Signals and Systems II

Exam SS 2023

Examiner: Prof. Dr.-Ing. Gerhard Schmidt

Date: 18.09.2023

Name: ______________________

Matriculation Number: ______________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________

Marking

Problem	1	2	3
Points	/35	/32	/33

Total number of points: ______________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ______________ Signature: ______________________

Signals and Systems II

Exam SS 2023

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	CAP3, Hörsaal 2
Date:	18.09.2023
Begin:	09:00 h
Reading Time:	10 minutes
Working Time:	90 minutes

Notes

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

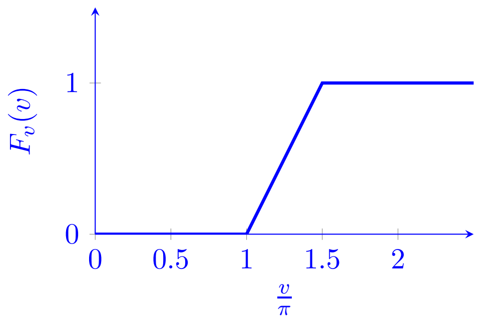
Task 1 (35 points)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

Let the probability distribution $F_v(v)$ of the ergodic random process with statistically independent values be given by:

$$F_v(v) = \begin{cases} 0, & \text{for } v < \pi, \\ \frac{2}{\pi}(v-\pi), & \text{für } \pi \leq v < \frac{3\pi}{2}, \\ 1, & \text{for } \frac{3\pi}{2} \leq v. \end{cases}$$

(a) Sketch the probability distribution $F_v(v)$. (2 P)



(b) Determine the probability density $f_v(v)$. (2 P)

$$f_v(v) = \begin{cases} 0, & \text{für } v < \pi, \\ \frac{2}{\pi}, & \text{für } \pi \leq v < \frac{3\pi}{2}, \\ 0, & \text{für } \frac{3\pi}{2} \leq v. \end{cases}$$

(c) Determine (3 P)

(i) the linear mean value m_v,

(ii) the variance σ_v^2 and

(iii) (as a numerical value) the root mean square value $m_v^{(2)}$

of the probability density $f_v(v)$.

(i)

$$m_v = \int_{-\infty}^{\infty} v f_v(v) dv$$

$$m_v = \frac{v_{max} + v_{min}}{2} = \frac{\frac{3\pi}{2} + \pi}{2} = \frac{5\pi}{4}$$

(ii)

$$\sigma_v^2 = \frac{1}{12}(v_{max} - v_{min})^2 = \frac{\pi^2}{48}$$

(iii)

$$m_v^{(2)} = m_v^2 + \sigma_v^2 \approx 4{,}1326$$

(d) Determine the power density spectrum $S_{vv}(e^{j\Omega})$. (4 P)

Since the process has statistically independent values, these are also uncorrelated and the result is for the autocorrelation function:

$$s_{vv}(\kappa) = m_v^2 + \sigma_v^2 \gamma_0(\kappa).$$

The following therefore applies to the power density spectrum:

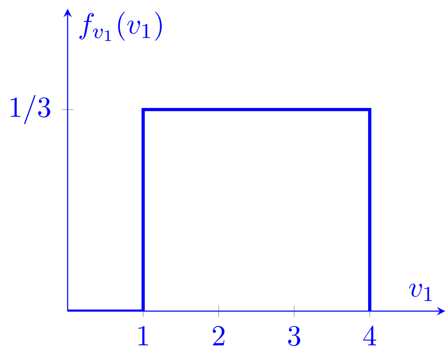
$$S_{vv}(e^{j\Omega}) = \mathcal{F}\{s_{vv}(\kappa)\} = 2\pi \cdot m_v^2 \sum_{\kappa=-\infty}^{\infty} \delta_0(\Omega - \lambda 2\pi) + \sigma_v^2.$$

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

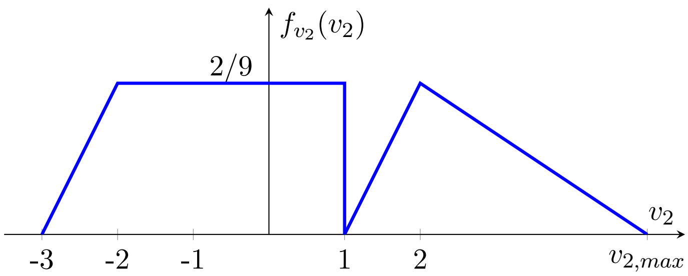
Given an equally distributed discrete random process $v_1(n)$ whose probability density function is defined to be non-zero within the range of values $v_1 \in [1, 4]$.

