Signals and Systems II

Exam SS 2023

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	18.09.2023
Name:	______________________
Matriculation Number:	______________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________

Marking

Problem	1	2	3
Points	/35	/32	/33

Total number of points: ____________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ____________ Signature: ______________________

Signals and Systems II

Exam SS 2023

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	CAP3, Hörsaal 2
Date:	18.09.2023
Begin:	09:00 h
Reading Time:	10 minutes
Working Time:	90 minutes

Notes

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Task 1 (35 points)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

Let the probability distribution $F_v(v)$ of the ergodic random process with statistically independent values be given by:

$$F_v(v) = \begin{cases} 0, & \text{for } v < \pi, \\ \frac{2}{\pi}(v - \pi), & \text{für } \pi \le v < \frac{3\pi}{2}, \\ 1, & \text{for } \frac{3\pi}{2} \le v. \end{cases}$$

(a) Sketch the probability distribution $F_v(v)$. (2 P)

(b) Determine the probability density $f_v(v)$. (2 P)

(c) Determine (3 P)

(i) the linear mean value m_v,

(ii) the variance σ_v^2 and

(iii) (as a numerical value) the root mean square value $m_v^{(2)}$

of the probability density $f_v(v)$.

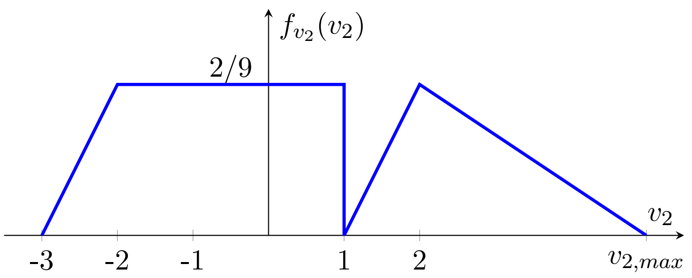
(d) Determine the power density spectrum $S_{vv}(e^{j\Omega})$. (4 P)

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

Given an equally distributed discrete random process $v_1(n)$ whose probability density function is defined to be non-zero within the range of values $v_1 \in [1, 4]$.

(e) Determine the associated probability density function $f_{v_1}(v_1)$ and draw it. Please label all axes! (2 P)

Furthermore, a second random process $v_2(n)$, which is statistically independent of $v_1(n)$, is to be considered. This is defined by the following probability density function:



(f) Determine the value $v_{2,max}$. (2 P)

(g) Give the probability density function $f_{v_2}(v_2)$ in mathematical form. (3 P)

(h) Determine the composite probability density $f_{v_1,v_2}(v_1, v_2)$ of the two processes $v_1(n)$ and $v_2(n)$. In a plane spanned by v_1 and v_2, mark the area in which $f_{v_1,v_2}(v_1, v_2) > 0$ holds. (6 P)

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

Given the following composite probability densities and probability density functions:

$$f_a(a) = \begin{cases} \frac{2}{5}a + \frac{1}{4}, & \text{for } 2 \leq a < 3, \\ 0, & \text{else.} \end{cases} \qquad f_b(b) = \begin{cases} \frac{1}{4}, & \text{for } -2 \leq b < -1, \\ -\frac{1}{4}b, & \text{for } -1 \leq b < 0, \\ \frac{1}{4}b, & \text{for } 0 \leq b < 1, \\ \frac{1}{4}, & \text{for } 1 \leq b < 3, \\ 0, & \text{else.} \end{cases}$$

$$F_c(c) = \begin{cases} 0, & \text{for } c < -\frac{\pi}{2}, \\ \frac{1}{\pi}c + \frac{1}{2}, & \text{for } -\frac{\pi}{2} \leq c < \frac{\pi}{2}, \\ 1, & \text{else.} \end{cases} \qquad F_d(d) = \begin{cases} 0, & \text{for } d < \frac{3}{4}, \\ d - \frac{3}{4}, & \text{for } \frac{3}{4} \leq d < \frac{3}{2}, \\ \frac{3}{4}, & \text{for } \frac{3}{2} \leq d < \frac{5}{2}, \\ \frac{1}{3}d - \frac{1}{9}, & \text{for } \frac{5}{2} \leq d < \frac{7}{2}, \\ 1, & \text{for } \frac{7}{2} \leq d. \end{cases}$$

(i) Which of the given probability densities or distribution functions are correct? State them and explain which conditions have to be fulfilled! (5 P)

(j) Sketch the associated distribution function $F_?(?)$ to the probability density $f_?(?)$ you identified as candidates in (i). (4 P)

(k) Give the associated probability density $f_?(?)$ to the distribution function $F_?(?)$ you identified as a candidate in (i) as a function! (2 P)

Task 2 **(32 points)**

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

Given the following difference equation of a linear time-invariant system with input $v(n)$ and output $y(n)$:

$$y(n+1) = 2y(n+2) - 2v(n+1) - v(n) + 2v(n-1) - 4v(n-2) + 4v(n-3).$$

(a) What is the transfer function $H(z)$ of the system? (3 P)

