Advanced Digital Signal Processing

Part 2: Digital Processing of Continuous-Time Signals

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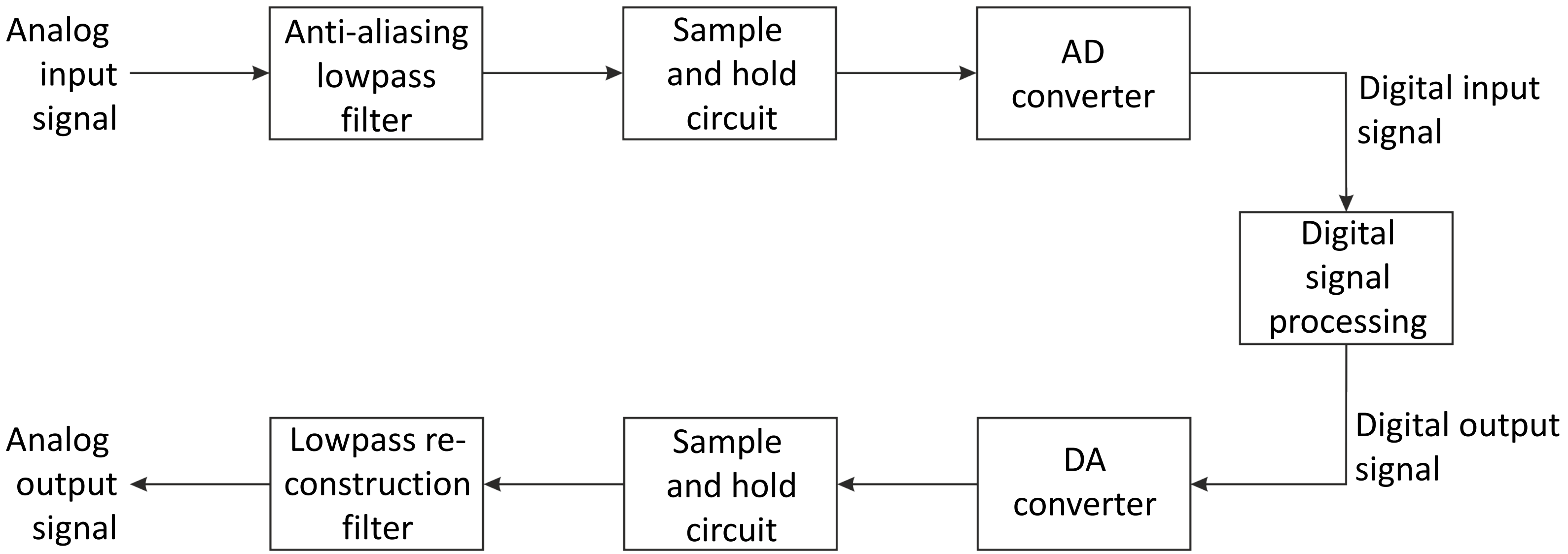
- Efficient FIR filter structures
- DFT and FFT
- Digital filters
- Multi-rate digital signal processing

Basic System

Refined digital signal processing system:



Sampling – Part 1

Basic idea:

- Generation of discrete-time signals from continuous-time signals.

Ideal sampling:

- An ideally sampled signal $v_{\mathrm{i}}(t)$ is obtained by multiplication of the continuous-time signal $v_{\mathrm{a}}(t)$ with a periodic impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta_0(t - nT),$$

where $\delta_0(t)$ is the Dirac delta function and T the sampling period. We obtain

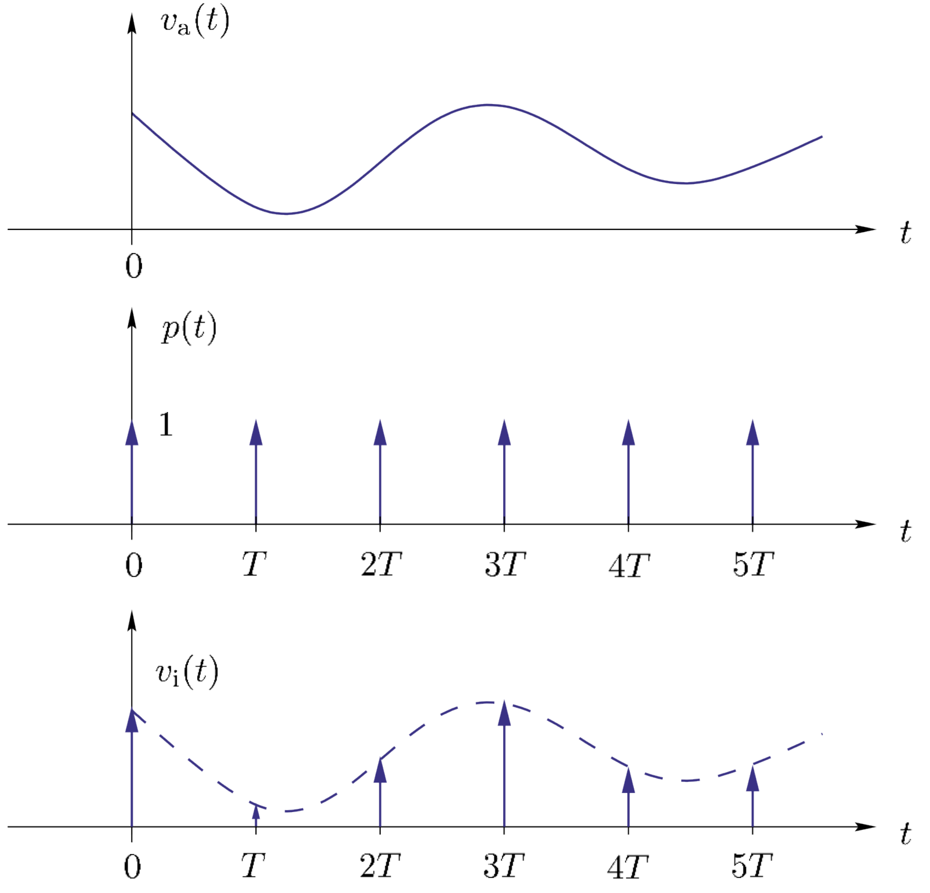
$$v_{\mathrm{i}}(t) = v_{\mathrm{a}}(t) \sum_{n=-\infty}^{\infty} \delta_0(t - nT)$$

… using the "gating" property of Dirac delta functions …

$$= \sum_{n=-\infty}^{\infty} v_{\mathrm{a}}(nT)\, \delta_0(t - nT).$$

Sampling – Part 2

Ideal sampling:



The lengths of Dirac deltas correspond to their weightings!

Sampling – Part 3

How does the Fourier transform $\mathcal{F}\{v_{\rm i}(t)\} = V_{\rm i}(j\omega)$ look like?

- ***Fourier transform*** of an impulse train

$$p(t) \circ\!\!-\!\!\bullet\; P(j\omega) \;=\; \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta_0(\omega - n\omega_{\rm S})$$

with $\omega_{\rm S} \;=\; \dfrac{2\pi}{T}$.

- A ***multiplication in the time domain*** represents a ***convolution in the Fourier domain***, thus we obtain for the spectrum of the signal $v_{\rm i}(t)$:

$$V_{\rm i}(j\omega) \;=\; \frac{1}{2\pi}\,V_{\rm a}(j\omega) \,*\, P(j\omega).$$

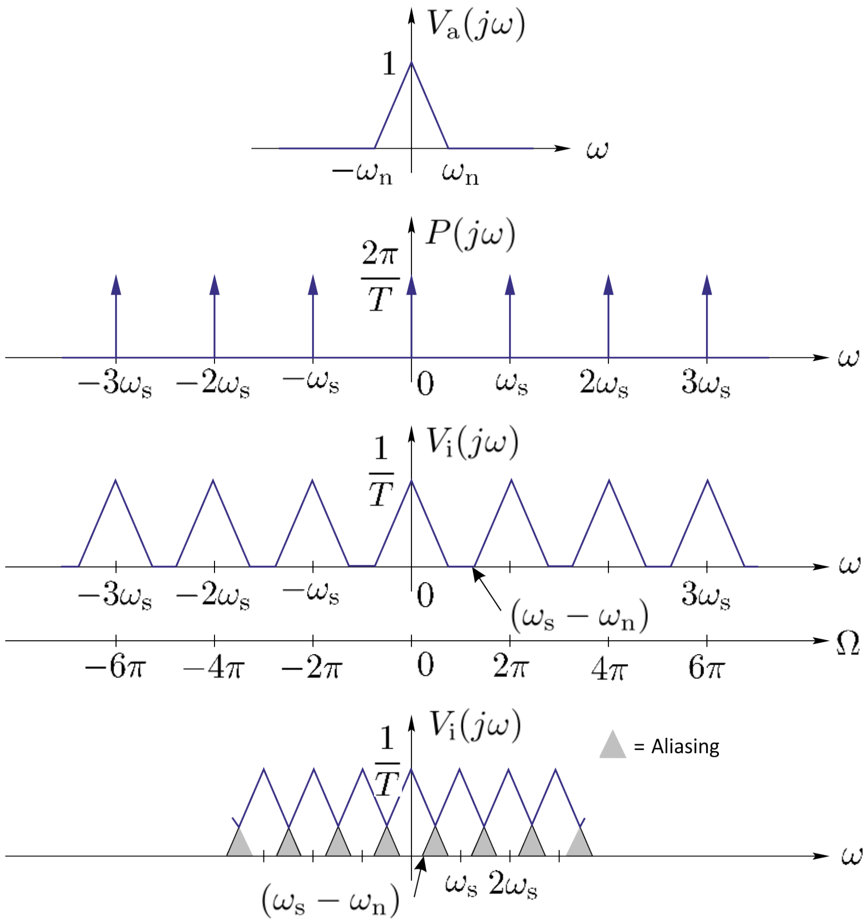
- ***Inserting the spectrum of an impulse train*** leads to

$$V_{\rm i}(j\omega) \;=\; \frac{1}{T}\sum_{n=-\infty}^{\infty} V_{\rm a}\big(j(\omega - n\omega_{\rm S})\big).$$

Periodically repeated copies of $V_{\rm a}(j\omega)$, shifted by integer multiples of the sampling frequency!

