

Signals and Systems II

Exam SS 2025

Examiner: Prof. Dr.-Ing. Gerhard Schmidt
 Date: 01.09.2025
 Name: _____
 Matriculation Number: _____

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.

I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: _____

Marking

Problem	1	2	3
Points	/31	/34	/35

Total number of points: _____ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

- The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated _____ Signature: _____

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Examiner: Prof. Dr.-Ing. Gerhard Schmidt
Room: LS1, Klausr-Murmann-Hörsaal
Date: 01.09.2025
Begin: 09:00 h
Reading Time: 10 minutes
Working Time: 90 minutes

Notes

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time**, **working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Task 1 (31 points)

Part 1 This part of the task can be solved independently of parts 2 and 3.

The following four functions are given:

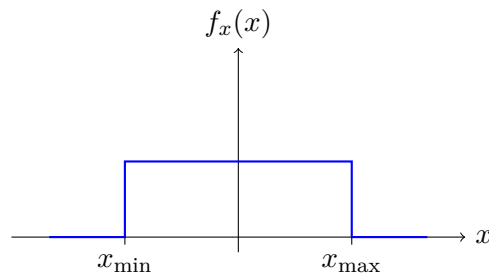
$$F_a(a) = \begin{cases} 1, & \text{for } 1 \leq a < 2 \\ \frac{1}{2}, & \text{for } 2 \leq a < 3 \\ 0, & \text{otherwise} \end{cases} \quad F_c(c) = \begin{cases} 0, & \text{for } c < -1 \\ c, & \text{for } -1 \leq c < 1 \\ 1, & \text{for } c \geq 1 \end{cases}$$

$$F_b(b) = \begin{cases} 0, & \text{for } b < 1 \\ \frac{1}{8}(b-1)^3, & \text{for } 1 \leq b < 3 \\ 1, & \text{for } b \geq 3 \end{cases} \quad F_d(d) = \begin{cases} d^2, & \text{for } 0 \leq d < 2 \\ 1, & \text{for } d \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

- (a) For which of the above functions can you rule out that they are distribution functions of continuous random variables? Justify your answer! Also identify the functions that are distribution functions. (4 P)
Hint: One or more distribution functions may be present.
- (b) Determine the probability density corresponding to the distribution function $F_b(b)$. State the relationship between the distribution function and the probability density. (4 P)
- (c) Determine the first moment, the second moment, and the second central moment of the distribution function $F_b(b)$. (7 P)

Part 2 This part of the task can be solved independently of parts 1 and 3.

The probability density is given as $f(x) = \begin{cases} \frac{1}{x_{\max} - x_{\min}}, & \text{for } x_{\min} \leq x \leq x_{\max} \\ 0, & \text{otherwise} \end{cases}$
 for uniformly distributed white continuous noise with $|x_{\max}| = |x_{\min}|$.



- (d) Specify the mean value m_x and the variance σ_x^2 of the signal $x(t)$. Also sketch its autocorrelation function $s_{xx}(\tau)$ with all necessary details. (6 P)
- (e) When are two processes orthogonal? What condition must the signal $y(t)$ satisfy so that the cross-correlation $s_{x,y}(t_1, t_2)$ is orthogonal? (2 P)
- (f) What does stationarity mean? (1 P)
- (g) What does ergodicity mean? What condition must hold for a signal to be ergodic? (2 P)
- (h) Explain the difference between a deterministic signal and a stochastic signal. (2 P)

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

The random variables x and y are given with the corresponding joint probability density $f_{x,y}(x, y)$:

$$f_{x,y}(x, y) = \begin{cases} 12x^2y & \text{for } 0 \leq x \leq 1, \frac{1}{3} \leq y \leq \frac{2}{3}, \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the corresponding marginal densities $f_x(x)$ and $f_y(y)$. (3 P)

Task 2 (34 points)

Part 1 This part of the task can be solved independently of part 2.