(b) What is the impulse response of the system? Draw it for $-3 < n < 7$. (6 P)

(c) Does the system have a direct pass? Justify both on the basis of your result from subtask *(a)* and on the basis of your result from subtask *(b)*. (1,5 P)

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

Let the state space model be described by the following equations of state:

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\,\boldsymbol{x}(n) + \boldsymbol{B}\,\boldsymbol{v}(n), \quad (1)$$

$$\boldsymbol{y}(n) = \boldsymbol{C}\,\boldsymbol{x}(n) + \boldsymbol{D}\,\boldsymbol{v}(n). \quad (2)$$

Furthermore, a system is defined that is parameterised with the following four matrices:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}, \; \boldsymbol{B} = \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}, \; \boldsymbol{C} = \begin{bmatrix} 0 & 2 \end{bmatrix}, \; \boldsymbol{D} = \begin{bmatrix} 0 \end{bmatrix}.$$

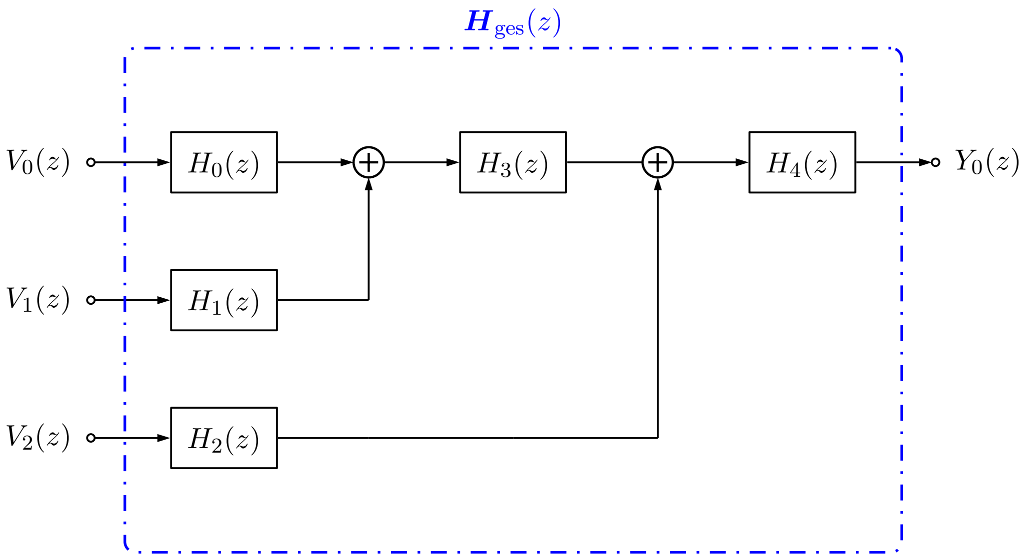
(d) Determine the transfer function $H(z)$. Simplify the result as much as possible. (6 P)
Note: In the last step, simplify the result so that the highest denominator degree in the transfer matrix is 1.

(e) Determine the impulse response $h_0(n)$. (2 P)

(f) Is the system $H(z)$ stable? Justify your answer. (1 P)

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

Let the following system be given:



(g) Determine $\boldsymbol{H}_{\text{ges}}(z)$ as a function of $H_i(z)$, $i \in [0,1,2,3,4]$. What do the individual elements of $\boldsymbol{H}_{\text{ges}}(z)$ describe? (5 P)

Only the transmission path from $V_0(z)$ to $Y_0(z)$ is now considered ($V_1(z)$ and $V_2(z)$ are zero and therefore negligible). Furthermore, let hold:

$$H_0(z) = \frac{z - \frac{1}{4}}{z + \frac{1}{4}}, \qquad H_1(z) = \frac{z}{z^2 - \frac{1}{4}}, \qquad H_2(z) = \frac{z + \frac{1}{4}}{z^2 + \frac{1}{4}z - \frac{1}{8}},$$

$$H_3(z) = \frac{z + \frac{1}{4}}{(z - \frac{1}{4})(z + 1)}, \qquad H_4(z) = \frac{z + 1}{z^2 - \frac{1}{16}}.$$

(h) Draw the pole/zero diagram for the transmission path under consideration. (4,5 P)

(i) Is the subsystem under consideration: (3 P)

(i) stable,

(ii) causal,

(iii) or minimum phase?

Give reasons for your answer in each case.

Task 3 (33 points)

Part 1 *This task part can be solved independently of part 2 and part 3.*

(a) What causes amplitude errors in angle-modulated signals and how can this signal distortion be compensated for on the receiving side? . (2 P)

Part 2 *This part of the task can be solved independently of part 1 and part 3.*

Given the system from Figure 1 for transmitting the signal $v(n)$.

