TECHNISCHE FAKULTÄT DER CHRISTIAN-ALBRECHTS-UNIVERSITÄT ZU KIEL

DIGITALE SIGNALVERARBEITUNG UND SYSTEMTHEORIE

Signal and Systems II

Exam Summer 2022

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	05.09.2022
Name:	____________________
Matriculation Number:	____________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ____________________

Marking

Problem	1	2	3
Points	/33	/33	/33

Total number of points: ____________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ____________ Signature: ____________________

Signal and Systems II

Exam Summer 2022

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	OS40, R. 201
Date:	05.09.2022
Begin:	09:00 h
Reading Time:	10 Minuten
Working Time:	90 Minuten

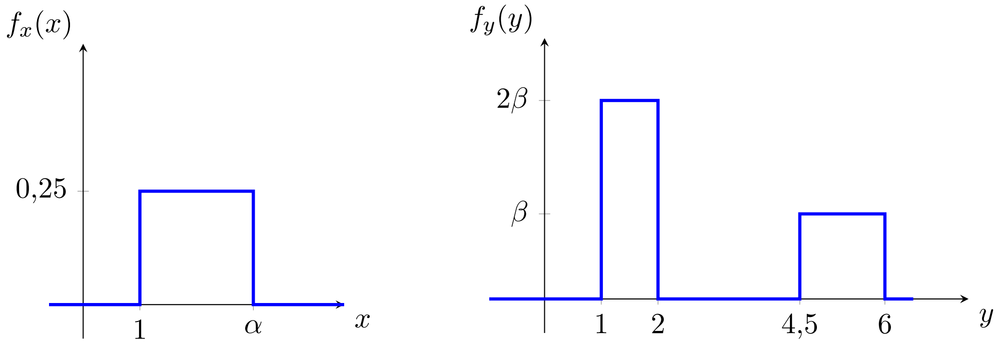
Notes

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Aufgabe 1 (33 Punkte)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

The stochastically independent, piecewise uniformly distributed random variables x and y with the probability densities $f_x(x)$ and $f_y(y)$ are given in graphical form:



(a) Specify the probability densities $f_x(x)$ and $f_y(y)$ depending on α and β. (3 P)

$$f_x(x) = \begin{cases} \frac{1}{4}, & \text{for } 1 \leq x < \alpha, \\ 0, & \text{else}, \end{cases}$$

$$f_y(y) = \begin{cases} 2\beta, & \text{for } 1 \leq y < 2, \\ \beta, & \text{for } 4{,}5 \leq y < 6, \\ 0, & \text{else}. \end{cases}$$

(b) Determine α and β. (3 P)
$\alpha = 5,\ \beta = \frac{2}{7}$

(c) Calculate the composite density $f_{x,y}(x, y)$ using your results from *(a)* and *(b)*. (3 P)
It applies $f_{x,y} = f_x(x) f_y(y)$.

$$f_{x,y}(x, y) = \begin{cases} \frac{1}{7}, & \text{for } 1 \leq x < 5 \wedge\ 1 \leq y < 2, \\ \frac{1}{14}, & \text{for } 1 \leq x < 5 \wedge\ 4{,}5 \leq y < 6, \\ 0, & \text{else}. \end{cases}$$

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

Given is the real random variable v_1 with the distribution function $F_{v_1}(v_1)$

$$F_{v_1}(v_1) = \begin{cases} a, & v_1 < b, \\ \frac{1}{4}v_1 + \frac{1}{4}, & b \leq v_1 < 3, \\ 6c, & 3 \leq v_1, \end{cases}$$

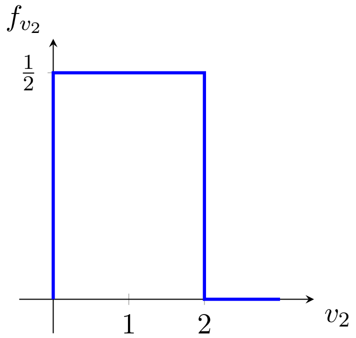
which is defined using the real constants a, b and c.

