Signals and Systems II
Exam SoSe 2024

Examiner: Prof. Dr.-Ing. Gerhard Schmidt

Date: 05.09.2024

Name: _____

Matriculation Number: _____

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.

I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: _____

Marking

Problem	1	2	3
Points	/33	/33,5	/33,5

Total number of points: _____ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐   The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated _____   Signature: _____

Digitale Signalverarbeitung und Systemtheorie, Prof. Dr.-Ing. Gerhard Schmidt, www.dss-kiel.de

Signals and Systems II

Signals and Systems II

Exam SoSe 2024

|	
Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	LS1, Klaus-Murmann-Hörsaal
Date:	05.09.2024
Begin:	09:00 h
Reading Time:	10 Minuten
Working Time:	90 Minuten

Notes

- Lay out your student or personal ID for inspection.

- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.

- Do **not** use **pencil or red pen**.

- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.

- The direct communication with any person who is not part of the exam supervision team is prohibited.

- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!

- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.

- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.

- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.

- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.

- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Aufgabe 1 (33 Punkte)

**Task 1** *This part of the task can be solved independently of parts 2 and 3.*

Given the stochastically independent random process $f_a(n)$ with the given distribution function:
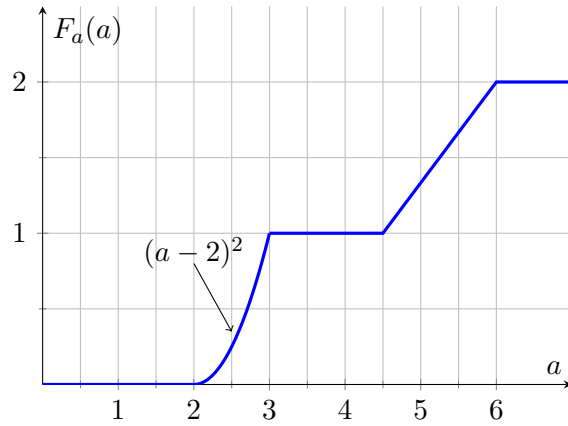
(a) What conditions must the distribution function generally satisfy? Show whether $F_a(a)$ satisfies the conditions and, if necessary, scale the distribution function so that it satisfies the condition. (4 P)

(b) Determine the corresponding probability density function $f_a(a)$. If scaling was done in part *(a)*, use the scaled distribution function. (2 P)

(c) Calculate the second statistical moment of a with your result from (b). (5 P)

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

The following block diagram is given with the constants $\alpha, \beta, \delta \in \mathbb{R} \setminus \{0\}$ and $\gamma \in \mathbb{N} \setminus \{0\}$, where $|\delta| < 1$ also applies. Assume that the system is excited with stationary, zero-mean white noise with power $m_s^{(2)}$.
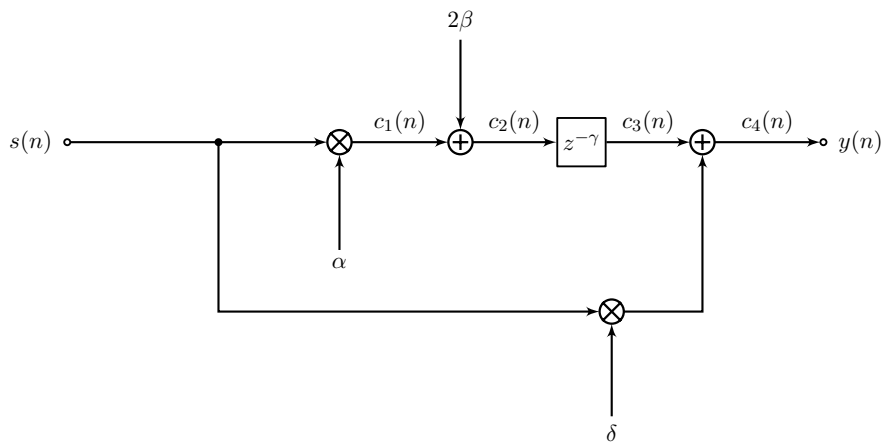
(d) Determine α as a function of the desired power $m_{c_1}^{(2)} > 0$. (2 P)

(e) Give the expected value of $c_2(n)$ as a function of α and β. (2 P)

(f) Show why an arbitrary choice of $\gamma \in \mathbb{N}$ still results in the second central moment of $c_3(n)$ being equal to the power of $c_1(n)$. (4 P)

