. $-a_1 = 1{,}1$



Stable IIR (Infinite Impulse Response)

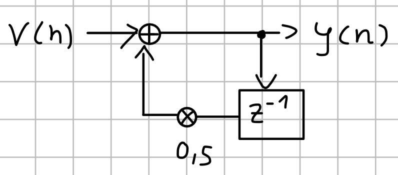


Unstable IIR

A note on impulse responses:

E.g.: $H(z) = \dfrac{1}{1-0{,}5\cdot z^{-1}}$ $\bullet\!\!-\!\!\circ$ $\underbrace{h(n) = 0{,}5^n \cdot \gamma_{-1}(n)}$ } System's reaction to input excitation with

1.order IIR



$v(n) = \gamma_0(n) = \begin{cases} 1, & n=0 \\ 0, & \text{else} \end{cases}$ } Kronecker-Delta

$\hookrightarrow\ y(n) = v(n) + 0{,}5 \cdot y(n-1)$

$= \gamma_0(n) + 0{,}5 \cdot y(n-1)$

$v(n) = \gamma_0(n) = [1\ 0\ 0\ 0\ 0 \ldots] \rightsquigarrow$

$n=0$: $y(0) = \underbrace{\gamma_0(0)}_{=1} + \underbrace{0{,}5 \cdot y(0-1)}_{=0} = 1$

$n=1$: $y(1) = \underbrace{\gamma_0(1)}_{=0} + 0{,}5 \cdot \overbrace{y(1-1)}^{=y(0)=1} = 0{,}5$

$n=2$: $y(2) = \ldots = 0{,}25$

$n \to \infty$: $\lim\limits_{n\to\infty} y(n) \Rightarrow 0$ } stable

$\Updownarrow$

$\rightarrow h(n) = 0{,}5^n \cdot \gamma_{-1}(n)$: $n=0$: $h(0) = \underbrace{0{,}5}_{1} \cdot \underbrace{\gamma_{-1}(0)}_{=1} = 1$

$n=1$: $h(1) = 0{,}5 \cdot \underbrace{\gamma_{-1}(1)}_{=1} = 0{,}5$

$\vdots$

$n\to\infty$: $\lim\limits_{n\to\infty} h(n) = 0$

$\underbrace{\text{for } \gamma_0(n) \text{ excitation}}_{\text{"impulse"}}$, $\underbrace{h(n) = y(n)}_{\text{"response"}}$

ADSP - Ex. 6

Problem 9 **(signal flow graph)**

The signal flow graph in figure 1 describes the input-output relationship of $v(k)$ and $y(k)$.

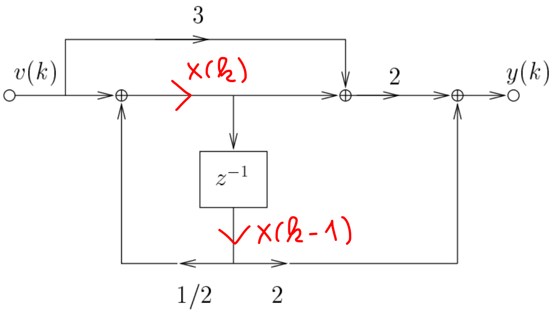
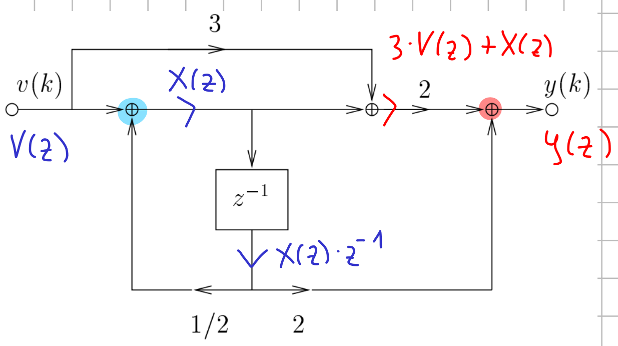