(e) Determine the associated probability density function $f_{v_1}(v_1)$ and draw it. Please label all axes! (2 P)

Since this is an equally distributed random process, the following probability density function results:



Furthermore, a second random process $v_2(n)$, which is statistically independent of $v_1(n)$, is to be considered. This is defined by the following probability density function:



(f) Determine the value $v_{2,max}$. (2 P)

With

$$\int_{-\infty}^{\infty} f_{v_2}(v_2) dv_2 = 1,$$

a closer look reveals $v_{2,max} = 3$

(g) Give the probability density function $f_{v_2}(v_2)$ in mathematical form. (3 P)

$$f_{v_2}(v_2) = \begin{cases} \frac{2}{9}v_2 + \frac{2}{3}, & \text{for } -3 \le v_2 < -2, \\ \frac{2}{9}, & \text{for } -2 \le v_2 < 1, \\ \frac{2}{9}v_2 - \frac{2}{9}, & \text{for } 1 \le v_2 < 2, \\ -\frac{2}{9}v_2 + \frac{2}{3}, & \text{for } 2 \le v_2 < 3, \\ 0, & \text{else.} \end{cases}$$

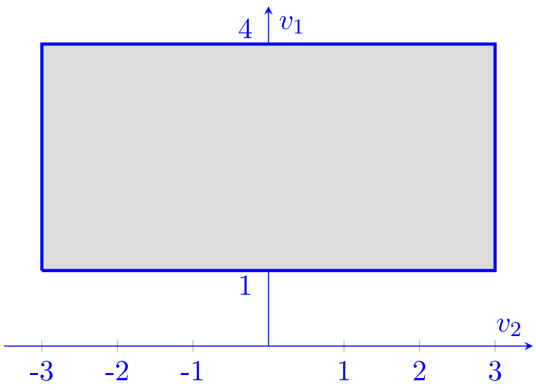
(h) Determine the composite probability density $f_{v_1,v_2}(v_1, v_2)$ of the two processes $v_1(n)$ and $v_2(n)$. In a plane spanned by v_1 and v_2, mark the area in which $f_{v_1,v_2}(v_1, v_2) > 0$ holds. (6 P)

Since statistical independence applies:

$$f_{v_1,v_2} = f_{v_1}(v_1) \cdot f_{v_2}(v_2).$$

Thus it follows:

$$f_{v_1,v_2} = \begin{cases} \frac{2}{27}v_2 + \frac{2}{9}, & \text{for } 1 \le v_1 < 4 \text{ and } -3 \le v_2 < -2, \\ \frac{2}{27}, & \text{for } 1 \le v_1 < 4 \text{ and } -2 \le v_2 < 1, \\ \frac{2}{27}v_2 - \frac{2}{27}, & \text{for } 1 \le v_1 < 4 \text{ and } 1 \le v_2 < 2, \\ -\frac{2}{27}v_2 + \frac{2}{9}, & \text{for } 1 \le v_1 < 4 \text{ and } 2 \le v_2 < 3, \\ 0, & \text{else.} \end{cases}$$



Part 3 *This part of the task can be solved independently of parts 1 and 2.*

Given the following composite probability densities and probability density functions:

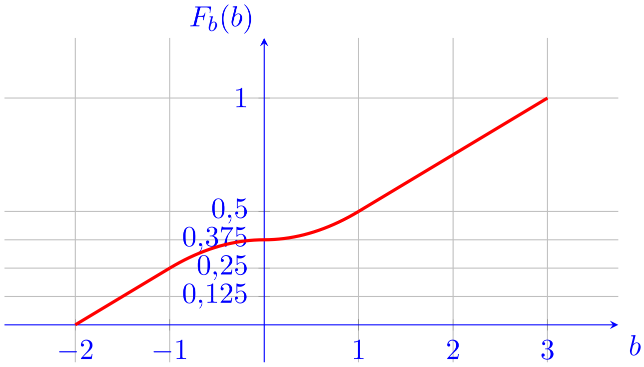
$$f_a(a) = \begin{cases} \frac{2}{5}a + \frac{1}{4}, & \text{for } 2 \leq a < 3, \\ 0, & \text{else.} \end{cases} \qquad f_b(b) = \begin{cases} \frac{1}{4}, & \text{for } -2 \leq b < -1, \\ -\frac{1}{4}b, & \text{for } -1 \leq b < 0, \\ \frac{1}{4}b, & \text{for } 0 \leq b < 1, \\ \frac{1}{4}, & \text{for } 1 \leq b < 3, \\ 0, & \text{else.} \end{cases}$$

$$F_c(c) = \begin{cases} 0, & \text{for } c < -\frac{\pi}{2}, \\ \frac{1}{\pi}c + \frac{1}{2}, & \text{for } -\frac{\pi}{2} \leq c < \frac{\pi}{2}, \\ 1, & \text{else.} \end{cases} \qquad F_d(d) = \begin{cases} 0, & \text{for } d < \frac{3}{4}, \\ d - \frac{3}{4}, & \text{for } \frac{3}{4} \leq d < \frac{3}{2}, \\ \frac{3}{4}, & \text{for } \frac{3}{2} \leq d < \frac{5}{2}, \\ \frac{1}{3}d - \frac{1}{9}, & \text{for } \frac{5}{2} \leq d < \frac{7}{2}, \\ 1, & \text{for } \frac{7}{2} \leq d. \end{cases}$$