Sampling – Part 4

How does the Fourier transform $\mathcal{F}\{v_\mathrm{i}(t)\} = V_\mathrm{i}(j\omega)$ look like?



- Fourier transform of a bandlimited analog input signal $V_\mathrm{a}(j\omega)$, highest frequency is ω_n.
- Fourier transform of the Dirac impulse train.
- Result of the convolution $P(j\omega) * V_\mathrm{a}(j\omega)$. It is evident that when $\omega_\mathrm{s} - \omega_\mathrm{n} > \omega_\mathrm{n}$ or $\omega_\mathrm{s} > 2\omega_\mathrm{n}$, the replicas of $V_\mathrm{a}(j\omega)$ do not overlap. ***In this case $v_\mathrm{a}(t)$ can be recovered by ideal lowpass filtering (later called "sampling theorem")***.
- If the condition above does not hold, i.e. if $\omega_\mathrm{s} \leq 2\omega_\mathrm{n}$, the copies of $V_\mathrm{a}(j\omega)$ overlap and the signal $v_\mathrm{a}(t)$ cannot be recovered by lowpass filtering. The distortion in the gray shaded areas are called ***aliasing***.

Sampling – Part 5

Non-ideal sampling

Modeling the sampling operation with the Dirac impulse train is not a feasible model in real life, since we always need a finite amount of time for acquiring a signal sample.

- Non-ideally sampled signals $v_{\mathrm{n}}(t)$ are obtained by multiplication of a continuous-time signal $v_{\mathrm{a}}(t)$ with a periodic rectangular window function $a_{\mathrm{n}}(t)$:

$$v_{\mathrm{n}}(t) = v_{\mathrm{a}}(t)\, a_{\mathrm{n}}(t),$$

with

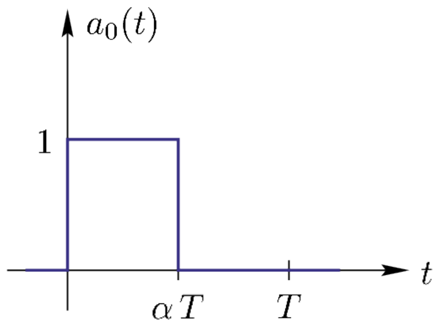
$$a_{\mathrm{n}}(t) = a_0(t) * \sum_{n=-\infty}^{\infty} \delta_0(t - nT) = \sum_{n=-\infty}^{\infty} a_0(t - nT)$$

- $a_0(t)$ denotes the rectangular prototype window

$$a_0(t) = \mathrm{rect}\left(\frac{t - \alpha T/2}{\alpha T}\right)$$

with

$$\mathrm{rect}(t) = \begin{cases} 0, & \text{for} \quad |t| > 1/2, \\ 1, & \text{else.} \end{cases}$$



Sampling – Part 6

Fourier transform of $a_0(t)$ *:*

The Fourier transform of the rectangular time window can be computed as a $\mathrm{si}(x) = \frac{\sin(x)}{x}$ function (see examples of the Fourier transform):

$$A_0(j\omega) = \mathcal{F}\{a_0(t)\} = \alpha T \, \mathrm{si}(\omega\alpha T/2) \, e^{-j\omega\alpha T/2}.$$

Using this result for computing the Fourier transform of $a_\mathrm{n}(t)$ leads to

$$A_\mathrm{n}(j\omega) = A_0(j\omega) \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta_0(\omega - n\omega_\mathrm{s})$$

… inserting the result from above and using the "gating" property of Dirac delta functions …

$$= 2\pi\alpha \sum_{n=-\infty}^{\infty} \mathrm{si}(n\omega_\mathrm{s}\alpha T/2) \, e^{-jn\omega_\mathrm{s}\alpha T/2} \, \delta_0(\omega - n\omega_\mathrm{s})$$

… inserting the definition of $\omega_\mathrm{s} = \frac{2\pi}{T}$ *…*

$$= 2\pi\alpha \sum_{n=-\infty}^{\infty} \mathrm{si}(n\pi\alpha) \, e^{-jn\pi\alpha} \, \delta_0(\omega - n\omega_\mathrm{s}).$$

Sampling – Part 7

Fourier transform of $v_{\mathrm{n}}(t)$*:*

Transforming the signal $v_{\mathrm{n}}(t)$ into the frequency domain leads to

$$v_{\mathrm{n}}(t) \;=\; v_{\mathrm{a}}(t)\, a_{\mathrm{n}}(t) \;\circ\!\!-\!\!\bullet\; V_{\mathrm{n}}(j\omega) \;=\; \frac{1}{2\pi} V_{\mathrm{a}}(j\omega) * A_{\mathrm{n}}(j\omega).$$

Using this result for computing the Fourier transform of $v_{\mathrm{n}}(t)$ leads to

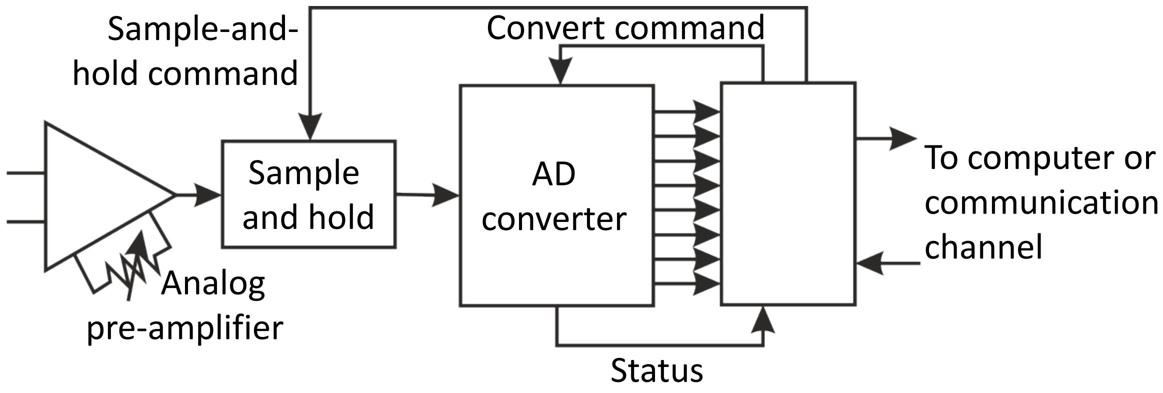
$$V_{\mathrm{n}}(j\omega) \;=\; \alpha \sum_{n=-\infty}^{\infty} \mathrm{si}(n\pi\alpha)\, e^{-jn\pi\alpha}\, V_{\mathrm{a}}\big(j(\omega - n\omega_{\mathrm{s}})\big).$$

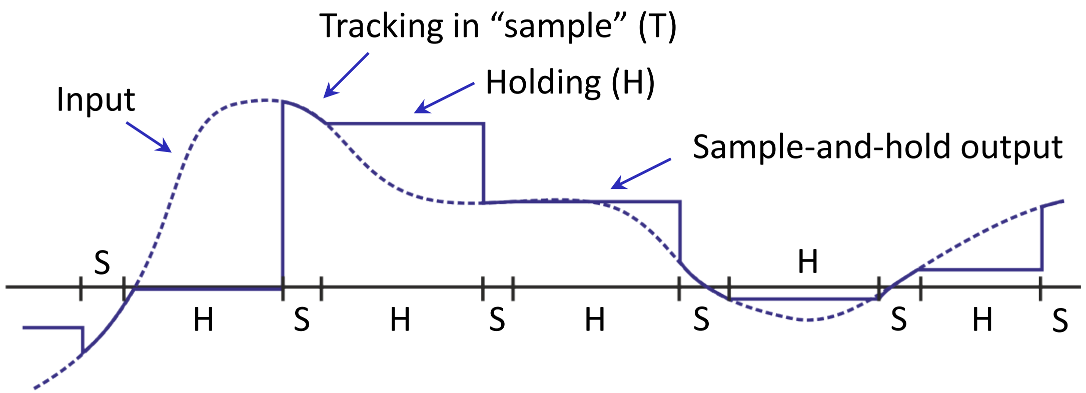
We can deduce the following:

- Compared to the result in the ideal sampling case here ***each repeated spectrum*** at the center frequency $n\omega_{\mathrm{s}}$ ***is weighted*** with the term $\alpha\,\mathrm{si}(n\pi\alpha)\, e^{-jn\pi\alpha}$.
- The energy $|V_{\mathrm{n}}(j\omega)|^2$ is proportional to α^2. This is problematic since in order to approximate the ideal case we would like to choose the parameter α as small as possible.

