Problem 13 (Digital IIR Filter Design)

The system function of a discrete-time system is

$$H(z) = \frac{2}{1 - e^{-0.2} z^{-1}} - \frac{1}{1 - e^{-0.4} z^{-1}}.$$

(a) Assume that this discrete-time filter was designed by the impulse invariance method with $T = 2$, i.e. $h_i = h_a(iT)$, where $h_a(t)$ is real. Find the system function $H_a(s)$ of a continuous-time filter that could have been the basis for the design. Is your answer unique? If not, find another system function $H_a(s)$.

(b) Assume that $H(z)$ was obtained by the bilinear transform with $T = 2$. Find the system function $H_a(s)$ that could have been the basis for the design. Is your answer unique? If not, find another $H_a(s)$.

Problem 14 (Digital IIR Filter Design)

A discrete-time lowpass filter is to be designed by applying the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function

$$|H(j\omega)|^2 = \frac{1}{1 + (\frac{\omega}{\omega_{cut}})^{2N}}$$

The specifications for the discrete-time signal are

$$\begin{aligned} 0.89125 \leq |H(e^{j\Omega})| \leq 1, \qquad & 0 \leq |\Omega| \leq 0.2\pi, \\ |H(e^{j\Omega})| \leq 0.17783, \qquad & 0.3\pi \leq |\Omega| \leq \pi. \end{aligned}$$

Assume that aliasing will not be a problem, i.e., design the continuous-time Butterworth filter to meet passband and stopband specifications as determined by the discrete-time filter.

(a) Sketch the tolerance bounds on the magnitude of the frequency response, $|H(j\omega)|$, of the continuous-time Butterworth filter such that after application of the impulse invariance method, the resulting discrete-time filter will satisfy the given design specifications. Do not assume that $T = 1$.

(b) Determine the integer order N and the quantity $\omega_{cut}T$ such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.