(i) Which of the given probability densities or distribution functions are correct? State them and explain which conditions have to be fulfilled! (5 P)
Density: Integral identical 1, no function values less than 0 -> satisfied by $f_b(b)$

Distribution function: Limit against $-\infty$ identical 0, limit against ∞ identical 1, monotonically increasing -> satisfied by $F_c(c)$

(j) Sketch the associated distribution function $F_?(?)$ to the probability density $f_?(?)$ you identified as candidates in (i). (4 P)



(k) Give the associated probability density $f_?(?)$ to the distribution function $F_?(?)$ you identified as a candidate in (i) as a function! (2 P)

$$f_c(c) = \begin{cases} 0, & \text{for } c < -\frac{\pi}{2}, \\ \frac{1}{\pi}, & \text{for } -\frac{\pi}{2} \leq c < \frac{\pi}{2}, \\ 0, & \text{else.} \end{cases}$$

Task 2 (32 points)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

Given the following difference equation of a linear time-invariant system with input $v(n)$ and output $y(n)$:

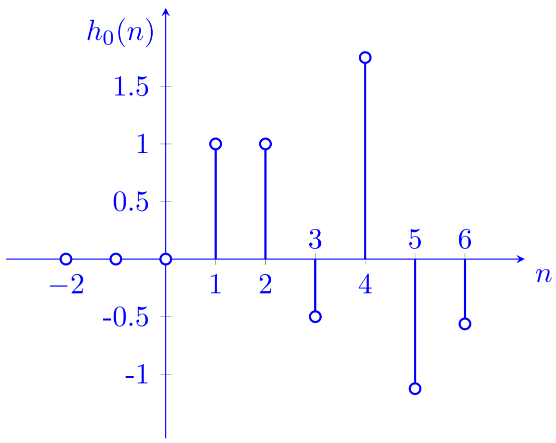
$$y(n+1) = 2y(n+2) - 2v(n+1) - v(n) + 2v(n-1) - 4v(n-2) + 4v(n-3).$$

(a) What is the transfer function $H(z)$ of the system? (3 P)

$$H(z) = \frac{Y(z)}{V(z)} = \frac{z^{-1} + \frac{1}{2}z^{-2} - z^{-3} + 2z^{-4} - 2z^{-5}}{1 - \frac{1}{2}z^{-1}}$$
$$= z^{-1}\frac{z}{z-\frac{1}{2}} + \frac{1}{2}z^{-2}\frac{z}{z-\frac{1}{2}} - z^{-3}\frac{z}{z-\frac{1}{2}} + 2z^{-4}\frac{z}{z-\frac{1}{2}} - 2z^{-5}\frac{z}{z-\frac{1}{2}}$$

(b) What is the impulse response of the system? Draw it for $-3 < n < 7$. (6 P)
Inverse z-transformation of the result from *(a)* (or excitation of the system with $v(n) = \gamma_0$) yields:

$$h_0(n) = \left(\frac{1}{2}\right)^{n-1}\gamma_{-1}(n-1) + \frac{1}{2}\left(\frac{1}{2}\right)^{n-2}\gamma_{-1}(n-2) - \left(\frac{1}{2}\right)^{n-3}\gamma_{-1}(n-3)$$
$$+ 2\left(\frac{1}{2}\right)^{n-4}\gamma_{-1}(n-4) - 2\left(\frac{1}{2}\right)^{n-5}\gamma_{-1}(n-5)$$



(c) Does the system have a direct pass? Justify both on the basis of your result from subtask *(a)* and on the basis of your result from subtask *(b)*. (1,5 P)
No, because the numerator degree in the transfer function $H(z)$ is smaller than the denominator degree and the impulse response $h(n)$ is at the position $h(0) = 0$.

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

Let the state space model be described by the following equations of state:

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\,\boldsymbol{x}(n) + \boldsymbol{B}\,\boldsymbol{v}(n), \tag{1}$$
$$\boldsymbol{y}(n) = \boldsymbol{C}\,\boldsymbol{x}(n) + \boldsymbol{D}\,\boldsymbol{v}(n). \tag{2}$$

Furthermore, a system is defined that is parameterised with the following four matrices:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 0 & 2 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 0 \end{bmatrix}.$$

(d) Determine the transfer function $H(z)$. Simplify the result as much as possible. (6 P)
Note: In the last step, simplify the result so that the highest denominator degree in the transfer matrix is 1.