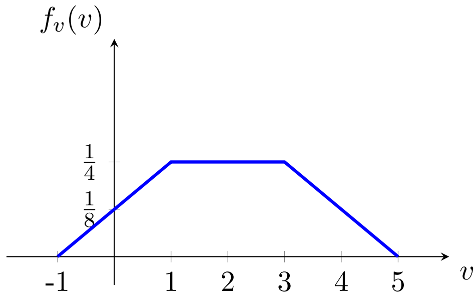
(d) Determine the constants a, b and c. Explain! (3 P)
$a = 0, b = -1, c = \frac{1}{6}$

(e) Calculate the associated probability density with the results from *(d)*. (3 P)

$$f_{v_1}(v_1) = \begin{cases} \frac{1}{4}, & -1 \leq v_1 < 3, \\ 0, & \text{else.} \end{cases}$$

(f) Given now also the stochastically independent process v_2, where v_2 is uniformly distributed on the interval [0,2]. Sketch the probability density of the process v_2, as well as the sum process $v = v_1 + v_2$. (5 P)





(g) Calculate the first moment and the second central moment of v. (4 P)
The mean can be read from the distribution density function or by

$$\begin{aligned} m_v &= m_{v_1} + m_{v_2} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

be calculated. The variance can be determined by

$$\sigma_v^2 = \sigma_{v_1}^2 + \sigma_{v_2}^2 + 2\psi_{v_1,v_2}(k).$$

Since both processes are statistically independent ($\psi_{v_1,v_2}(k) = 0$), it follows

$$\sigma_v^2 = \frac{4}{3} + \frac{1}{3} = \frac{5}{3},$$

because for the variance of uniform distributions

$$\sigma_{v_i}^2 = \frac{1}{12}(v_{i_{max}} - v_{i_{min}})^2$$

applies.

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

The following composite probability density is given:

$$f_{a,b}(a,b) = \begin{cases} \frac{1}{9}, & \text{for } 3 \leq a < 5 \wedge \; 1 \leq b < 4, \\ \frac{1}{12}, & \text{for } 0 \leq a < 2 \wedge \; -1 \leq b < 1, \\ 0, & \text{else.} \end{cases}$$

(h) Calculate the marginal density $f_a(a)$. (3 P)

$$f_a(a) = \begin{cases} \int_1^4 \frac{1}{9} db, & \text{for } 3 \leq a < 5, \\ \int_{-1}^1 \frac{1}{12} db, & \text{for } 0 \leq a < 2, \\ 0, & \text{else,} \end{cases}$$
$$= \begin{cases} \frac{1}{3}, & \text{for } 3 \leq a < 5, \\ \frac{1}{6}, & \text{for } 0 \leq a < 2, \\ 0, & \text{else.} \end{cases}$$

(i) Calculate the corresponding distribution function with the result from *(h)*. (3 P)

$$F_a(a) = \begin{cases} 0, & a < 0, \\ \frac{1}{6}a, & 0 \leq a < 2, \\ \frac{1}{3}, & 2 \leq a < 3, \\ \frac{1}{3}a - \frac{2}{3}, & 3 \leq a < 5, \\ 1, & 5 \leq a. \end{cases}$$

(j) Calculate $f_c(c = 9)$ of the resulting density if the mapping rule $c = 2a + 5$ holds. (3 P)
Note: Perform a point density transformation.