Figure 1: Signal flow graph of a filter

Determine the differential equation, the transfer function $H(z) = \frac{Y(z)}{V(z)}$ and the impulse response $h(k)$ of the system.



$$X(z) = V(z) + \frac{1}{2} \cdot X(z) \cdot z^{-1}$$

$$X(z) \cdot \left(1 - \frac{1}{2} \cdot z^{-1}\right) = V(z)$$

$$X(z) = \frac{V(z)}{1 - \frac{1}{2} z^{-1}}$$

&

$$Y(z) = 2 \cdot X(z) \cdot z^{-1} + 2 \cdot (3V(z) + X(z))$$

$$= 6 \cdot V(z) + 2X(z) \cdot (z^{-1} + 1)$$

$$= 6 \cdot V(z) + 2 \frac{V(z)}{1 - \frac{1}{2} z^{-1}} (z^{-1} + 1)$$

$$= V(z) \cdot \left(6 + \frac{2z^{-1}}{1 - \frac{1}{2} z^{-1}} + \frac{2}{1 - \frac{1}{2} z^{-1}}\right)$$

$$\cdot 1 = \frac{1 - \frac{z^{-1}}{2}}{1 - \frac{z^{-1}}{2}}$$

$$= V(z) \cdot \left(\frac{6 - 3z^{-1} + 2z^{-1} + 2}{1 - \frac{1}{2} z^{-1}}\right)$$

$$\Rightarrow Y(z) = V(z) \cdot \left(\frac{8 - z^{-1}}{1 - \frac{1}{2} z^{-1}}\right)$$

Transfer function:

of the causal LTI-System

$$\Rightarrow H(z) = \frac{Y(z)}{V(z)} = \frac{8 - z^{-1}}{\underbrace{1}_{a_0} - \underbrace{\frac{1}{2}}_{a_1} z^{-1}} \quad \begin{cases} b_0 = 8,\ b_1 = -1 \\ a_0 = 1,\ a_1 = -\frac{1}{2} \end{cases}$$

$$H(z) = \frac{\sum_{i=0}^{N} b_{N-i} z^i}{\sum_{i=0}^{N} a_{N-i} z^i} = \frac{\sum_{i=0}^{N} b_i z^{-i}}{1 + \sum_{i=1}^{N} a_i z^{-i}}$$

Filter order $N = 1$

Impulse response: $h(k) \circ\!\!-\!\!\bullet\ H(z)$

Formulary: z-transform pair

I. $\dfrac{z}{z-a} = \dfrac{1}{1-a\cdot z^{-1}} \bullet\!\!-\!\!\circ\ a^n \cdot \gamma_{-1}(n)$

II. $V(z)\cdot z^{-n_0} \bullet\!\!-\!\!\circ\ v(n-n_0)$

Linearity: $a_1 \cdot V_1(z) + a_2 V_2(z) \ \bullet\!\!-\!\!\circ\ a_1 v_1(n) + a_2 v_2(n)$

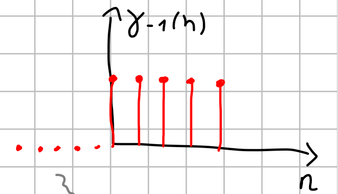
$$\rightsquigarrow H(z) = \frac{8 - z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$= 8 \cdot \underbrace{\frac{1}{1-\frac{1}{2}\cdot z^{-1}}}_{\text{I}} - \underbrace{z^{-1}}_{\text{II}} \cdot \underbrace{\frac{1}{1-\frac{1}{2}\cdot z^{-1}}}_{\text{I}}$$

$$\bullet\!\!-\!\!\circ \quad 8 \cdot \left(\tfrac{1}{2}\right)^k \cdot \gamma_{-1}(k) - 1 \cdot \left(\tfrac{1}{2}\right)^{k-1} \gamma_{-1}(k-1)$$

$$\Rsh\ h(k) = 8 \cdot (0{,}5)^k\, \gamma_{-1}(k) - (0{,}5)^{k-1}\, \gamma_{-1}(k-1)$$