$$\begin{aligned}
H(z) &= \boldsymbol{C} \cdot [z \cdot \boldsymbol{I} - \boldsymbol{A}]^{-1} \cdot \boldsymbol{B} + \boldsymbol{D} \\
&= \begin{bmatrix} 0 & 2 \end{bmatrix} \cdot \left[z \cdot \boldsymbol{I} - \begin{bmatrix} 1 & 0 \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix} \right]^{-1} \cdot \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \\
&= \frac{1}{(z-1)(z-\frac{1}{4})} \cdot \begin{bmatrix} 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} z-\frac{1}{4} & 0 \\ \frac{3}{4} & z-1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix} \\
&= \frac{1}{(z-1)(z-\frac{1}{4})} \cdot \begin{bmatrix} \frac{3}{2} & 2z-2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix} \\
&= \frac{3+z-1}{(z-1)(z-\frac{1}{4})} \\
&= \frac{z+2}{(z-1)(z-\frac{1}{4})}
\end{aligned}$$

(e) Determine the impulse response $h_0(n)$. (2 P)
Due to an error, the original solution of task d) was:

$$H(z) = \frac{3}{z-\frac{1}{4}}$$

Accordingly, the solution for e) would have been:

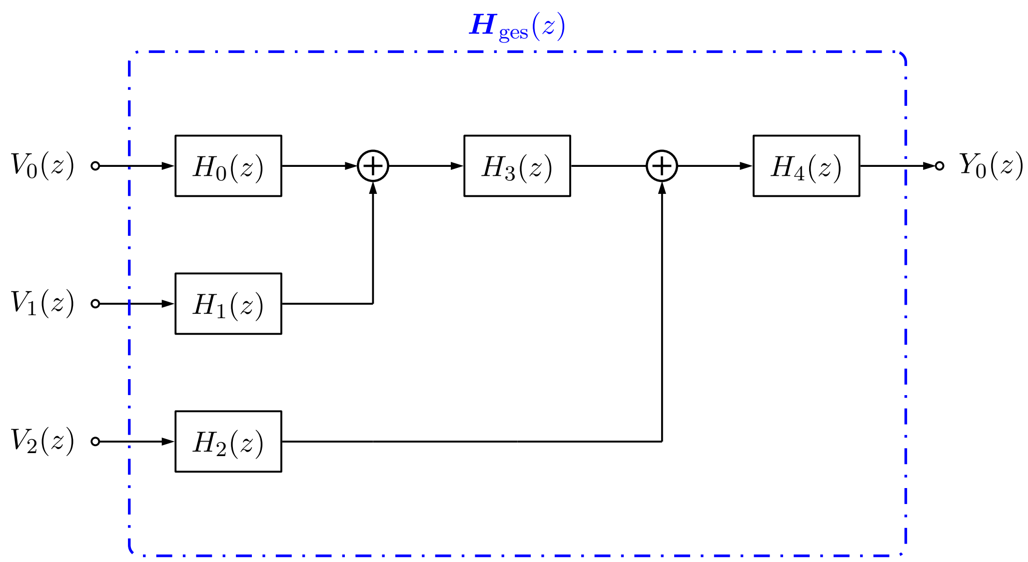
$$\begin{aligned}
h_0(n) &= \mathcal{Z}^{-1}\{H(z)\} \\
&= \mathcal{Z}^{-1}\left\{ \frac{3}{z-\frac{1}{4}} \right\} \\
&= 3\left(\frac{1}{4}\right)^{n-1} \gamma_{-1}(n-1), \qquad \forall\, |z| > \left|\frac{1}{4}\right|
\end{aligned}$$

Since the more complex solution in d) has made task e) more extensive, the correction has been adjusted accordingly.

(f) Is the system $H(z)$ stable? Justify your answer. (1 P)
Yes, since the system $H(z)$ has a pole of $z_{\infty,0} = \frac{1}{4}$ and thus satisfies the stability condition $|z_\infty| < 1$.

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

Let the following system be given:



(g) Determine $\boldsymbol{H}_{\text{ges}}(z)$ as a function of $H_i(z)$, $i \in [0,1,2,3,4]$. What do the individual elements of $\boldsymbol{H}_{\text{ges}}(z)$ describe? (5 P)

Let $\boldsymbol{V}(z) = [V_0(z), V_1(z), V_2(z)]^{\mathrm{T}}$, where $V_i(z)$ describes the z-transformation of $v_i(n)$ with $i \in [0, 1, 2]$.

$$Y_0(z) = \boldsymbol{H}_{\text{ges}}(z)\ \boldsymbol{V}(z) = \begin{bmatrix} H_{0,0}(z) & H_{1,0}(z) & H_{2,0}(z) \end{bmatrix} \cdot \begin{bmatrix} V_0(z) \\ V_1(z) \\ V_2(z) \end{bmatrix}$$

$H_{i,0}$ thus describes the influence of input v_i on output y_0. This results in:

$$\boldsymbol{H}_{\text{ges}}(z) = \begin{bmatrix} H_0(z)H_3(z)H_4(z) & H_1(z)H_3(z)H_4(z) & H_2(z)H_4(z) \end{bmatrix}.$$