It generally applies

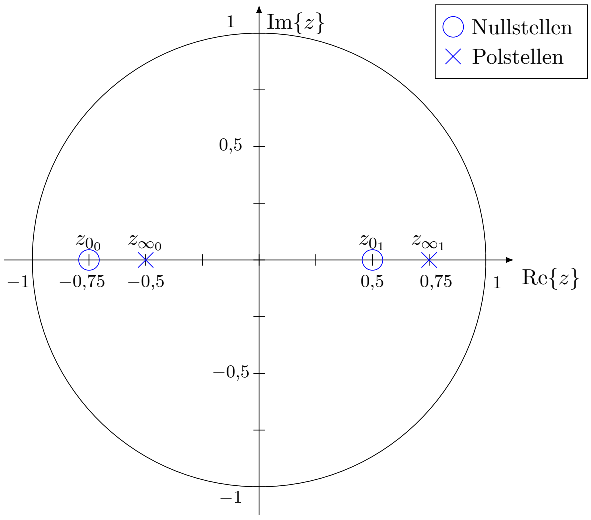
$$f_c(c_0) = \sum_{i=0}^{N} \frac{f_a(a_{0,i})}{|g'(a_{0,i})|},$$

where $g(a) = c$ is the mapping rule from a to c. Here $N = 1$. Therefore, the following development point is necessary: $c_0 = 2$.

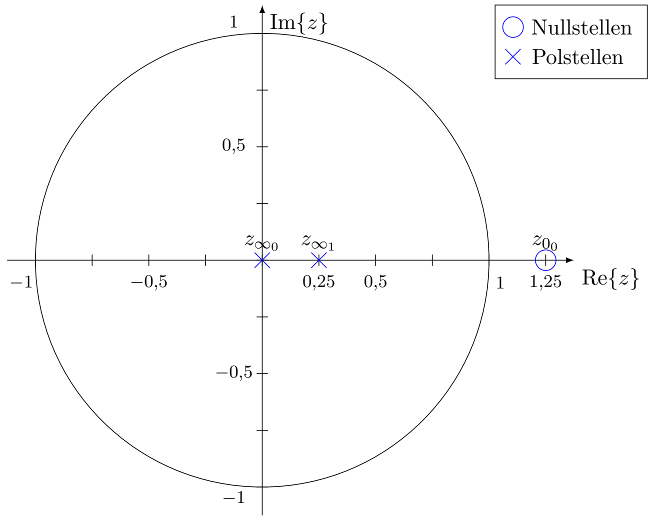
$$f_c(c = 9) = \frac{f_a(2)}{|g'(2)|} = \frac{f_a(2)}{2} = 0$$

Aufgabe 2 (33 Punkte)

Part 1 *This part of the task can be solved independently of Part 2.* The following systems are given, described using pole-zero diagrams:



(a) System $H_I(z)$



(b) System $H_{II}(z)$

(a) Determine both transfer functions $H_I(z)$ and $H_{II}(z)$ and give them in polynomial representation in which z is the largest has the first power of 0. (6 P)

$$H_I(z) = K_I \, \frac{(z - \frac{1}{2})(z + \frac{3}{4})}{(z + \frac{1}{2})(z - \frac{3}{4})}$$
$$= K_I \, \frac{z^2 + \frac{1}{4}z - \frac{3}{8}}{z^2 - \frac{1}{4}z - \frac{3}{8th}}$$
$$= K_I \, \frac{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}$$
$$H_{II}(z) = K_{II} \, \frac{(z - \frac{5}{4})}{z(z - \frac{1}{4})}$$
$$= K_{II} \, \frac{z - \frac{5}{4}}{z^2 - \frac{1}{4}z}$$
$$= K_{II} \, \frac{z^{-1} - \frac{5}{4}z^{-2}}{1 - \frac{1}{4}z^{-1}}$$

(b) Are the systems $H_I(z)$ and $H_{II}(z)$ respectively: (4,5 P)

(i) minimal phase?