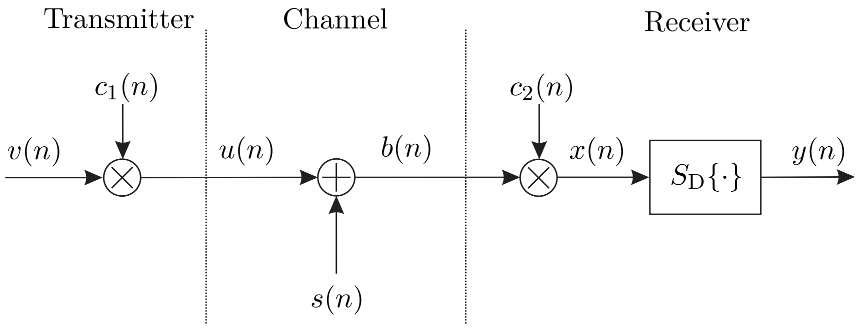


Figure 1: Transmission link

Let the spectrum $V(e^{j\Omega}) = \mathcal{F}\{v(n)\}$ be given by

$$V(e^{j\Omega}) = \begin{cases} -1 - \frac{\left(\frac{\pi}{4} - |\Omega + \lambda \cdot 2\pi|\right)}{\frac{\pi}{4}} & , \text{ falls } \frac{-\pi}{4} \leq \Omega + \lambda \cdot 2\pi \leq \frac{\pi}{4}, \\ 0 & , \text{ sonst} \end{cases}$$

(b) Sketch the spectrum $V(e^{j\Omega}) = \mathcal{F}\{v(n)\}$ in the range $-\pi < \Omega < \pi$. Label all axes! (2 P)

(c) For modulation, the carrier signal $c_1(n) = 2\cos(\Omega_c n)$ with $\Omega_c = \frac{3}{4}\pi$ is used. Calculate the spectrum $U(e^{j\Omega}) = \mathcal{F}\{u(n)\}$ as a function of $V(e^{j\Omega})$ and draw the real part of the spectrum $U(e^{j\Omega})$ in the range of $-2\pi < \Omega < 2\pi$. Label all axes! What is the modulation type? (5 P)

(d) Due to a non-interference-free signal transmission, the signal $s(n)$ couples additively with the real-valued spectrum from Figure 2. Calculate the spectrum $B(e^{j\Omega}) = \mathcal{F}\{b(n)\}$ as a function of $U(e^{j\Omega})$ and $S(e^{j\Omega})$ and sketch in the region of $-\pi < \Omega < \pi$ the real part of the spectrum $B(e^{j\Omega})$. Label all axes! (3 P)

(e) The demodulation is done with the signal $c_2(n) = \cos(\Omega_c n + \Delta)$ where Δ describes a phase error. Calculate the spectrum $X(e^{j\Omega}) = \mathcal{F}\{x(n)\}$ as a function of $V(e^{j\Omega})$, $S(e^{j\Omega})$, Δ and Ω_c. (10 P)

(f) For the ideal reconstruction the system $S_D\{\cdot\}$ shall be used. The phase error is assumed to be $\Delta = 40°$. What properties must this filter have in terms of gain/attenuation and cut-off frequencies? (2 P)

(g) Can the phase error lead to a complete cancellation of the demodulated useful signal in the baseband? Justify your answer! (2 P)

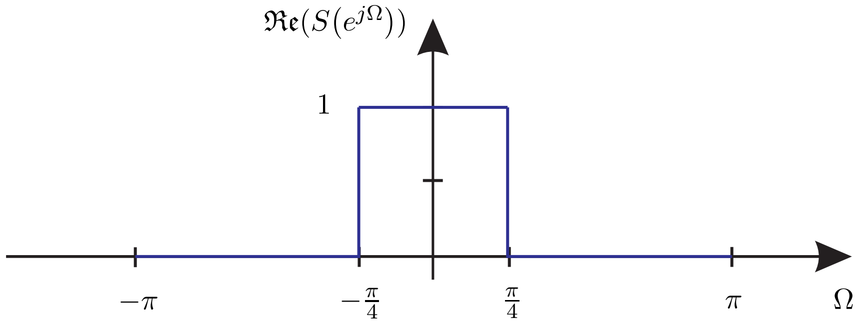


Figure 2: Spectrum

Part 3 *This part of the task can be solved independently of part 1 and part 2.*

Given is the instantaneous angular frequency $\Omega_m(t)$ of a frequency modulated carrier:

$$\Omega_m(t) = \begin{cases} \omega_0 + k_{\text{FM}} \sin(2\pi f_0 t), & \text{for } t \geq 0, \\ 0, & \text{else.} \end{cases}$$

(h) Calculate the associated instantaneous phase $\Phi(t)$. (3 P)

(i) Name the advantages and disadvantages of amplitude and angle modulation respectively. (4 P)

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