(g) Determine δ so that $c_4(n)$ corresponds to the specified power $m_{c_4}^{(2)}$. (6 P)

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

Given the random variables x and y with the associated composite probability density $f_{xy}(x, y)$:

$$f_{xy}(x, y) = \begin{cases} P \cdot \sin(2x - y), & \text{for } 0 \leq x < \frac{\pi}{2} \text{and} - \frac{\pi}{2} \leq y < 0 \\ 0, & \text{else} \end{cases}$$

(h) Determine the value of the variable P. (4 P)

(i) Calculate the corresponding boundary densities $f_x(x)$ and $f_y(y)$. (4 P)

Aufgabe 2 (33,5 Punkte)

Part 1   *This part of the task can be solved independently of parts 2 and 3.*

Given a system which is described by the following signal flow graph:
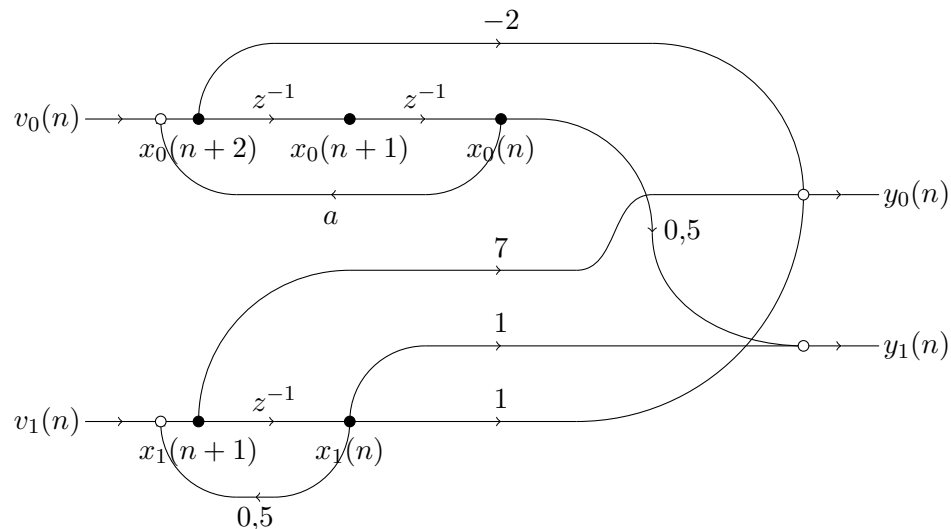
(a) Enter the number L of inputs, N of states and R of outputs of the system. (1,5 P)

(b) Determine the matrices/vectors/scalars $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ for the above system. (4 P)

(c) Determine the characteristic polynomial of the system matrix $\mathbf{A}$. (3 P)

(d) Define a suitable range of values for the parameter a, so that the system is stable. (2,5 P)

Teil 2   *This part of the task can be solved independently of parts 1 and 3.*

Given is the difference equation of the system

$$y(n) = -2y(n-1) + 5v(n) + 10v(n-1) + v(n-4) \quad .$$

(e) Give the formula for the relationship between transfer function $H(z)$, input $V(z)$ and output $Y(z)$. (1 P)

(f) Determine the transfer function $H(z)$ using the difference equation. (3 P)

(g) Give the equation for calculating the transfer function $H(z)$ using the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ of the state space description. (1 P)

(h) Given the matrices (2 P)

$$\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 3 \end{bmatrix} \text{ and } \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}.$$