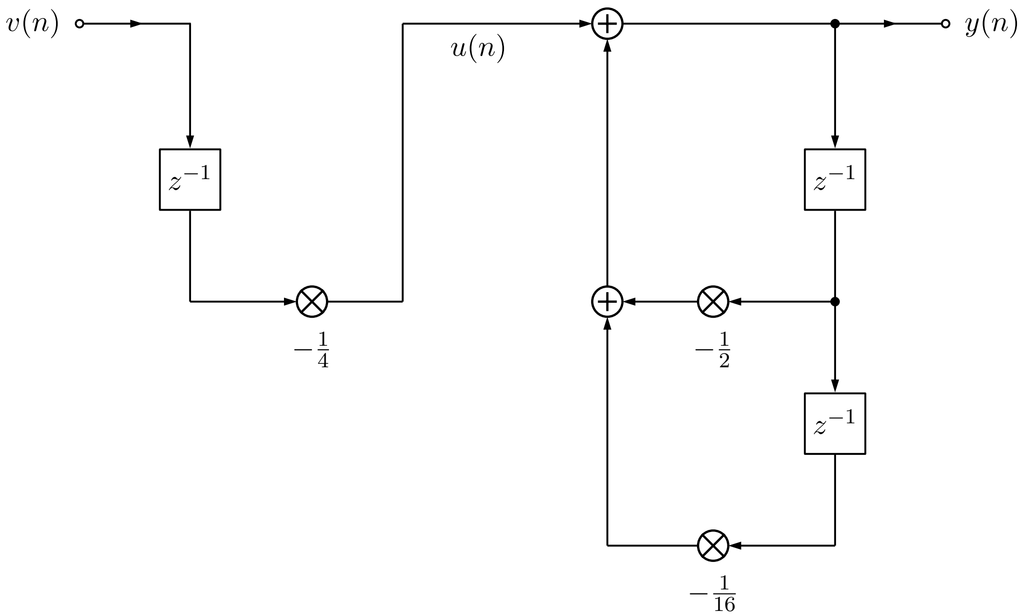
(ii) stable?

(iii) causal?

Give reasons for your answers.
$H_I(z)$ is minimum phase because all zeros are in the unit circle
$H_{II}(z)$ is not in minimum phase because not all zeros are in the unit circle
$H_I(z)$ is stable because all poles lie in the unit circle
$H_{II}(z)$ is stable because all poles lie in the unit circle
$H_I(z)$ is causal because the denominator degree is equal to the numerator degree
$H_{II}(z)$ is causal because the denominator degree is greater than the numerator degree

Part 2 *This part of the task can be solved independently of Part 1.*

The following block diagram is given. Let all memories for $n < 0$ be initialized with 0.



(c) What form does the block diagram above have? (1 P)
Direct form I

(d) Which part of the block diagram above has an FIR characteristic and which part has an IIR characteristic? Explain! (4 P)

- **Part 1** with input $v(n)$ and output $u(n)$ (transversal structure) has an *FIR* characteristic, since there is no feedback of the output signal $u(n)$ occurs.
- **Part 2** with input $u(n)$ and output $y(n)$ (recursive structure) has an *IIR* characteristic, since a feedback of the output signal $y(n)$ occurs.

Furthermore, the input signal is now defined as follows:

$$v(n) = [\gamma_{-1}(n-3) - \gamma_0(n)] \cdot \gamma_0(n+1) + 2\gamma_0(n) - 2\gamma_{-1}(n-2) + \gamma_0(n-3) + \gamma_{-1}(n-5) + 2\gamma_{-1}(n-7) + \gamma_{-1}(n-7) \cdot \gamma_0(n-8) - \gamma_{-1}(n-9).$$

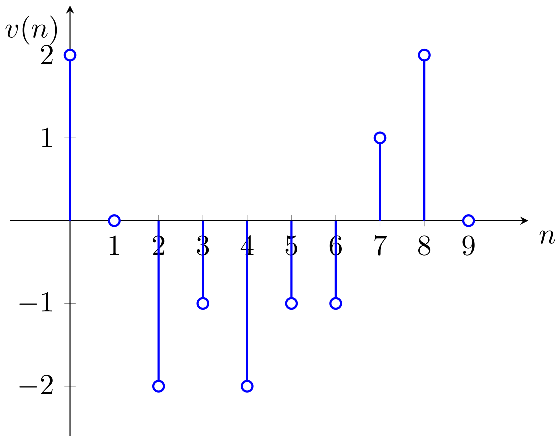
(e) Draw $v(n)$ for $0 \leq n < 10$. (4.5 P)