Sampling – Part 8

Sampling performed by a sample-and-hold (S/H) circuit:



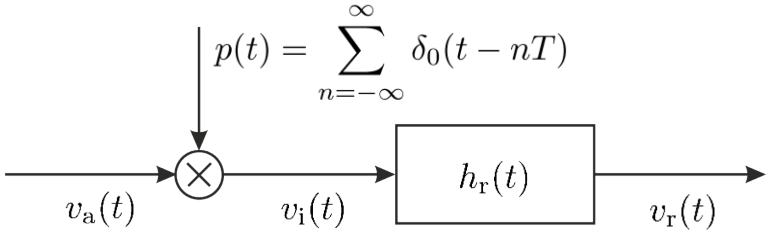


- The goal is to continuously sample the input signal and to hold that value constant as long as it takes for the AD converter to obtain its digital representation.
- Ideal S/H circuit introduces no distortion and can be modeled as an ideal sampler.
- As a result: drawbacks for the non-ideal sampling case can be avoided (all results for the ideal case hold here as well).

… Figure following [Proakis, Manolakis, 1996] …

Sampling Theorem – Part 1

Reconstruction of an ideally sampled signal by ideal lowpass filtering:



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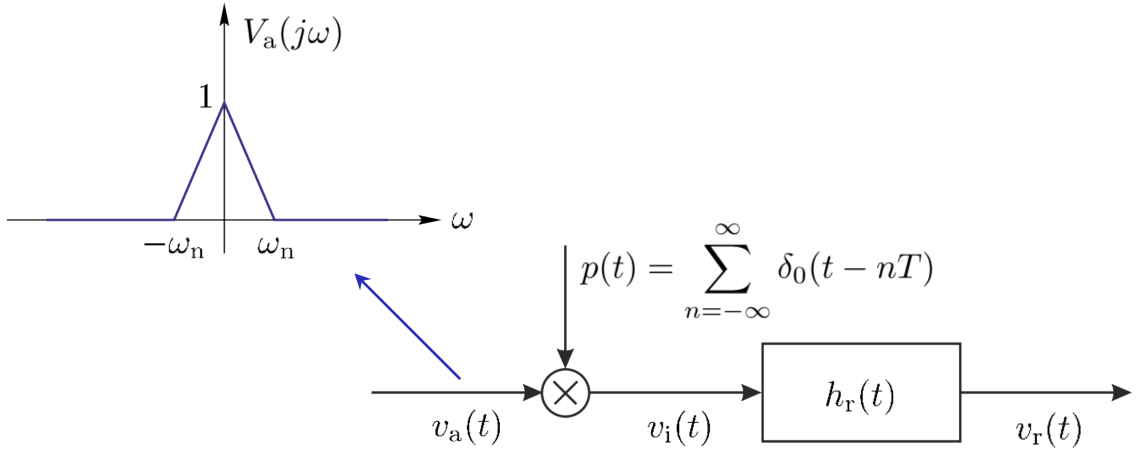

Sampling Theorem – Part 1

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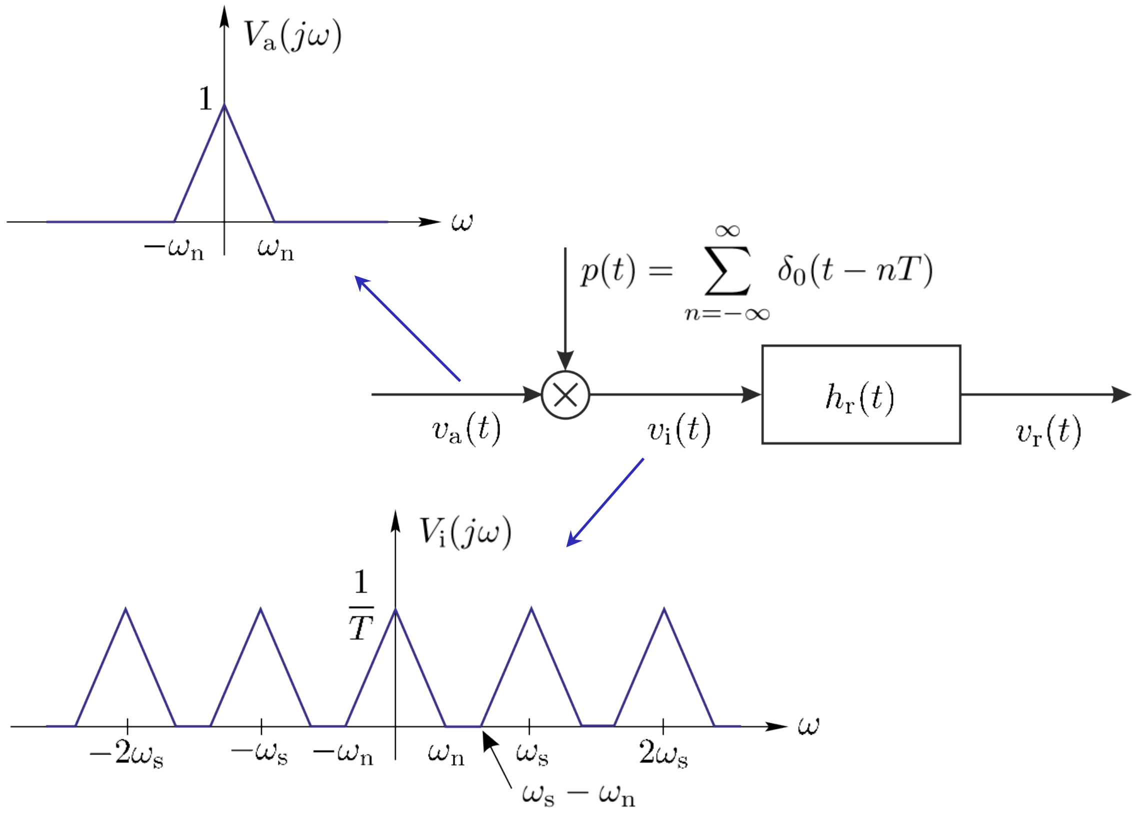

Sampling Theorem – Part 1

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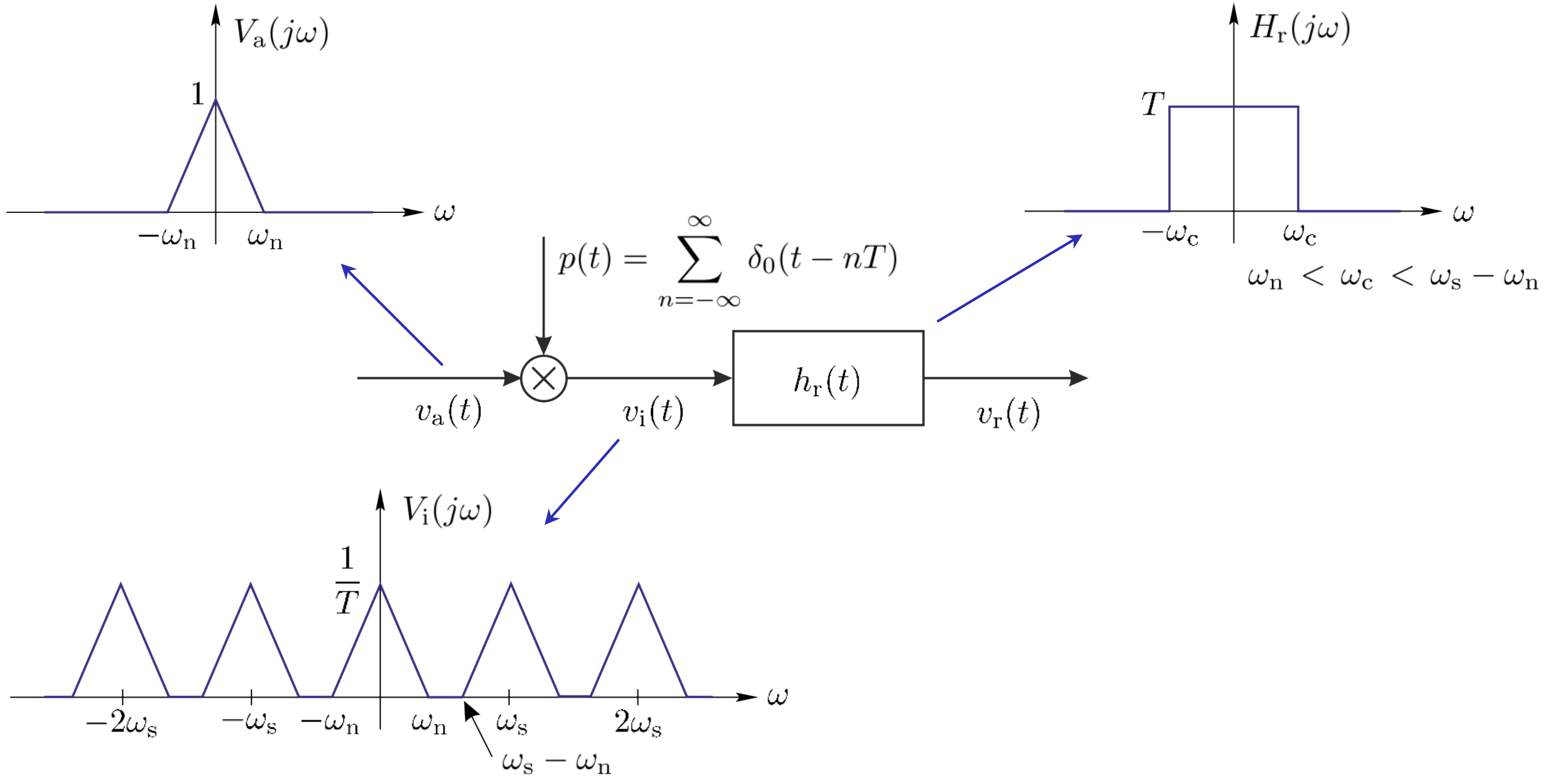