Given the following difference equation of a linear time-invariant system with input $v(n)$ and output $y(n)$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = v(n) + 2v(n-1) .$$

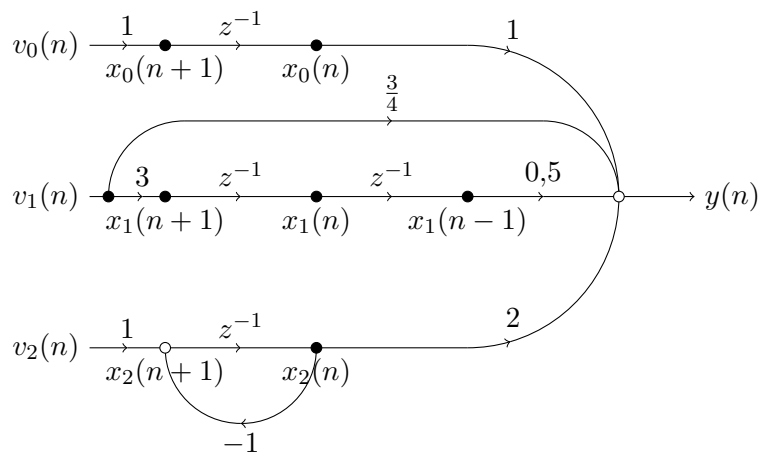
- (a) Is the given system causal? Explain! (1 P)
- (b) What is the transfer function $H(z)$ of the system? (4 P)
- (c) What is the impulse response of the system? (10 P)

Part 2 This part of the task can be solved independently of Part 1.

The state space model is described by the following state equations:

$$\begin{aligned} \mathbf{x}(n+1) &= \mathbf{A}\mathbf{x}(n) + \mathbf{B}\mathbf{v}(n), \\ \mathbf{y}(n) &= \mathbf{C}\mathbf{x}(n) + \mathbf{D}\mathbf{v}(n). \end{aligned}$$

The following system is given in the form of the signal flow graph:



- (d) State the number of inputs, outputs, and states and determine the dimensions of the dynamics matrix \mathbf{A} , the input matrix \mathbf{B} , the output matrix \mathbf{C} , and the dimensions of the pass-through matrix \mathbf{D} . Define the state vector $\mathbf{x}(n)$ for this purpose. (4,5 P)
- (e) Determine the dynamic matrix \mathbf{A} , the input matrix \mathbf{B} , the output matrix \mathbf{C} , and the pass-through matrix \mathbf{D} . (4,5 P)
- (f) Determine the transfer function $H(z)$. (10 P)

Note: If you cannot calculate the inverse $(z\mathbf{I} - \mathbf{A})^{-1}$, you can continue with the

following result:

$$(z\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} \frac{1}{z^2} & 0 & 0 & 2 \\ 0 & \frac{1}{z} & \frac{1}{z} & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 1 & 0 & 0 & \frac{1}{2z-1} \end{bmatrix}$$

Solving the task using the above result as a guide leads to a maximum achievable score of (6 P) instead of the original (10 P) for this part of the task!

Task 3 (35 points)

Part 1 This part of the task can be solved independently of parts 2 and 3.

- (a) What is meant by the modulation of a signal? Name at least three different types of modulation. (2.5 P)

Part 2 This part of the task can be solved independently of parts 1 and 3.

Given is the band-limited signal $V(e^{j\Omega})$ shown in Fig. 1 with a maximum frequency of f_v [Hz], which was sampled at a sampling rate of f_S [Hz]. The discrete spectrum was normalized with $\Omega = 2\pi f/f_S$ [rad].

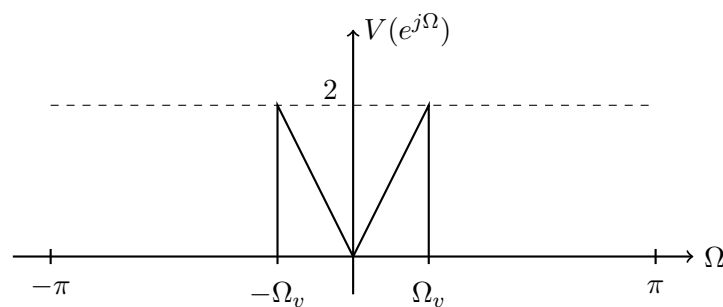


Figure 1: Spectrum of $V(e^{j\Omega})$.