$v(n)$ can be simplified by the masking property of the momentum function $\gamma_0(n)$:

$v(n) = 2\gamma_0(n) - 2\gamma_{-1}(n-2) + \gamma_0(n-3) + \gamma_{-1}(n-5) + 2\gamma_{-1}(n-7) + \gamma_0(n-8) - \gamma_{-1}(n-9)$

For $0 \leq n < 10$ we get for $\boldsymbol{v} = [v(0), v(1), v(2), \ldots, v(9)]^{\mathrm{T}}$ the values:

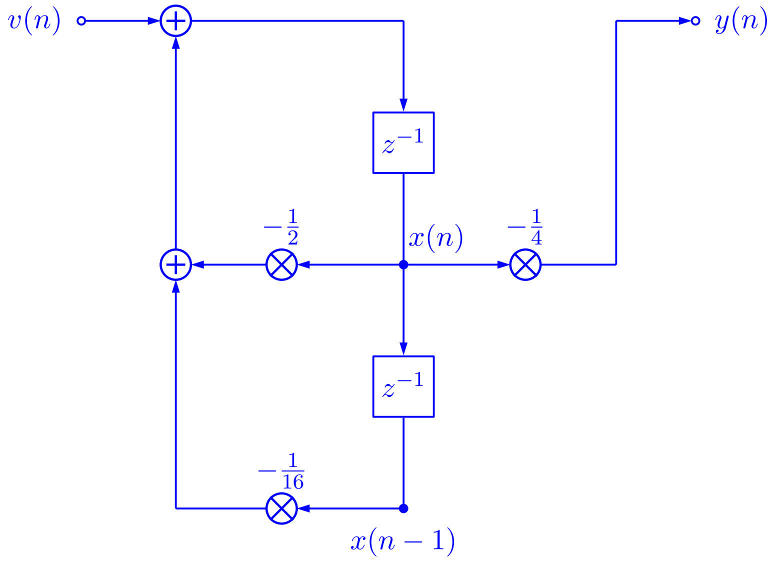
$$\boldsymbol{v} = [2,\ 0,\ -2,\ -1,\ -2,\ -1,\ -1,\ 1,\ 2,\ 0]^{\mathrm{T}}$$



(f) Give the sequence $u(n)$ depending on $v(n)$. (1 P)
From the block diagram $u(n)$ results in:

$$u(n) = -\frac{1}{4}\, v(n-1).$$

(g) Draw the direct form II and state an advantage over the form used above. (4 P)
The direct form II requires less state memory than the direct form I (in this case two) and results in:



(h) How many states does the system have? Mark these in your solution from *(g)*. (1 P)
Two states (for labeling see solution for previous part of the exercise, $x(n), x(n-1)$).

(i) Determine the impulse response for the entire system. (7 P)

$$y(n) = u(n) - \frac{1}{2}y(n-1) - \frac{1}{16}y(n-2)$$

$$y(n) + \frac{1}{2}y(n-1) + \frac{1}{16}y(n-2) = -\frac{1}{4}v(n-1)$$

In the z area follows:

$$\left(1 + \frac{1}{2}z^{-1} + \frac{1}{16}z^{-2}\right)\ Y(z) = \left(-\frac{1}{4}z^{-1}\right)\ V(z).$$

And thus:

$$\begin{aligned} H(z) &= \frac{Y(z)}{V(z)} \\ &= \frac{-\frac{1}{4}z^{-1}}{1 + \frac{1}{2}z^{-1} + \frac{1}{16}z^{-2}} \\ &= \frac{-\frac{1}{4}z}{z^2 + \frac{1}{2}z + \frac{1}{16}} \\ &= \frac{-\frac{1}{4}z}{\left(z + \frac{1}{4}\right)^2}. \end{aligned}$$