Only the transmission path from $V_0(z)$ to $Y_0(z)$ is now considered ($V_1(z)$ and $V_2(z)$ are zero and therefore negligible). Furthermore, let hold:

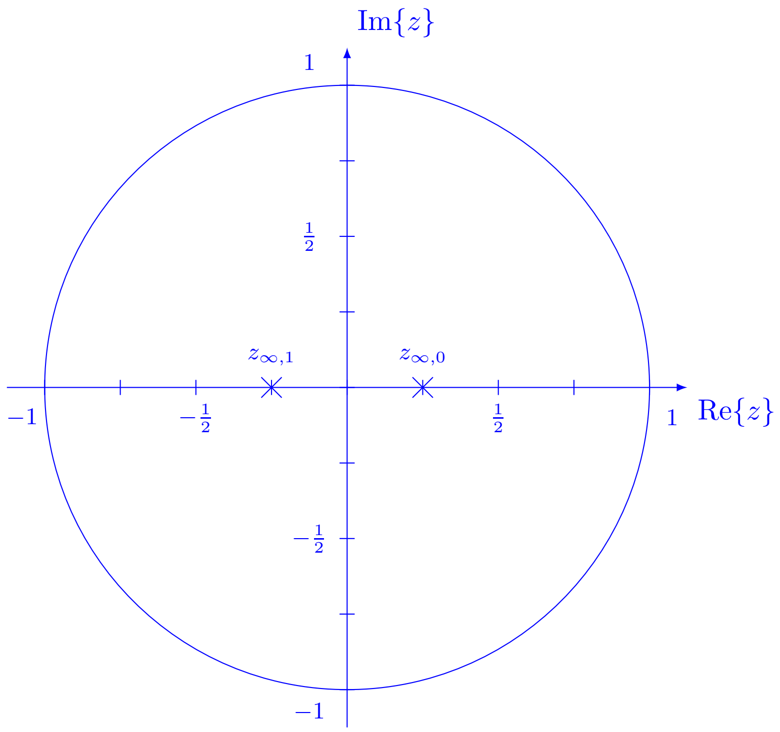
$$H_0(z) = \frac{z - \frac{1}{4}}{z + \frac{1}{4}}, \qquad H_1(z) = \frac{z}{z^2 - \frac{1}{4}}, \qquad H_2(z) = \frac{z + \frac{1}{4}}{z^2 + \frac{1}{4}z - \frac{1}{8}},$$

$$H_3(z) = \frac{z + \frac{1}{4}}{(z - \frac{1}{4})(z + 1)}, \qquad H_4(z) = \frac{z + 1}{z^2 - \frac{1}{16}}.$$

(h) Draw the pole/zero diagram for the transmission path under consideration. (4,5 P)

$$\begin{aligned}
H_{0,0}(z) &= H_0(z)\ H_3(z)\ H_4(z) \\
&= \frac{z-\frac{1}{4}}{z+\frac{1}{4}}\ \frac{z+\frac{1}{4}}{(z-\frac{1}{4})(z+1)}\ \frac{z+1}{z^2-\frac{1}{16}} \\
&= \frac{(z-\frac{1}{4})(z+\frac{1}{4})(z+1)}{(z+\frac{1}{4})(z-\frac{1}{4})(z+1)(z^2-\frac{1}{16})} \\
&= \frac{1}{z^2-\frac{1}{16}} \\
&= \frac{1}{(z-\frac{1}{4})(z+\frac{1}{4})}
\end{aligned}$$

The system has two poles. The first pole is at $z_{\infty,0} = \frac{1}{4}$ and the second at $z_{\infty,1} = -\frac{1}{4}$. This results in the following pole/zero diagram:



(i) Is the subsystem under consideration: (3 P)

(i) stable,
Yes the system is stable because all pole positions are within the unit circle.

(ii) causal,
Yes, because the numerator degree of the system is smaller than the denominator degree of the system and thus the output $y(n)$ does not depend on future input values $v(n)$.

(iii) or minimum phase?
Yes, because no zero lies outside of the unit circle.

Give reasons for your answer in each case.

Task 3 (33 points)

Part 1 *This task part can be solved independently of part 2 and part 3.*

(a) What causes amplitude errors in angle-modulated signals and how can this signal distortion be compensated for on the receiving side? (2 P)
Amplitude errors are not critical with angle modulation. The distortion can be (largely) compensated for by a limiter amplifier followed by a bandpass filter .

Part 2 *This part of the task can be solved independently of part 1 and part 3.*

Given the system from Figure 1 for transmitting the signal $v(n)$.