- (b) Which condition must the angular frequency Ω_v fulfill so that no aliasing occurs and which frequency range is then sufficient to consider the signal in order to describe it uniquely? (2 P)
- (c) Look at Fig. 1 and determine the expression $V(e^{j\Omega})$, i.e., the DTFT of this signal as a function of the normalized angular frequency Ω . (2 P)
Tip: Remember the periodicity in the spectrum of the DTFT.
- (d) The signal $v(n) \circ \bullet V(e^{j\Omega})$ is now modulated with the carrier signal $c_1(n) = \cos(\Omega_T n)$. The carrier frequency is $\Omega_v \ll \Omega_T$ and $\Omega_T + \Omega_v \leq \pi$. The resulting spectrum is denoted by $U(e^{j\Omega})$. What type of modulation is this? (1 P)
- (e) Calculate the spectrum $U(e^{j\Omega})$ of the modulated signal and sketch it in the range $-\pi < \Omega < \pi$. (5 P)
*Tip: Simply sketching the **correctly labeled** axis system will earn you points. Pay particular attention to the type of transformation and the notation of all corresponding quantities.*

The signal is now to be transmitted over an ideal channel with unlimited bandwidth i.e., $H(e^{j\Omega}) = 1$. At the same time, another participant transmits the signal shown in Fig. 2, which couples additively into the received signal as an interference signal $b(n)$. The angular frequency of the interference signal is $\Omega_b < \Omega_T - \Omega_v$.

- (f) Calculate the spectrum of the disturbed received signal $S(e^{j\Omega}) = \mathcal{F}\{s(n)\}$ and sketch this in the range of $-\pi < \Omega < \pi$. (3.5 P)

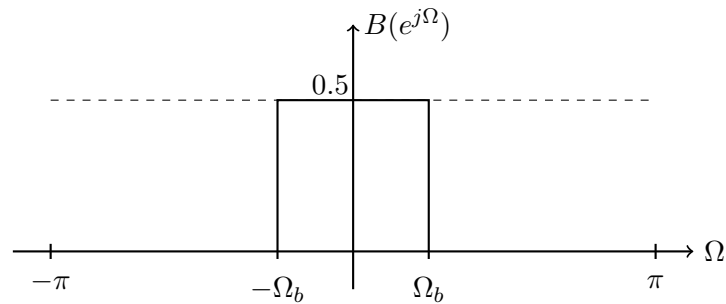


Figure 2: Interference signal from another participant in the channel.

The other channel participant is not adhering to the specifications and is now transmitting in such a way that everything below the carrier frequency Ω_T is severely interfered with. The spectrum of the interference signal is no longer based on Fig. 2, but can be considered arbitrary, since it is unknown at the receiving end.

- (g) How can you theoretically reconstruct your transmitted signal ideally? Describe your procedure step by step and briefly comment on your steps. If you use filters, they should be defined in the range $-\pi < \Omega < \pi$ – outside this range, periodicity is assumed. Make sure that scaling must be performed in the last step of your processing at the latest, so that the desired signal $V(e^{j\Omega})$ is available again without attenuation. (10 P)

Tip: The modulation type used does not need to be changed. The spectrum of the baseband signal can be represented by $V(e^{j\Omega}) = V_{neg}(e^{j\Omega}) + V_{pos}(e^{j\Omega})$, where $V_{neg}(e^{j\Omega})$ describes all negative frequency components of $V(e^{j\Omega})$ and $V_{pos}(e^{j\Omega})$ describes all positive frequency components of $V(e^{j\Omega})$.

Part 3 This part of the task can be solved independently of parts 1 and 2.

The following phase-modulated signal is given:

$$c(t) = \hat{c}_T \cos[\Phi_T(t)] = \hat{c}_T \cos[2\pi f_T t + \eta \cos(2\pi f_N t)].$$

Here, f_N is the frequency of the wanted signal.

- (h) Determine the instantaneous angular frequency $\Omega_T(t)$ of the signal $c(t)$. (3 P)
- (i) Give the frequency deviation Δf for the signal $c(t)$. Explain in your own words the meaning of the frequency deviation. (3 P)
- (j) Give the minimum and maximum FM bandwidth f_{Bmin} and f_{Bmax} for a frequency deviation $\Delta f = 40$ kHz for a useful signal in the range from 100 Hz to 10 kHz. (3 P)

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