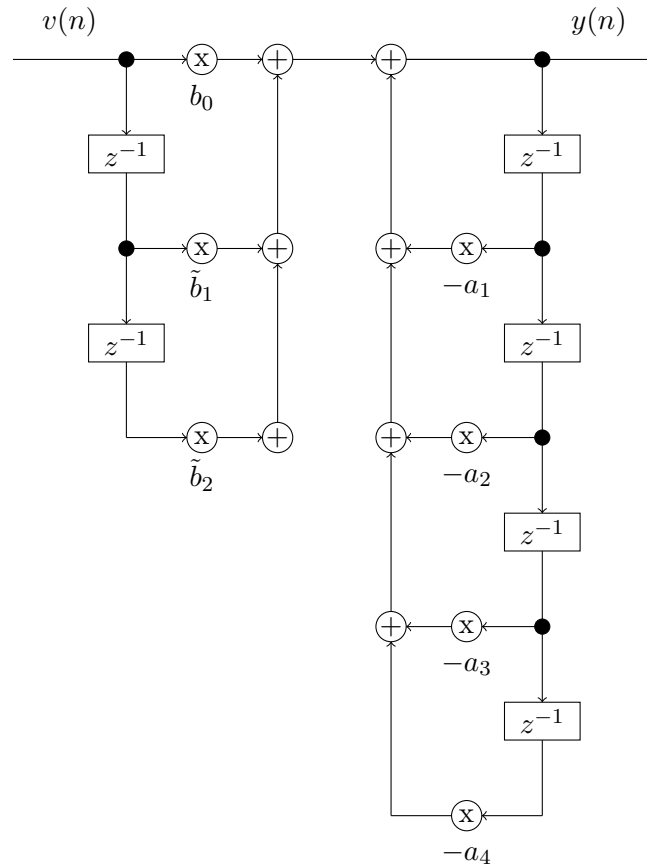Make a statement as to whether this is the same system.

(i) Determine the impulse response $h_0(n)$ of the transfer function (4 P)

$$H(z) = \frac{3z - 0{,}75 + 0{,}5z - 0{,}25}{z^2 - 0{,}75z + 0{,}125} + 1 \quad .$$

Teil 3 *This part of the task can be solved independently of parts 1 and 2.*

For this part of the task, the difference equation and a representation in one of the two direct forms are given.

$$y(n) = -2y(n-1) + y(n-3) - 0{,}5y(n-4) + 0{,}3v(n) - 7v(n-1) - 3v(n-2)$$

(j) Determine all coefficients a_i, and b_i. (3,5 P)

(k) In addition to the direct form shown, there is another direct form. Convert the given system into a realization in the other direct form. (4 P)
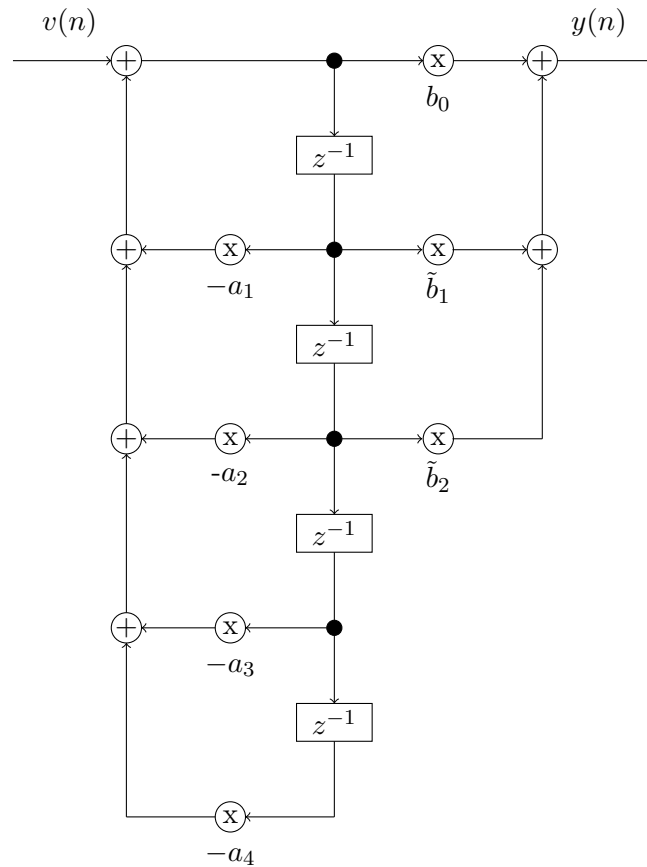
*(l)* Convert the difference equation into a state space description. (4 P)

Aufgabe 3 (33,5 Punkte)

Teil 1 *This part of the task can be solved independently of Part 2 and Part 3.*

(a) What is understood by the modulation of a signal? Name at least three different types of modulation. (2 P)

(b) How does single-sideband modulation differ from double-sideband modulation? Which of the two modulations is preferable to the other and why? (2 P)