By applying the known correspondence of the z-transformation from the formula collection of the lecture ($x(n) = na^n\gamma_{-1}(n)$, $X(z) = \frac{za}{(z-a)^2}$) follows:

$$h(n) = n\ \left(-\frac{1}{4}\right)^n\ \gamma_{-1}(n)$$

Aufgabe 3 (33 Punkte)

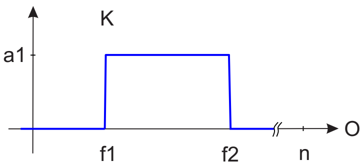
Part 1 *This part of the task can be solved independently of parts 2 and 3.*

(a) What purpose does the modulation of a signal serve, according to the lecture? What types of modulation do you know? Name at least three types of modulation. (2 P)
The purpose of modulation is to adapt the signal spectrum to the frequency range of the transmission, storage or processing medium to be used. Amplitude, phase and frequency modulation.

(b) Give a practical example of an additive and a multiplicative channel interference, which influences the signal-to-noise ratio, especially in amplitude modulation. (2 P)
An example of an additive disturbance is mains hum at 50 Hz; a multiplicative disturbance would be, for example, an amplifier fluctuation.

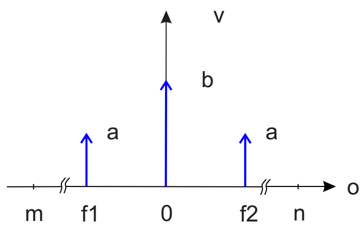
Part 2 *This part of the task can be solved independently of parts 1 and 3.*

A signal $v(n) = 2\cos^2\left(\frac{\Omega_0}{2}n\right)$ is to be transmitted over a channel with a real impulse response and the magnitude frequency response shown below. Assume that $\Omega_0 < \Omega_1$.



(c) Determine the Fourier transform $V\left(e^{j\Omega}\right)$ $\bullet\!\!-\!\!\circ$ $v(n)$ and sketch $V\left(e^{j\Omega}\right)$ in the range from $-\pi$ to π. (3 P)

$$V\left(e^{j\Omega}\right) = \pi\left[\sum_{\lambda=-\infty}^{\infty} 2\delta_0(\Omega - 2\pi\lambda) + \delta_0(\Omega + \Omega_0 - 2\pi\lambda) + \delta_0(\Omega - \Omega_0 - 2\pi\lambda)\right].$$



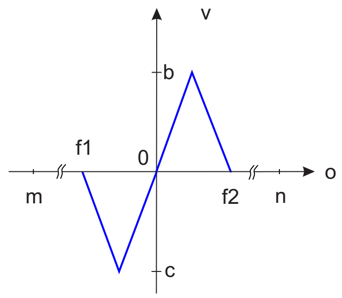
(d) Define the frequency Ω_0 such that the signal $v(n)$ would be transmittable across the channel in terms of its bandwidth using linear modulation. (1 P)

$$\Omega_0 \leq \frac{\Omega_2 - \Omega_1}{2}$$

(e) Define the range for the carrier frequency Ω_T so that the modulated or upmixed version of $v(n)$ lies in the passband of the channel. Assume that Ω_0 is in the range required by *(f)*. (1 P)

$$\Omega_1 + \Omega_0 \leq \Omega_T \leq \Omega_2 - \Omega_0.$$

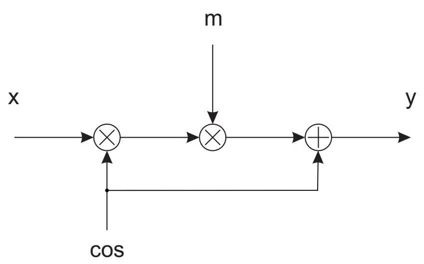
Now the signal $X\left(e^{j\Omega}\right) \bullet\!\!-\!\!\circ\, x(n)$ are sent via the channel defined above.