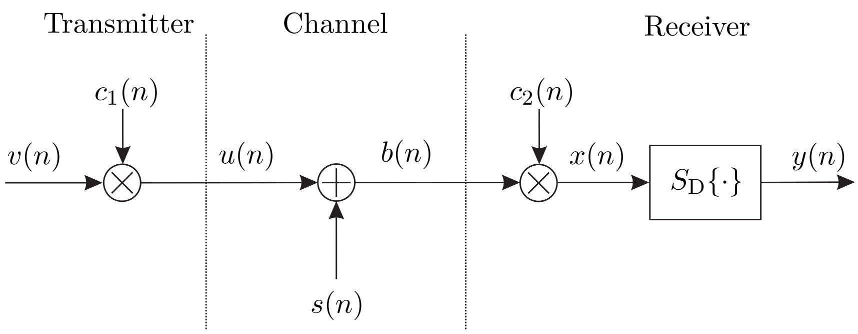


Figure 1: Transmission link

Let the spectrum $V(e^{j\Omega}) = \mathcal{F}\{v(n)\}$ be given by

$$V(e^{j\Omega}) = \begin{cases} -1 - \frac{(\frac{\pi}{4} - |\Omega + \lambda \cdot 2\pi|)}{\frac{\pi}{4}} & , \text{ falls } \frac{-\pi}{4} \leq \Omega + \lambda \cdot 2\pi \leq \frac{\pi}{4}, \\ 0 & , \text{ sonst} \end{cases}$$

(b) Sketch the spectrum $V(e^{j\Omega}) = \mathcal{F}\{v(n)\}$ in the range $-\pi < \Omega < \pi$. Label all axes! (2 P)

The spectrum $V(e^{j\Omega}) = \mathcal{F}\{v(n)\}$ is shown in Figure 2.

(c) For modulation, the carrier signal $c_1(n) = 2\cos(\Omega_\mathrm{c} n)$ with $\Omega_\mathrm{c} = \frac{3}{4}\pi$ is used. Calculate the spectrum $U(e^{j\Omega}) = \mathcal{F}\{u(n)\}$ as a function of $V(e^{j\Omega})$ and draw the real part of the spectrum $U(e^{j\Omega})$ in the range of $-2\pi < \Omega < 2\pi$. Label all axes! What is the modulation type? (5 P)

It is a two-sideband modulation. The following applies to the spectrum:

$$\begin{aligned}
U(e^{j\Omega}) &= \mathcal{F}\{u(n)\} \\
&= \mathcal{F}\{v(n) \cdot c_1(n)\} \\
&= \mathcal{F}\{v(n) \cdot 2\cos(2\pi\Omega_\mathrm{c} n)\} \\
&= \frac{1}{2\pi} V(e^{j\Omega}) \circledast \left[2\pi \sum_{\lambda=-\infty}^{\infty} [\delta_0(\Omega + \Omega_\mathrm{c} - 2\pi\lambda) + \delta_0(\Omega - \Omega_\mathrm{c} - 2\pi\lambda)] \right] \\
&= \left[V(e^{j(\Omega+\Omega_\mathrm{c})}) + V(e^{j(\Omega-\Omega_\mathrm{c})}) \right]
\end{aligned}$$

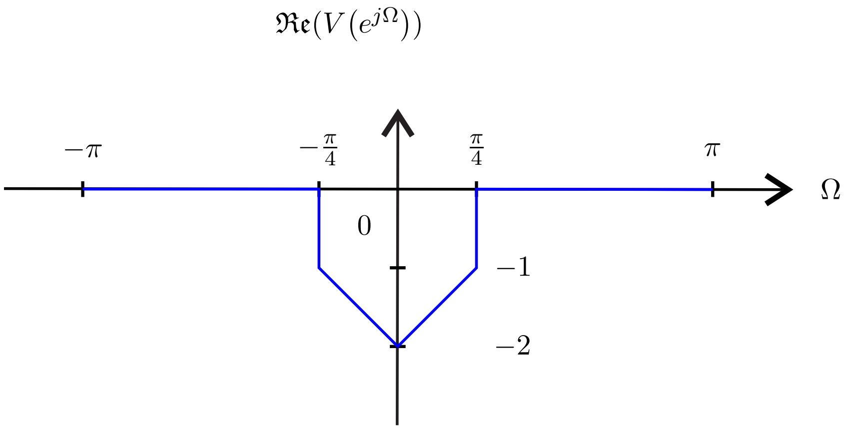


Figure 2: Spectrum

For the spectrum it follows:

$$U(e^{j\Omega}) = \left[V(e^{j(\Omega+\Omega_c)}) + V(e^{j(\Omega-\Omega_c)})\right]$$

The sketch is shown in Figure 3.