Step function: $\gamma_{-1}(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$



* Check note on next page for practical explanation

Differential equation:

$$\frac{Y(z)}{V(z)} = \frac{8 - z^{-1}}{1 - \frac{1}{2} z^{-1}} \quad \Big|\cdot V(z) \ \Big| \cdot \left(1 - \tfrac{1}{2} z^{-1}\right)$$

$$Y(z) - \tfrac{1}{2} Y(z) z^{-1} = 8 \cdot V(z) - V(z) \cdot z^{-1} \quad \Big| + \tfrac{1}{2} Y(z) z^{-1}$$

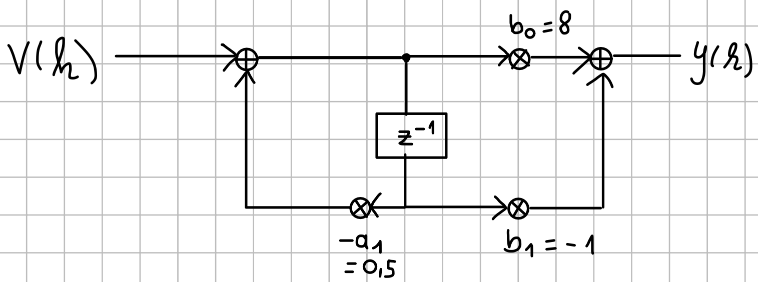
$$Y(z) = 8 \cdot V(z) - V(z)\, z^{-1} + \tfrac{1}{2} Y(z) \cdot z^{-1}$$

$$\bullet\!\!-\!\!\circ$$

$$y(k) = \underbrace{8}_{b_0} \cdot v(k) - \underbrace{1}_{b_1} \cdot v(k-1) + \underbrace{\tfrac{1}{2}}_{-a_1} y(k-1)$$

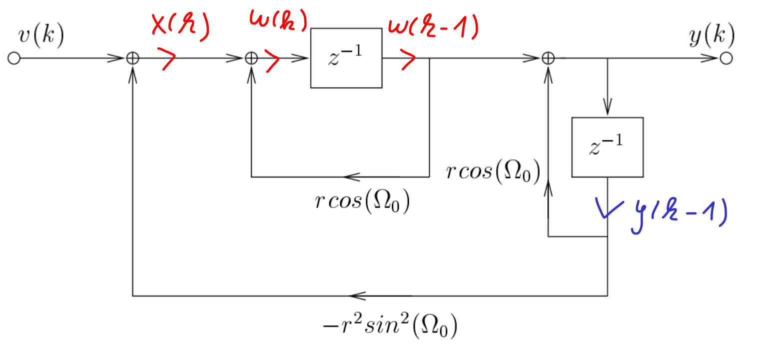
$$= \sum_{i=0}^{N} b_i \cdot v(k-i) - \sum_{i=1}^{N} a_i\, y(k-i)$$

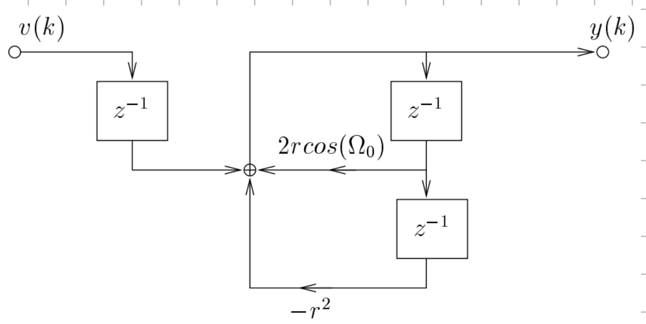
(Canonical) direct form II



Problem 10 (signal flow graph)

Show that the systems in figure 2 are equivalent.