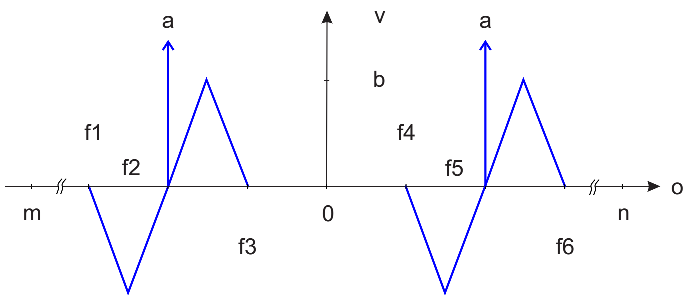
(f) The signal $x(n)$ is to be amplitude modulated. Write down the modulation equation in general terms. The carrier signal should also be transmitted. (2 P)

$$y(n) = A_T\left[1 + mx(n)\right]\cos(\Omega_T n).$$

(g) Draw a simple block diagram according to the equation from *(f)*. (3 P)



(h) Sketch the spectrum of the modulated signal from *(h)* in the range from $-\pi$ to π. Assume that $\Omega_0 + \Omega_T < \pi$. (3 P)



Part 3 *This part of the task can be solved independently of parts 1 and 2.*

The continuous signal $s(t)$ is to be transmitted over a channel using FM modulation.

$$s(t) = \begin{cases} -1\text{V}, & \text{fÃ¼r } 0s \leq t < 1s, \\ 2\text{V}, & \text{fÃ¼r } 2s \leq t < 4s, \\ -1\text{V}, & \text{fÃ¼r } 5s \leq t < 8s, \\ 0\text{V}, & \text{else.} \end{cases}$$

(i) Describe the general function of angle modulation and the meaning of the individual variables and terms. (2 P)

According to the lecture *Signals and Systems II*, the continuous angle modulation has the following form:

$$\begin{aligned} c_\text{T} &= \hat{c}_\text{T} \cos(\omega_\text{T} t + \phi_\text{T}(t)) = \hat{c}_\text{T} \cos(\Phi_\text{T}(t)), \\ \text{TrÃďger}: &\, c_\text{T}, \\ \text{TrÃďgeramplitude}: &\, \hat{c}_\text{T}, \\ \text{TrÃďgerfrequenz}: &\, \omega_\text{T}, \\ \text{TrÃďgerphase}: &\, \phi_\text{T}(t), \\ \text{TrÃďger} - \text{Winkel} - \text{Momentanphase}: &\, \Phi_\text{T}(t). \end{aligned}$$

(j) Enter the instantaneous frequency $\Omega_\text{T}(t)$ and the instantaneous phase $\Phi_\text{T}(t)$ in Dependence on $s(t)$ and the modulation constant k_FM. (2 P)

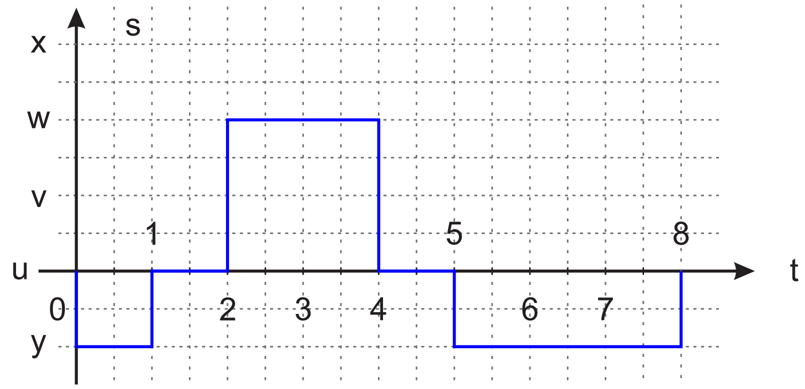
$$\begin{aligned} \Omega_\text{T}(t) &= \omega_\text{T} + k_\text{FM} 2\pi s(t), \\ \Phi_\text{T}(t) &= \omega_\text{T} t + k_\text{FM} 2\pi \int_{-\infty}^{t} s(\tau) d\tau, \\ &= \omega_\text{T} t + \phi_\text{T}(t). \end{aligned}$$