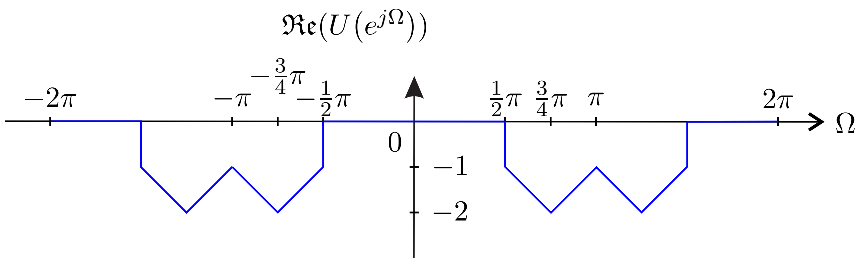


Figure 3: Spectrum

(d) Due to a non-interference-free signal transmission, the signal $s(n)$ couples additively with the real-valued spectrum from Figure 4. Calculate the spectrum $B(e^{j\Omega}) = \mathcal{F}\{b(n)\}$ as a function of $U(e^{j\Omega})$ and $S(e^{j\Omega})$ and sketch in the region of $-\pi < \Omega < \pi$ the real part of the spectrum $B(e^{j\Omega})$. Label all axes! (3 P)

For $B(e^{j\Omega}) = \mathcal{F}\{b(n)\}$ we get

$$B(e^{j\Omega}) = U(e^{j\Omega}) + S(e^{j\Omega}).$$

The following follows for the spectrum

$$B(e^{j\Omega}) = U(e^{j\Omega}) + S(e^{j\Omega}).$$

The sketch is shown in Figure 5.

(e) The demodulation is done with the signal $c_2(n) = \cos(\Omega_c n + \Delta)$ where Δ describes a phase error. Calculate the spectrum $X(e^{j\Omega}) = \mathcal{F}\{x(n)\}$ as a function of $V(e^{j\Omega})$, $S(e^{j\Omega})$, Δ and Ω_c. (10 P)

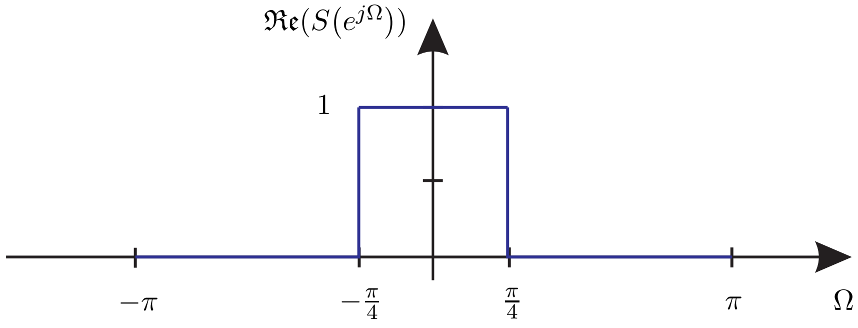


Figure 4: Spectrum

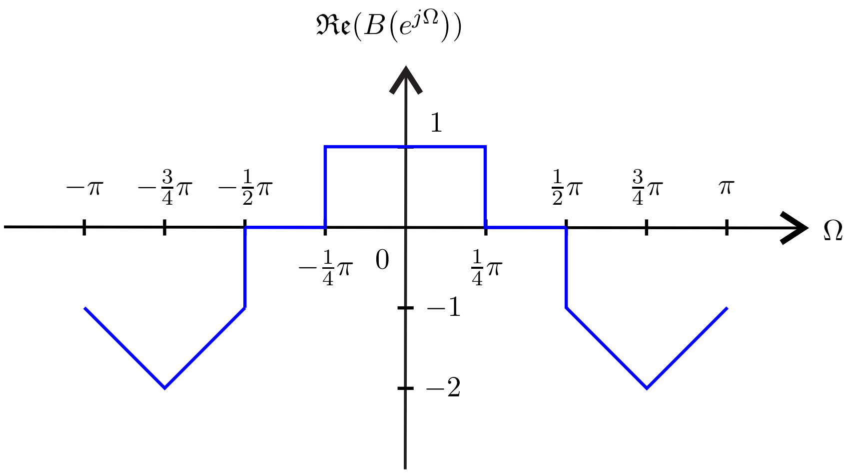


Figure 5: Spectrum

$$
\begin{aligned}
X(e^{j\Omega}) &= \mathcal{F}\{x(n)\} \\
&= \mathcal{F}\{b(n) \cdot c_2(n)\} \\
&= \mathcal{F}\{(v(n) \cdot c_1(n) + s(n)) \cdot c_2(n)\} \\
&= \mathcal{F}\{v(n) \cdot c_1(n) \cdot c_2(n) + s(n) \cdot c_2(n)\} \\
&= \mathcal{F}\{v(n) \cdot 2\cos(\Omega_c n) \cdot \cos(\Omega_c n + \Delta) + s(n) \cdot \cos(\Omega_c n + \Delta)\} \\
&= \underbrace{\mathcal{F}\{v(n) \cdot [\cos(-\Delta) + \cos(2\Omega_c n + \Delta)]\}}_{\widehat{V}(e^{j\Omega})} + \underbrace{\mathcal{F}\{s(n) \cdot \cos(\Omega_c n + \Delta)\}}_{\widehat{S}(e^{j\Omega})} \\
&= \widehat{V}(e^{j\Omega}) + \widehat{S}(e^{j\Omega})
\end{aligned}
$$