↳ $X(z) = V(z) - r^2\sin^2(\Omega_0)\, Y(z)\cdot z^{-1}$

insert

$W(z) = X(z) + r\cdot\cos(\Omega_0)\, W(z)\cdot z^{-1} \leadsto W(z)\cdot(1 - r\cos(\Omega_0)z^{-1}) = X(z)$

$$W(z) = \frac{X(z)}{(1 - r\cos(\Omega_0)z^{-1})}$$

$Y(z) = W(z)\cdot z^{-1} + r\cos(\Omega_0)\cdot Y(z)\cdot z^{-1}$

insert

$$W(z) = \frac{V(z) - r^2\sin^2(\Omega_0)\, Y(z)\cdot z^{-1}}{(1 - r\cos(\Omega_0)z^{-1})}$$

$Y(z)(1 - r\cos(\Omega_0)\, z^{-1}) = W(z)\cdot z^{-1}$

$$Y(z) = \frac{W(z)\cdot z^{-1}}{(1 - r\cos(\Omega_0)\, z^{-1})}$$

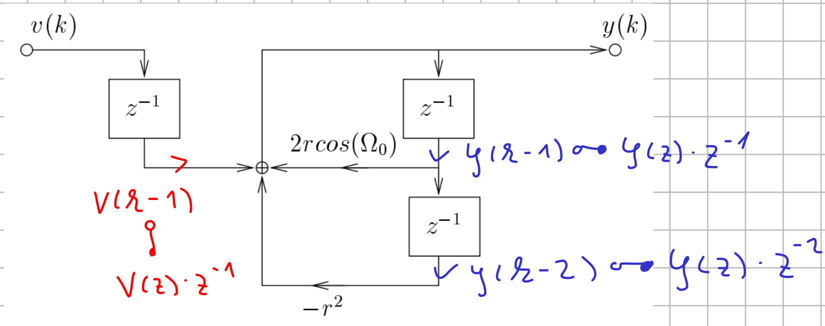
$$Y(z) = \frac{(V(z) - r^2\sin^2(\Omega_0)\, Y(z)\cdot z^{-1})\, z^{-1}}{\underbrace{(1 - r\cos(\Omega_0)z^{-1})\cdot(1 - r\cos(\Omega_0)z^{-1})}_{=(1 - r\cos(\Omega_0)z^{-1})^2}}$$

↳ $Y(z)\cdot(1 - r\cos(\Omega_0)z^{-1})^2 = V(z)\cdot z^{-1} - r^2\sin^2(\Omega_0)\cdot Y(z)\cdot z^{-2}$

$$Y(z)\cdot(1 - 2r\cos(\Omega_0)z^{-1} + \underbrace{r^2\cos^2(\Omega_0)z^{-2} + r^2\sin^2(\Omega_0)z^{-2}}_{r^2\cdot z^{-2}(\underbrace{\cos^2(\Omega_0)+\sin^2(\Omega_0)}_{=1}) = r^2 z^{-2}}) = V(z)\cdot z^{-1}$$

$$Y(z)\cdot(1 - 2r\cdot\cos(\Omega_0)\cdot z^{-1} + r^2 z^{-2}) = V(z)\cdot z^{-1}$$

$$H_1(z) = \frac{Y(z)}{V(z)} = \frac{z^{-1}}{1 - 2r\cdot\cos(\Omega_0)\cdot z^{-1} + r^2 z^{-2}}$$



$$y(z) = V(z) \cdot z^{-1} + 2r\cos(\Omega_0) \cdot y(z) \cdot z^{-1} - r^2\, y(z) \cdot z^{-2}$$

$$y(z) \cdot \left(1 - 2r\cos(\Omega_0) \cdot z^{-1} + r^2 z^{-2}\right) = V(z)\, z^{-1}$$

$$\hookrightarrow H_2(z) = \frac{y(z)}{V(z)} = \frac{z^{-1}}{1 - 2r \cdot \cos(\Omega_0) \cdot z^{-1} + r^2 z^{-2}} = H_1(z)$$