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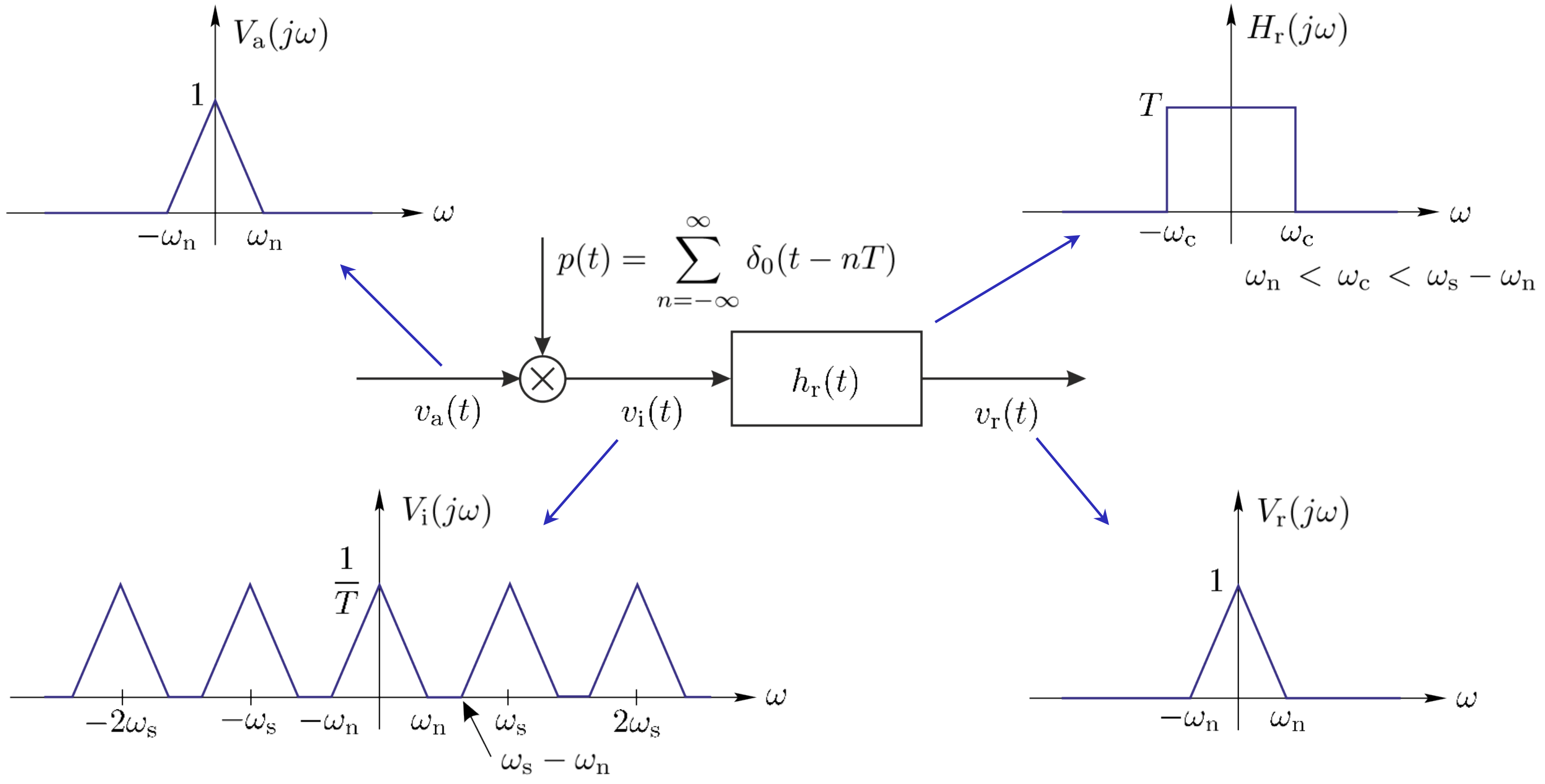
Sampling Theorem – Part 1

Reconstruction of an ideally sampled signal by ideal lowpass filtering:



Sampling Theorem – Part 2

Reconstruction of an ideally sampled signal by ideal lowpass filtering:

In order to get the input signal $v_{\mathrm{a}}(t)$ back after reconstruction, i.e. $V_{\mathrm{r}}(j\omega) = V_{\mathrm{a}}(j\omega)$, the conditions

$$\omega_{\mathrm{n}} < \frac{\omega_{\mathrm{s}}}{2} \qquad \text{and} \qquad \omega_{\mathrm{n}} < \omega_{\mathrm{c}} < \omega_{\mathrm{s}} - \omega_{\mathrm{n}}$$

have both to be satisfied. In this case, we get

$$V_{\mathrm{r}}(j\omega) = V_{\mathrm{a}}(j\omega) = V_{\mathrm{i}}(j\omega)\, H_{\mathrm{r}}(j\omega) \;\bullet\!\!-\!\!\!-\!\!\circ\; v_{\mathrm{r}}(r) = v_{\mathrm{a}}(r) = v_{\mathrm{i}}(t) * h_{\mathrm{r}}(t).$$

We now choose the cutoff frequency ω_{c} of the lowpass filter as $\omega_{\mathrm{c}} = \omega_{\mathrm{s}}/2$. This satisfies both conditions from above. An ideal lowpass filter (see before) can be described by its time and frequency response:

$$H_{\mathrm{r}}(j\omega) = T\,\mathrm{rect}\left(\frac{\omega}{\omega_{\mathrm{s}}}\right) \;\bullet\!\!-\!\!\!-\!\!\circ\; h_{\mathrm{r}}(t) = \mathrm{si}\left(\frac{\omega_{\mathrm{s}} t}{2}\right).$$

Sampling Theorem – Part 3

Reconstruction of an ideally sampled signal by ideal lowpass filtering:

Combining everything leads to:

$$v_\mathrm{a}(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} v_\mathrm{a}(nT)\,\delta_0(\tau - nT)\,\mathrm{si}\left(\frac{1}{2}\,\omega_\mathrm{s}(t-\tau)\right)\,d\tau$$

… changing the order of the summation and the integration …

$$= \sum_{n=-\infty}^{\infty} v_\mathrm{a}(nT) \int_{-\infty}^{\infty} \delta_0(\tau - nT)\,\mathrm{si}\left(\frac{1}{2}\,\omega_\mathrm{s}(t-\tau)\right)\,d\tau$$

… inserting the properties of the Dirac distribution …

$$= \sum_{n=-\infty}^{\infty} v_\mathrm{a}(nT)\,\mathrm{si}\left(\frac{1}{2}\,\omega_\mathrm{s}(t-nT)\right).$$

Result: *Every band-limited continuous-time signal* $v_\mathrm{a}(t)$ *with* $\omega_\mathrm{n} < \frac{\omega_\mathrm{s}}{2}$ *can be uniquely recovered from its samples* $v_\mathrm{a}(nT)$ *according to*

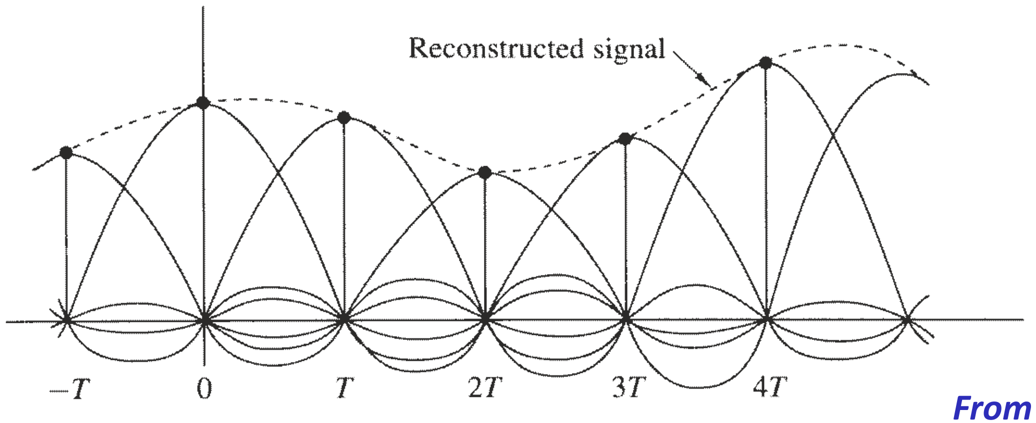
$$\boxed{v_\mathrm{a}(t) = \sum_{n=-\infty}^{\infty} v_\mathrm{a}(nT)\,\mathrm{si}\left(\frac{1}{2}\,\omega_\mathrm{s}(t-nT)\right).}$$

This is called the ideal interpolation formula, and the si-function is named ideal interpolation function!