For a cosine oscillation with a phase error we get:

$$
\begin{aligned}
\mathcal{F}\{\cos(\Omega_c n + \Delta)\} &= \mathcal{F}\left\{\frac{1}{2}(e^{j(\Omega_c n+\Delta)} + e^{-j(\Omega_c n+\Delta)})\right\} \\
&= \mathcal{F}\left\{\frac{1}{2}(e^{j\Omega_c n}e^{j\Delta} + e^{-j\Omega_c n}e^{-j\Delta})\right\} \\
&= \frac{1}{2}\left(e^{j\Delta}\mathcal{F}\left\{e^{j\Omega_c n}\right\} + e^{-j\Delta}\mathcal{F}\left\{e^{-j\Omega_c n}\right\}\right) \\
&= \frac{e^{j\Delta}}{2}2\pi \sum_{\lambda=-\infty}^{\infty} \delta_0(\Omega - \Omega_c - 2\pi\lambda) + \frac{e^{-j\Delta}}{2}2\pi \sum_{\lambda=-\infty}^{\infty} \delta_0(\Omega + \Omega_c - 2\pi\lambda) \\
&= e^{j\Delta}\,\pi \sum_{\lambda=-\infty}^{\infty} \delta_0(\Omega - \Omega_c - 2\pi\lambda) + e^{-j\Delta}\,\pi \sum_{\lambda=-\infty}^{\infty} \delta_0(\Omega + \Omega_c - 2\pi\lambda)
\end{aligned}
$$

Thus it follows for $\widehat{S}(e^{j\Omega})$:

$$
\begin{aligned}
\widehat{S}(e^{j\Omega}) &= \frac{1}{2\pi}\left[e^{j\Delta}\,\pi S(e^{j(\Omega-\Omega_c)}) + e^{-j\Delta}\,\pi S(e^{j(\Omega+\Omega_c)})\right] \\
&= \frac{1}{2}\left[e^{j\Delta} S(e^{j(\Omega-\Omega_c)}) + e^{-j\Delta} S(e^{j(\Omega+\Omega_c)})\right]
\end{aligned}
$$

For $\widehat{V}(e^{j\Omega})$:

$$
\begin{aligned}
\widehat{V}(e^{j\Omega}) =& V(e^{j\Omega})\cos(-\Delta) + \\
& \frac{1}{2\pi}\left(e^{j\Delta}\pi V(e^{j(\Omega-2\Omega_c)}) + e^{-j\Delta}\pi V(e^{j(\Omega+2\Omega_c)})\right) \\
=& V(e^{j\Omega})\cos(\Delta) + \frac{1}{2}\left[e^{j\Delta} V(e^{j(\Omega-2\Omega_c)}) + e^{-j\Delta} V(e^{j(\Omega+2\Omega_c)})\right]
\end{aligned}
$$

(f) For the ideal reconstruction the system $S_D\{\cdot\}$ shall be used. The phase error is assumed to be $\Delta = 40°$. What properties must this filter have in terms of gain/attenuation and cut-off frequencies? (2 P)

With the help of an ideal low-pass filter, the original signal can be recovered. The cut-off frequency should be $\frac{\pi}{4}$ and in the passband there should be a gain of $\frac{1}{\cos(40° \cdot \frac{\pi}{180})}$.

(g) Can the phase error lead to a complete cancellation of the demodulated useful signal in the baseband? Justify your answer! (2 P)

The phase error scales the demodulated useful signal. For a phase error of $\Delta = \frac{\pi}{2} + \lambda\pi$ with $\lambda \in \mathbb{Z}$ the demodulated signal is zero and the transmission does not work!

Part 3 *This part of the task can be solved independently of part 1 and part 2.*

Given is the instantaneous angular frequency $\Omega_m(t)$ of a frequency modulated carrier:

$$\Omega_m(t) = \begin{cases} \omega_0 + k_{\mathrm{FM}} \sin(2\pi f_0 t), & \text{for } t \geq 0, \\ 0, & \text{else.} \end{cases}$$

(h) Calculate the associated instantaneous phase $\Phi(t)$. (3 P)

$$\begin{aligned} \Phi(t) = \int_0^t \Omega_m(\tau) d\tau &= \omega_0 t + k_{\mathrm{FM}} \int_0^t \sin(2\pi f_0 \tau) d\tau \\ &= \omega_0 t + k_{\mathrm{FM}} \Big(1 - \cos(2\pi f_0 t)\Big) \frac{1}{2\pi f_0}. \end{aligned}$$

(i) Name the advantages and disadvantages of amplitude and angle modulation respectively. (4 P)

Amplitude modulation is simple to implement, requires a small bandwidth and is more susceptible to interference. Angle modulation requires a large bandwidth and is more complicated to implement.

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