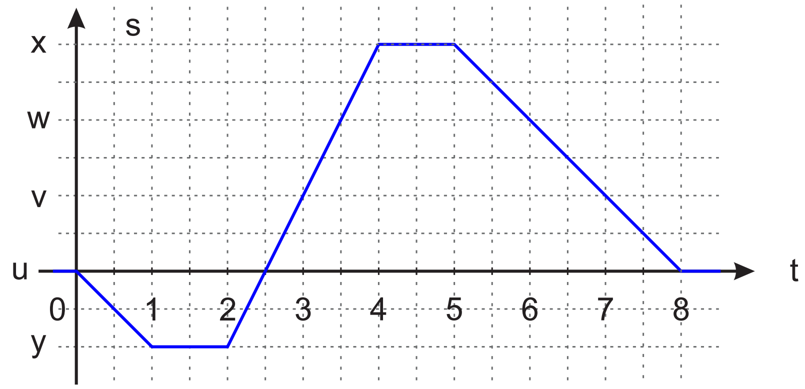
(k) Sketch $s(t)$ in the range from 0 to 8. (3 P)

(l) Calculate the carrier phase (not the instantaneous phase!) $\phi_\text{T}(t)$ and sketch it! (4 P)

$$\phi_\text{T}(t) = k_\text{FM} 2\pi \int_{-\infty}^{t} s(\tau) d\tau = \begin{cases} -k_\text{FM} 2\pi 1\text{V} t, & \text{fÃ¼r } 0s \leq t < 1s, \\ -k_\text{FM} 2\pi 1\text{V}, & \text{fÃ¼r } 1s \leq t < 2s, \\ k_\text{FM} 2\pi 2\text{V}(t - 2{,}5s), & \text{fÃ¼r } 2s \leq t < 4s, \\ k_\text{FM} 2\pi 3\text{V}, & \text{fÃ¼r } 4s \leq t < 5s, \\ -k_\text{FM} 2\pi 1\text{V}(t - 8s), & \text{fÃ¼r } 5s \leq t < 8s, \\ 0, & \text{else.} \end{cases}$$

(m) What unit of measurement has the constant k_FM if the signal $s(t)$ is a voltage signal? (3 P)





The signal $s(t)$ has the unit of measurement Volt [V]. Frequency is given in $\frac{1}{\text{s}}$ or in Hz. This allows the unit of measurement of k_{FM} to be determined. The corresponding size equation results from the equation for the instantaneous frequency:

$$\begin{aligned}[\Omega_{\text{T}}(t)] &= [\omega_{\text{T}}] + [k_{\text{FM}}] \cdot [s(t)]\,, \\ \text{Hz} &= \text{Hz} + [k_{\text{FM}}] \cdot \text{V}, \\ \text{Hz} &= [k_{\text{FM}}] \cdot \text{V}, \\ [k_{\text{FM}}] &= \frac{\text{Hz}}{\text{V}} = \frac{1}{\text{s} \cdot \text{V}}.\end{aligned}$$

(n) Determine the modulation constant k_{FM} so that the frequency deviation Δf corresponds to ± 50 kHz. (2 P)

According to the lecture, the frequency deviation corresponds to:

$$\Delta f = k_{\text{FM}} \hat{s},$$

where $\hat{s}$ is the maximum or minimum value of the signal $s(t)$. This means that the modulation constant is:

$$k_{\text{FM}} = \frac{\pm 50 \text{ kHz}}{\pm 2 \text{ V}} = 25 \frac{\text{kHz}}{\text{V}}.$$

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