Sampling Theorem – Part 4

Reconstruction of a continuous-time signal using ideal interpolation:

Basic principle:



From [Proakis, Manolakis, 1996]

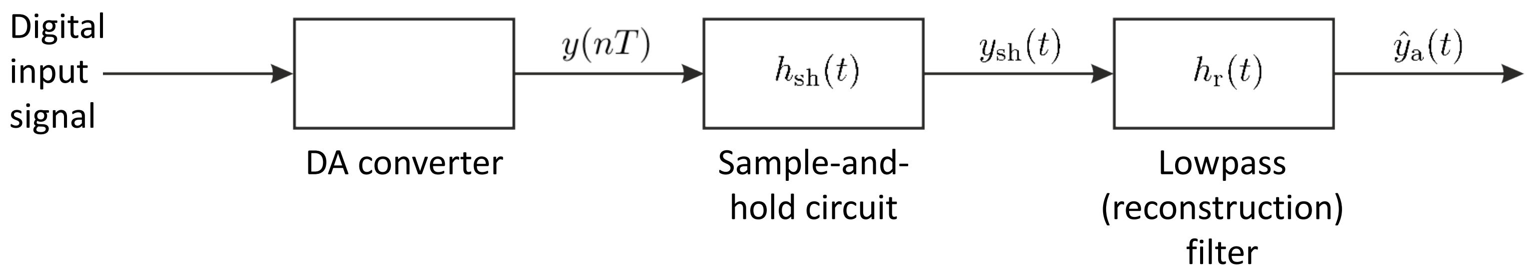
Anti-aliasing lowpass filtering:

- In order to avoid aliasing, the continuous-time input signal has to be bandlimited by means of an anti-aliasing lowpass-filter with cut-off frequency $\omega_{\mathrm{c}} \leq \omega_{\mathrm{s}}/2$ prior to sampling, such that the sampling theorem is satisfied.

Reconstruction with Sample-and-Hold Circuits – Part 1

Signal reconstruction:

In practice, a reconstruction is carried out by combining a DA converter with a sample-and-hold circuit, followed by a lowpass reconstruction filter.



- A DA converter accepts electrical signals that correspond to binary words as input, and delivers an output voltage or current being proportional to the value of the binary word for every clock interval nT.
- Often, the application on an input code word yields a ***high-amplitude transient*** at the output of the DA converter (”glitch”). Thus, the sample-and-hold circuit serves as a ”deglitcher”.

Reconstruction with Sample-and-Hold Circuits – Part 2

Analysis:

The ***sample and hold circuit*** has the ***impulse response***

$$h_{\mathrm{sh}}(t) = \mathrm{rect}\left(\frac{t - T/2}{T}\right),$$

which can be transformed into the ***frequency response***

$$H_{\mathrm{sh}}(j\omega) = T\,\mathrm{si}\left(\frac{\omega T}{2}\right) e^{-j\omega T/2}.$$

Consequences:

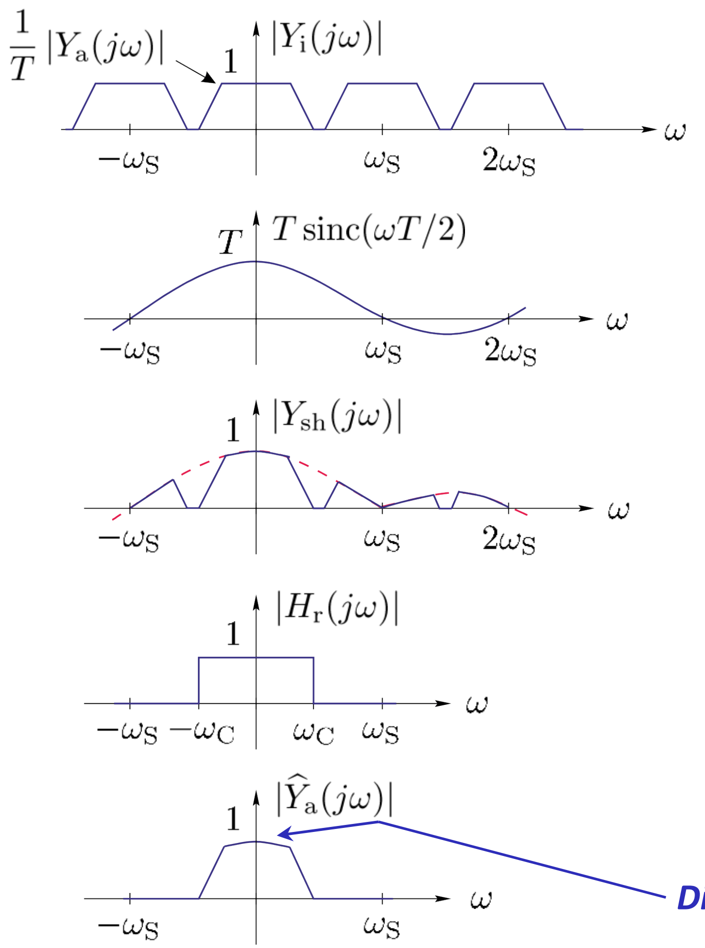
- No sharp cutoff frequency response characteristics. Thus, we have undesirable frequency components (above $\omega_{\mathrm{s}}/2$), which can be removed by passing $y_{\mathrm{sh}}(t)$ through a lowpass reconstruction filter $h_{\mathrm{r}}(t)$. This operation is equivalent to ***smoothing the staircase-like signal*** $y_{\mathrm{sh}}(t)$ after the sample-and-hold operation.
- When we now suppose that the reconstruction filter $h_{\mathrm{r}}(t)$ is an ideal lowpass with cutoff frequency $\omega_{\mathrm{c}} = \omega_{\mathrm{s}}/2$ and an amplification of one, the only distortion in the reconstructed signal $\hat{y}_{\mathrm{a}}(t)$ is due to the sample-and-hold operation:

$$\left|\widehat{Y}_{\mathrm{a}}(j\omega)\right| = \left|Y_{\mathrm{a}}(j\omega)\right|\,\left|\mathrm{si}(\omega T/2)\right|.$$

However, in case of non-ideal reconstruction filters we have additional distortions.

Reconstruction with Sample-and-Hold Circuits – Part 3

Spectral interpretation of the reconstruction process:



- Magnitude frequency response of the ideally sampled continuous-time signal.
- Frequency response of the sample-and-hold circuit (phase factor $e^{-j\omega T/2}$ omitted).
- Magnitude frequency response after the sample-and-hold circuit.
- Magnitude frequency response of the lowpass reconstruction filter.
- Magnitude frequency response of the reconstructed continuous-time signal.

Distortion due to the sinc function may be corrected by pre-biasing the reconstruction filter.

Quantization – Part 1

Basics:

Conversion carried out by an AD converter involves quantization of the sampled input signal and the encoding of the resulting binary representation.

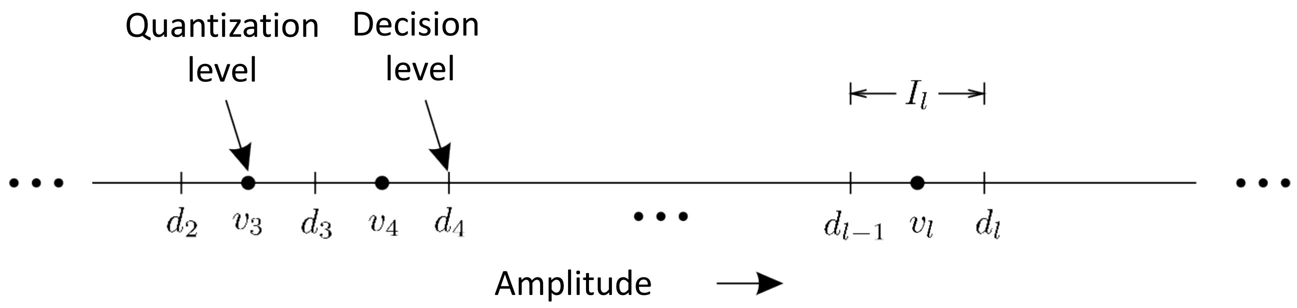
- Quantization is a ***non-linear*** and ***non-invertible*** process which realizes the mapping

$$v_a(nT) = v(n) \longrightarrow v_l \in \mathcal{I},$$

where the amplitude v_l is taken from a finite alphabet $\mathcal{I}$.

