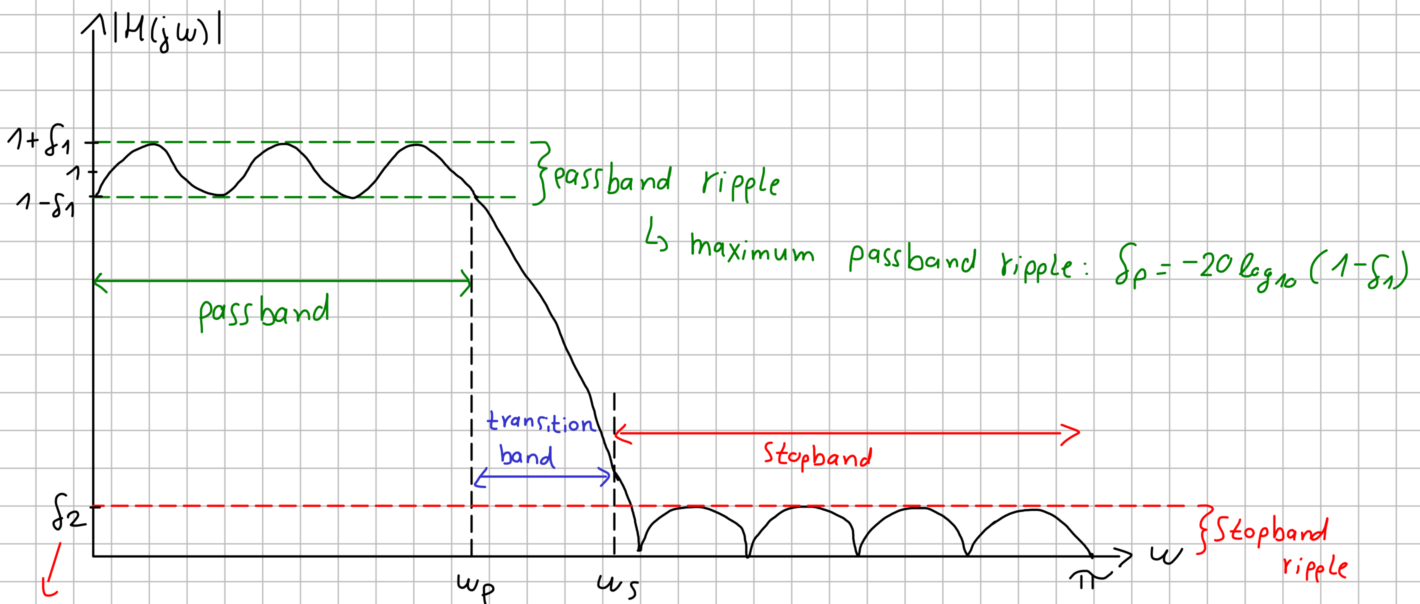
ADSP - Ex.8: FIR filter design

Problem 12 (filter design)

Digital filter specifications are often given in terms of the loss function, $H_l(\Omega) = -20log_{10}(|H(e^{j\Omega})|)$, in dB. In this problem the peak passband ripple δ_p and the minimum stopband attenuation δ_s are given in dB, i.e., the loss specifications of the digital filter are given by

$$\delta_p = -20log_{10}(1-\delta_1)dB,$$
$$\delta_d = -20log_{10}(\delta_2)dB.$$

(a) Estimate the order of an optimal equiripple linear-phase lowpass FIR filter with the following specifications: passband edge $F_p = 1.8kHz$, stopband edge $F_s = 2kHz$, $\delta_p = 0.1dB$, $\delta_s = 35dB$, and sampling frequency $F_T = 12kHz$.



$\delta_s = -20\log_{10}(\delta_2)$ — Minimum stopband attenuation

ω_p — passband edge frequency

ω_s — Stopband edge frequency

transition bandwidth: $\Delta\Omega = \Omega_s - \Omega_p$

Filter order estimation:

↳ Optimum equiripple design (Chebyshev approx) with Remez exchange requires wanted filter order length

↳ approximation formula:

$$N = \frac{-10 \cdot \log_{10}(\delta_1 \cdot \delta_2) - 13}{2{,}324 \cdot \Delta\Omega}$$

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$$\begin{aligned}\delta_p &= -20log_{10}(1-\delta_1)dB,\\ \delta_d &= -20log_{10}(\delta_2)dB.\end{aligned}$$

(a) Estimate the order of an optimal equiripple linear-phase lowpass FIR filter with the following specifications: passband edge $F_p = 1.8kHz$, stopband edge $F_s = 2kHz$, $\delta_p = 0.1dB$, $\delta_s = 35dB$, and sampling frequency $F_T = 12kHz$.