Part 2 *This part of the task can be solved independently of Part 1 and Part 3.*

Given is the system from the figure below for transmitting the signal
$v_a(n) = v_1(n) + v_2(n)$ with $v_1(n) = \cos(\Omega_1 n)$ and $v_2(n) = \cos(\Omega_2 n)$, where $\Omega_1 < \Omega_2 < \pi$.
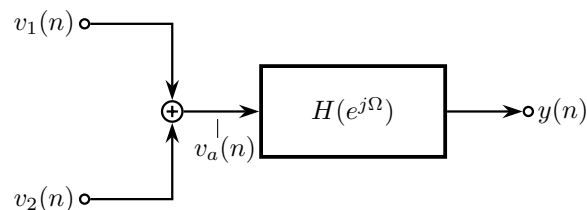
(c) Determine the Fourier transform $V_a\big(e^{j\Omega}\big) \bullet\!\!-\!\!\circ v_a(n)$ and sketch $V_a\big(e^{j\Omega}\big)$ in the range from $-\pi$ to π. (4,5 P)

The unmodulated signal $v_a(n)$ should now be sent via a channel with the real-valued transfer function $H\big(e^{j\Omega}\big)$ from the figure below.
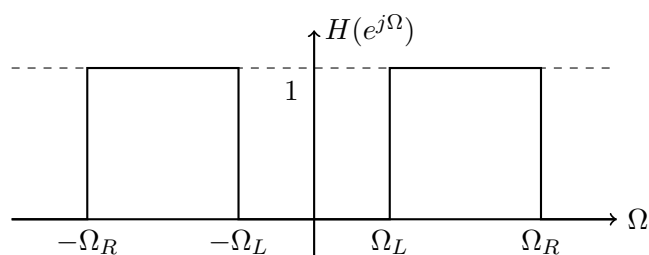
(d) How must the bandwidth of $H\big(e^{j\Omega}\big)$ be and which conditions must the frequencies Ω_L and Ω_R fulfill so that $v_a(n)$ can be completely transmitted with respect to the bandwidth by means of linear modulation over the channel $H\big(e^{j\Omega}\big)$? (3 P)

A modulator $c(n) = 2\cos(\Omega_T n)$ with carrier frequency Ω_T is now inserted between the adder and the transmission channel, resulting in the system shown on the next page.
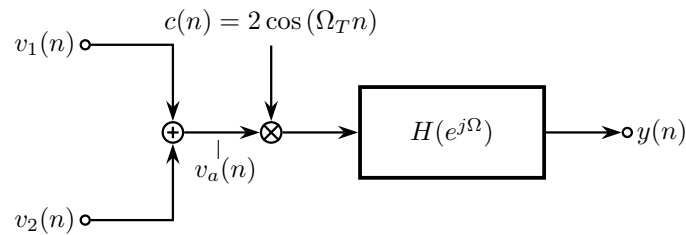
The transmission channel was adjusted accordingly so that all frequencies are still in the passband of the channel. This means that $B = \Omega_R - \Omega_L \geq 2\,\Omega_2$ and $\Omega_2 \leq \frac{\Omega_R - \Omega_L}{2}$.

(e) How must the carrier frequency Ω_T be chosen so that the modulated signal lies in   (2 P) the passband of the channel?

(f) Calculate the Fourier transform of the modulated signal $y(n) = c(n) \cdot [v_1(n) + v_2(n)]$.   (5 P) *Tip: First calculate the Fourier transform of $c(n) \cdot v_1(n)$.*

(g) Sketch the spectrum of the signal $y(n)$.   (3 P)
Tip: Even if you did not successfully determine $y(n)$, you can at least depict the axis system.

(h) By appropriate demodulation, a spectrum is created as already calculated and sketched   (1 P) in part *(c)*. Explain in general terms how you can recover the individual signals $v_1(n)$ and $v_2(n)$ after demodulation.

Teil 3   *This part of the task can be solved independently of Part 1 and Part 2.*

(i) Draw the block diagram for frequency modulation. Also explain which processing   (3 P) step you would have to change to convert the frequency modulation into a phase modulation.

Given is the frequency modulated signal $c_T(t)$:

$$c_T(t) = \hat{c}_T \cos\left(2\pi f_T t + k 2\pi \frac{\hat{v}}{\omega_1} \sin\left(\omega_1 t\right)\right).$$

(j) Calculate the corresponding instantaneous frequency $\Omega(t)$ and specify the signal to   (4 P) be transmitted and modulation index η.

(k) Name the advantages and disadvantages of amplitude and angle modulation.   (4 P)

Dies ist eine leere Seite.