- The signal amplitude range is divided into L intervals I_l using the $L+1$ decision levels $d_0, d_1, ..., d_L$:

$$I_l = \{d_{l-1} < v(n) \leq d_l\}, \qquad l = 1, 2, ..., L.$$



Quantization – Part 2

Basics (continued):

- The mapping is denoted as

$$v_q(n) \;=\; Q\big[v(n)\big].$$

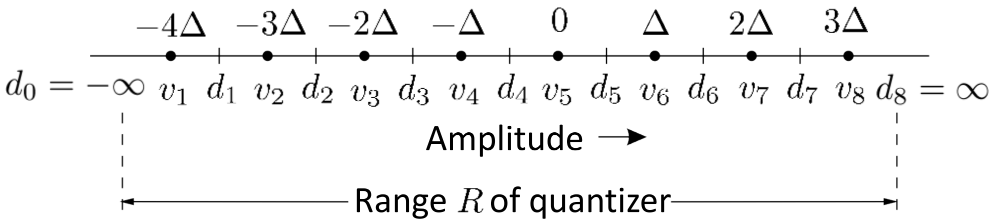
- ***Uniform*** or ***linear*** quantizers with constant quantization step size Δ are very often used in signal processing applications:

$$\Delta \;=\; v_l - v_{l-1} \;=\; \text{const.}, \qquad \forall\, l \;=\; 2,\, 3,\, \ldots,\, L.$$

- Two main types of linear quantizers:
    - ***Midtread*** quantizer - zero is assigned as a quantization level.
    - ***Midrise*** quantizer - zero is assigned as a decision level.

Example:

Midtread quantizer with $L \;=\; 8$ levels and range $R \;=\; 8\Delta$



Quantization – Part 3

Basics (continued):

- The ***quantization error*** signal (with respect to the unquantized signal) is defined as

  $$e_\mathrm{q}(n) \;=\; v(n) - v_\mathrm{q}(n).$$

  Without reaching the limits of the quantizer we get for the quantization error

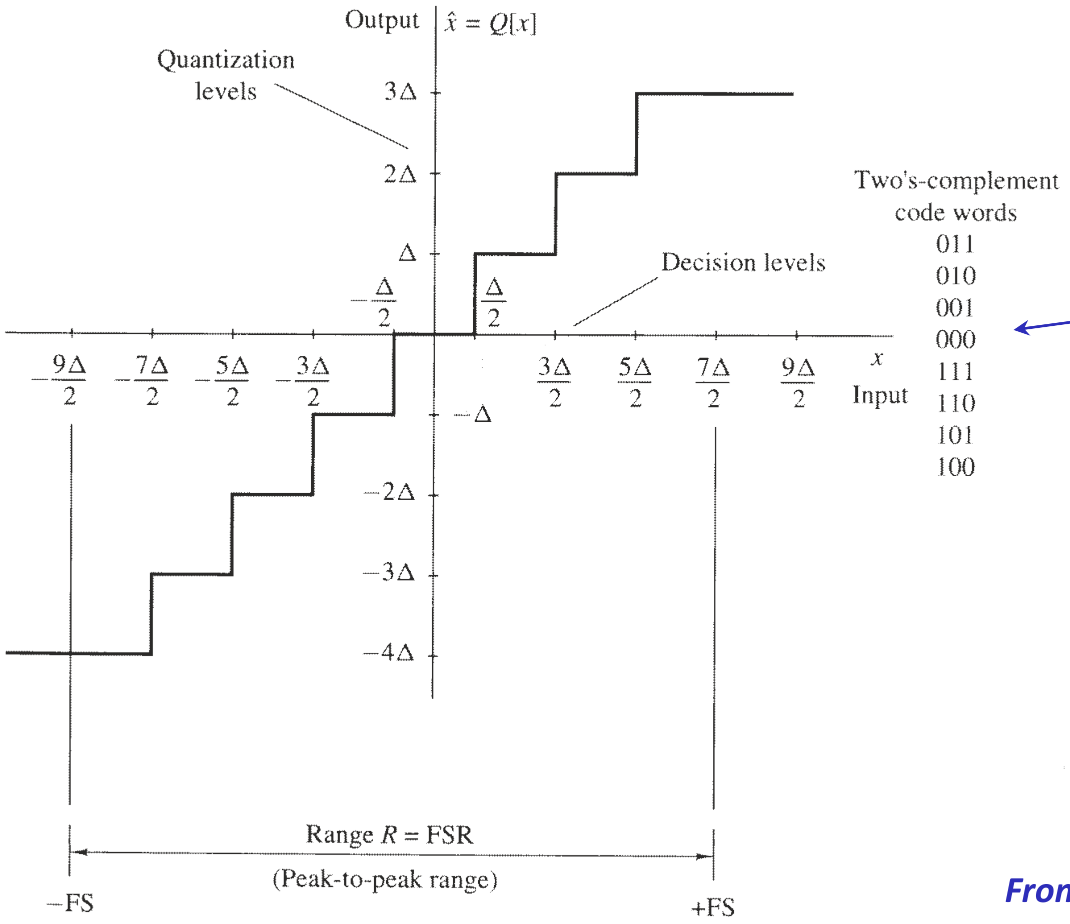
  $$-\frac{\Delta}{2} \;<\; e_\mathrm{q}(n) \;\leq\; \frac{\Delta}{2}.$$

  If the dynamic range of the input signal $v_\mathrm{max} - v_\mathrm{min}$ is larger than the range of the quantizer, the samples exceeding the quantizer range are clipped, which leads to

  $$\big|e_\mathrm{q}(n)\big| \;>\; \frac{\Delta}{2}.$$

Quantization – Part 4

Quantization characteristic for a midtread quantizer with $L = 8$ (3 bits):



"Mirrored" at zero and inverted

From [Proakis, Manolakis, 1996]

Quantization – Part 5

Coding:

The coding process in an AD converter assigns a binary number to each quantization level.

- With a word length of b bits we can represent $2^b \geq L$ binary numbers, which yields

$$b \geq \log_2(L).$$

- The step size or the ***resolution*** of the AD converter is given as

$$\Delta = \frac{R}{2^b},$$

with the range R of the quantizer.

- Two's complement representation is used in most fixed-point DSPs: A b-bit binary fraction

$$[\beta_0, \beta_1, \ldots, \beta_{b-1}]$$

with β_0 denoting the ***most significant bit*** (MSB) and β_{b-1} the ***least significant bit*** (LSB), represents the value

$$v = -\beta_0 + \sum_{l=1}^{b-1} \beta_l \, 2^{-l}.$$

Quantization – Part 6

Commonly used bipolar codes:

Number	*Positive reference*	*Negative reference*	*Sign and magnitude*	*Two's complement*	*Offset binary*	*One's complement*
+7	+7/8	-7/8	0 1 1 1	0 1 1 1	1 1 1 1	0 1 1 1
+6	+3/4	-3/4	0 1 1 0	0 1 1 0	1 1 1 0	0 1 1 0
+5	+5/8	-5/8	0 1 0 1	0 1 0 1	1 1 0 1	0 1 0 1
+4	+1/2	-1/2	0 1 0 0	0 1 0 0	1 1 0 0	0 1 0 0
+3	+3/8	-3/8	0 0 1 1	0 0 1 1	1 0 1 1	0 0 1 1
+2	+1/4	-1/4	0 0 1 0	0 0 1 0	1 0 1 0	0 0 1 0
+1	+1/8	-1/8	0 0 0 1	0 0 0 1	1 0 0 1	0 0 0 1
+0	0+	0-	0 0 0 0	0 0 0 0	1 0 0 0	0 0 0 0
-0	0-	0+	1 0 0 0	(0 0 0 0)	(1 0 0 0)	1 1 1 1
-1	-1/8	+1/8	1 0 0 1	1 1 1 1	0 1 1 1	1 1 1 0
-2	-1/4	+1/4	1 0 1 0	1 1 1 0	0 1 1 0	1 1 0 1
-3	-3/8	+3/8	1 0 1 1	1 1 0 1	0 1 0 1	1 1 0 0
-4	-1/1	+1/1	1 1 0 0	1 1 0 0	0 1 0 0	1 0 1 1
-5	-5/8	+5/8	1 1 0 1	1 0 1 1	0 0 1 1	1 0 1 0
-6	-3/4	+3/4	1 1 1 0	1 0 1 0	0 0 1 0	1 0 0 1
-7	-7/8	+7/8	1 1 1 1	1 0 0 1	0 0 0 1	1 0 0 0
-8	-1	+1		1 0 0 0	0 0 0 0	

Quantization – Part 7

Commonly used bipolar codes:

Conversions to integer numbers:

- Sign and magnitude:

$$v \;=\; (-1)^{\beta_0} \cdot \sum_{l=1}^{b-1} \beta_l \, 2^{-l}. \qquad 1011 \;\Longrightarrow\; -(1)^1 \cdot \left(0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}\right) \;=\; -\frac{3}{8}$$

- Two's complement:

$$v \;=\; -\beta_0 + \sum_{l=1}^{b-1} \beta_l \, 2^{-l}. \qquad 1011 \;\Longrightarrow\; -1 + \left(0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}\right) \;=\; -\frac{5}{8}$$

- Offset binary:

$$v \;=\; (\beta_0 - 1) + \sum_{l=1}^{b-1} \beta_l \, 2^{-l}. \qquad 1011 \;\Longrightarrow\; (1-1) + \left(0 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}\right) \;=\; \frac{3}{8}$$

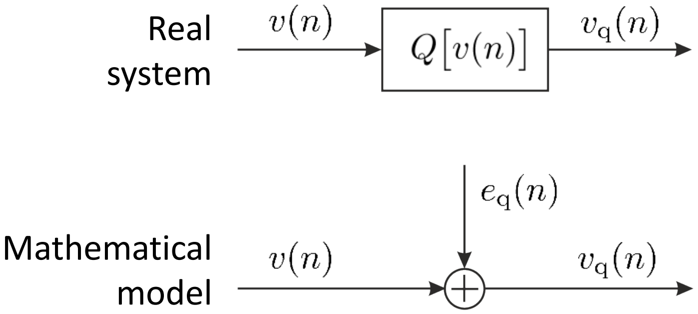
- One's complement:

$$v \;=\; \beta_0 \left(-1 + 2^{-b+1}\right) + \sum_{l=1}^{b-1} \beta_l \, 2^{-l}. \qquad 1011 \;\Longrightarrow\; -\frac{7}{8} + \left(\cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}\right) \;=\; -\frac{1}{2}$$

Quantization – Part 8

Quantization error:

The quantization error is modeled as noise, which is added to the unquantized signal:



Assumptions:

- The quantization error range $-\Delta/2 < e_{\mathrm{q}}(n) \leq \Delta/2$.
- The error sequence $e_{\mathrm{q}}(n)$ is modeled as a stationary white noise.
- The error sequence $e_{\mathrm{q}}(n)$ is uncorrelated with the signal sequence $v(n)$.
- The signal sequence is assumed to have zero mean.

The assumptions do not hold in general, but they are fairly well satisfied for large quantizer word lengths b .

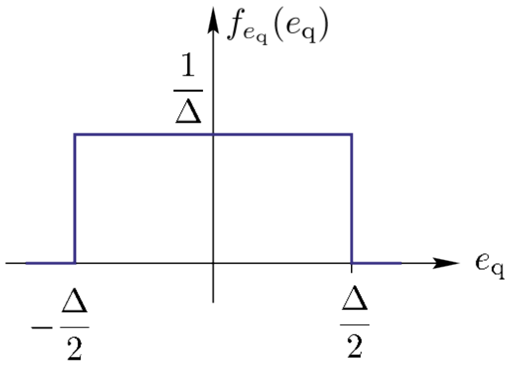
Quantization – Part 9

Quantization error (continued):

The effect of quantization errors (or quantization noise) on the resulting signal $v_{\mathrm{q}}(n)$ can be evaluated in terms of the signal- to-noise ratio (SNR) in decibels (dB):

$$SNR/\mathrm{dB} \;=\; 10\,\log_{10}\left(\frac{P_v}{P_n}\right),$$

where P_v denotes the signal power and P_n the power of the quantization noise. Quantization noise is assumed to be uniformly distributed in the range $(-\Delta/2,\,\Delta/2)$:



The variance of the quantization noise can be computed as

$$P_n \;=\; \sigma_{e_{\mathrm{q}}}^2 \;=\; \int\limits_{-\Delta/2}^{\Delta/2} e^2\,f_e(e)\,de \;=\; \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2\,de \;=\; \frac{\Delta^2}{12}.$$

Inserting our definition of the resolution of an AD converter yields

$$\sigma_{e_{\mathrm{q}}}^2 \;=\; \frac{2^{-2b}\,R^2}{12}.$$

Quantization – Part 10

Quantization error (continued):

Inserting the result from the last slide into our SNR formula yields

$$SNR/\mathrm{dB} \;=\; 10\log_{10}\left(\frac{\sigma_v^2}{\sigma_{e_\mathrm{q}}^2}\right) \;=\; 10\log_{10}\left(\frac{12\,2^{2b}\,\sigma_v^2}{R^2}\right) \;=\; 6.02\,b + 10.8 - 20\log_{10}\left(\frac{R}{\sigma_v}\right).$$

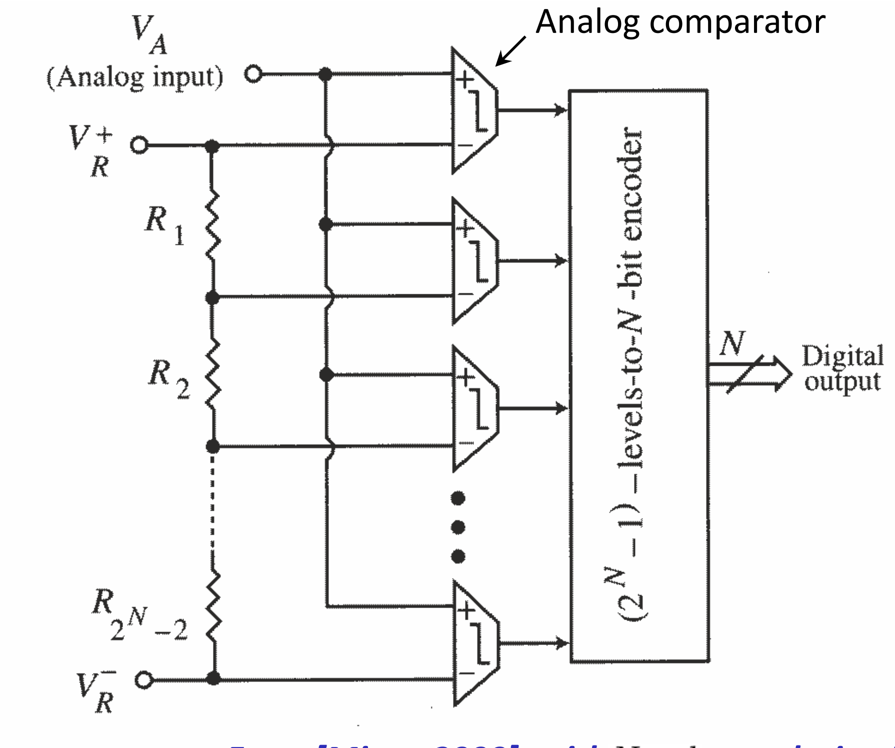
Remarks:

- σ_v denotes here the root-mean-square (RMS) amplitude of the signal $v(t)$.
- If σ_v is too small, the SNR drops as well.
- If σ_v is too large, the range R of the AD converter might be exceeded.

The signal amplitude has to be matched carefully to the range of the AD converter!

Analog-to-Digital Converter Realizations – Part 1

Flash AD converters:



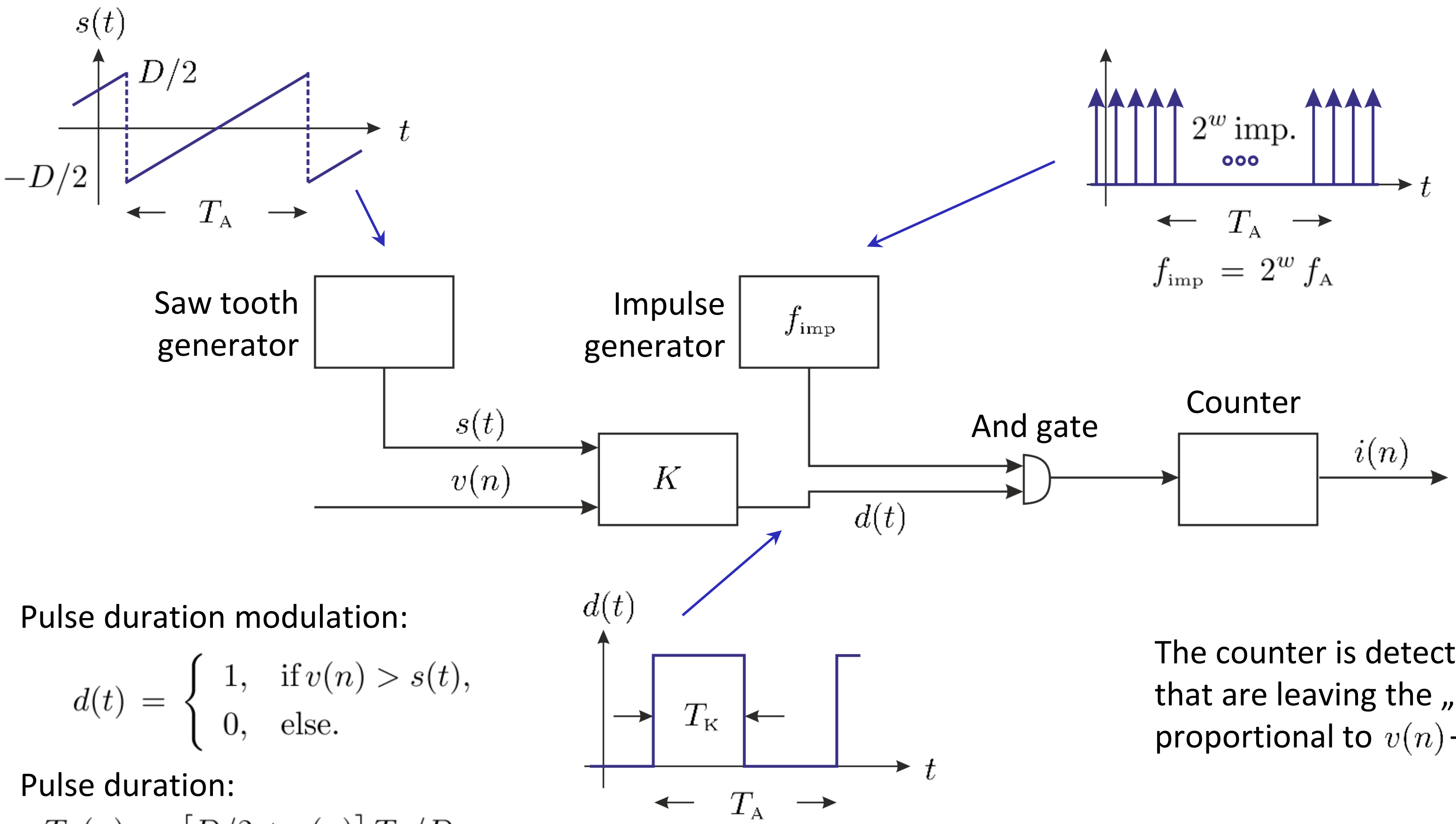
From [Mitra, 2000], with $N = b$: resolution in bits

- Analog input voltage V_{A} is simultaneously compared with a set of $2^b - 1$ separated reference voltage levels by means of a set of $2^b - 1$ analog comparators. The locations of the comparator circuits indicate the range of the input voltage.
- All output bits are developed simultaneously. Thus, a very fast conversion is possible.
- The hardware requirements for this type of converter increase exponentially with an increase in resolution.

Flash converters are used for low-resultion and high-speed conversion applications.

Analog-to-Digital Converter Realizations – Part 2

Serial AD converters:



Pulse duration modulation:

$$d(t) = \begin{cases} 1, & \text{if } v(n) > s(t), \\ 0, & \text{else.} \end{cases}$$

Pulse duration:

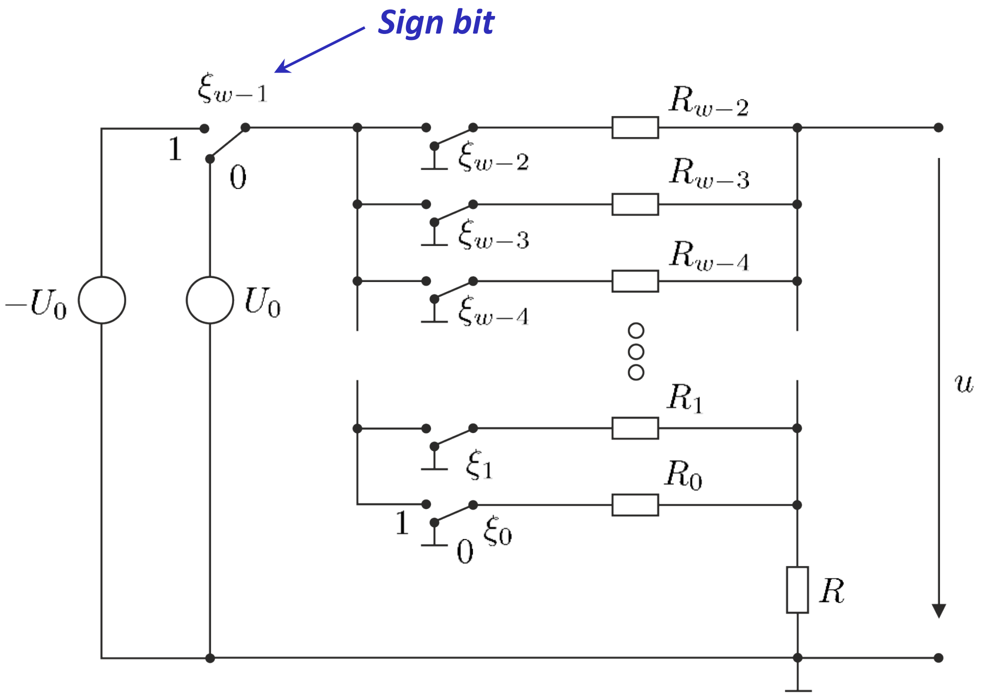
$$T_K(n) = \left[D/2 + v(n)\right] T_A/D$$

The counter is detecting the amount of ones T_K that are leaving the „and gate“. This amount is proportional to $v(n)$ – it results in $i(n)$.

Digital-to-Analog Converter Realizations – Part 1

Possible realization:

Switches, that are „controlled“ by bits:



Analysis for $\xi_\mu = 1,\ \xi_\nu = 0\ \forall \nu \neq \mu$:

$$u = u_\mu = \pm \frac{U_0}{R_\mu\, G_\Sigma} \sim \frac{1}{R_\mu},$$

actually $1 - 2\xi_{w-1}$

with

$$G_\Sigma = \sum_{\nu=0}^{w-2} \frac{1}{R_\nu}.$$

(conductance)

Due to linearity we get:

$$u = \pm \frac{U_0}{G_\Sigma} \sum_{\nu=0}^{w-2} \xi_\nu \frac{1}{R_\nu} = v_Q.$$

This results in (if we neglect the offset):

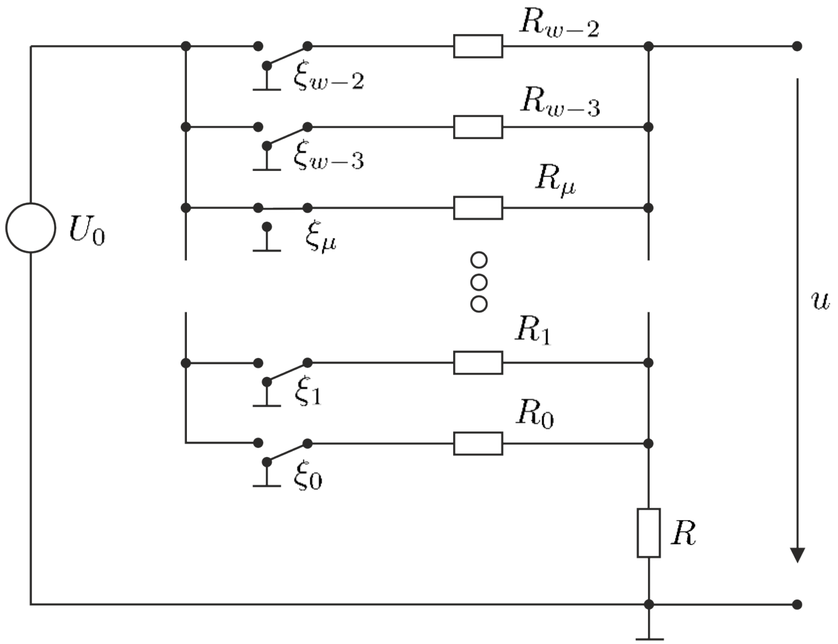
$$R_\nu\, G_\Sigma = 2^{-\nu},\ \nu \in \{0, ..., w-2\}.$$

It‘s important to meet the accuracy requirements for the resistors (especially due to the large range of values).

Digital-to-Analog Converter Realizations – Part 2

Circuit analysis:

Understanding of the circuit if only a single bit is set to one on the one hand and for an arbitrary bit setup on the other hand at the blackboard. However, for reason of simplicity we assume that the sign bit is set such, that we have a positive output voltage.



Details at the blackboard!

Partner Work – Part 1

Questions – Part 1:

Partner work – Please think about the following questions and try to find answers
(first group discussions, afterwards broad discussion in the whole group).

- What are the necessary components if you want to replace an analog system by a digital one?
  What to you need to know about the involved systems and signals?

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- What kind of converter type would you use for different applications?
  Which system properties are important to make this decision?

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Partner Work – Part 2

Questions – Part 2:

- What is meant by "digital"?

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- Can you think of applications where analog processing would be beneficial compared to digital processing? Give examples for such applications!

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- What happens if you neglect anti-aliasing filtering before sampling?

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Summary

- Introduction
- ***Digital processing of continuous-time signals***
  - ***Sampling and sampling theorem (repetition)***
  - ***Quantization***
  - ***Analog-to-digital (AD) and digital-to-analog (DA) conversion***
- Efficient FIR filter structures
- DFT and FFT
- Digital filters
- Multi-rate digital signal processing