$\hookrightarrow \Omega_p = 2\pi \frac{1{,}8\,kHz}{12\,kHz}, \quad \Omega_s = 2\pi \frac{2\,kHz}{12\,kHz}$

$\hookrightarrow$ Transition bandwidth: $\Delta\Omega = \Omega_s - \Omega_p = 2\pi \cdot \frac{0{,}2\,kHz}{12\,kHz}$

$= 2\pi \cdot \frac{1}{60} = \pi \frac{1}{30}$

passband ripple: $\delta_p = 0{,}1\,dB = -20\log_{10}(1-\delta_1)$

$\log_{10}(1-\delta_1) = -\frac{\delta_p}{20} \quad \Big|\, 10^{\cdots}$

$1-\delta_1 = 10^{-\frac{\delta_p}{20}}$

$\delta_1 = 1 - 10^{-\frac{\delta_p}{20}} = 1 - 10^{-\frac{0{,}1}{20}} \approx 0{,}011469$

Stopband ripple: $\delta_s = -20\log_{10}(\delta_2) \rightsquigarrow \delta_2 = 10^{-\frac{\delta_s}{20}} = 10^{-\frac{35}{20}} \approx 0{,}017782$

N increases for decreasing ripples

$\hookrightarrow N = \frac{-10\cdot\log_{10}(\delta_1\cdot\delta_2) - 13}{2{,}324\cdot\Delta\Omega} \approx 98{,}225 \Rightarrow N = 99$

N increases for narrow $\Delta\Omega$ (faster transitions from pass $\rightarrow$ stopband)

round to next integer value

& decreases for wider $\Delta\Omega \rightsquigarrow$ slower transitions

Wanted: $\underbrace{\text{linear-phase}}_{\text{Type 1-4}}$ low-pass, which filter type?

impulse response symmetry

		even	odd
filter length $L = N+1$	even	2	4
	odd	1	3

Here: $N=99 \rightsquigarrow L=\underbrace{100}_{\text{even}} \rightsquigarrow$ $\underbrace{\text{Type } 2}$ or $\underbrace{4}$

$\underbrace{H(e^{j\pi})=0}$ — Suitable, attenuation at $f_s/2$ — Would be unsuitable for a HP filter!!

$H(\underbrace{e^{j0}})=0$? Unwanted attenuation at 0Hz → not suitable for LP

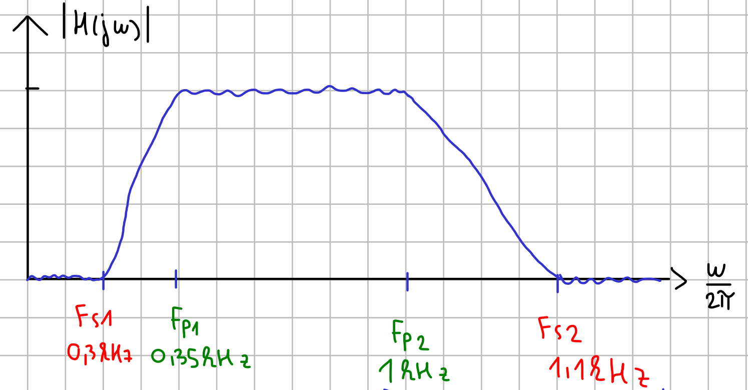
We could also increase the filter order by 1 $\rightsquigarrow N=100 \rightsquigarrow L=\underbrace{101}_{\text{odd length}}$

Type 1 ✓ or $\underbrace{3}$ $H(e^{j0})=0$?

↳ type 1 realization also suitable

The estimation formula can also be used to estimate the length of highpass, bandpass, and bandstop optimal equiripple FIR filters. Then the width of the smallest transition band is used to estimate the filter order.

(b) Estimate the order of an optimal equiripple linear-phase bandpass FIR filter with the following specifications: passband edges $F_{p1} = 0.35kHz$ and $F_{p2} = 1kHz$, stopband edges $F_{s1} = 0.3kHz$ and $F_{s2} = 1.1kHz$, passband ripple $\delta_1 = 0.002$, stopband ripple $\delta_2 = 0.001$, and sampling frequency $F_T = 10kHz$.



$\Delta\Omega_1 = 2\pi \cdot \left(\frac{F_{P1}-F_{S1}}{F_T}\right) \approx \frac{1}{100}\pi$ $\qquad$ $\Delta\Omega_2 = 2\pi \cdot \left(\frac{F_{S2}-F_{P2}}{F_T}\right) = \frac{1}{50}\pi$

$\underbrace{\Delta\Omega_1} < \Delta\Omega_2$

faster transition ⇝ requires higher filter order than $\Delta\Omega_2$ transition

↓

Use $\Delta\Omega_1$ to calculate the total filter order

$$N = \frac{-10\cdot\log_{10}(\delta_1\cdot\delta_2)-13}{2{,}324\cdot\Delta\Omega_1} \approx 602{,}51 \Rightarrow N=603 \rightsquigarrow L=604$$

↳ Type 2 realization (Type 4 too, but 90° phase shift is introduced)