Signals and Systems II

Exam SS 2025

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	01.09.2025
Name:	______________________________
Matriculation Number:	______________________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________________

Marking

Problem	1	2	3
Points	/31	/34	/35

Total number of points: __________________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated __________________ Signature: ______________________________

Signals and Systems II

Exam SS 2025

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	LS1, Klausr-Murmann-Hörsaal
Date:	01.09.2025
Begin:	09:00 h
Reading Time:	10 minutes
Working Time:	90 minutes

Notes

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Task 1 (31 points)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

The following four functions are given:

$$F_a(a) = \begin{cases} 1, & \text{for } 1 \leq a < 2 \\ \frac{1}{2}, & \text{for } 2 \leq a < 3 \\ 0, & \text{otherwise} \end{cases} \qquad F_c(c) = \begin{cases} 0, & \text{for } c < -1 \\ c, & \text{for } -1 \leq c < 1 \\ 1, & \text{for } c \geq 1 \end{cases}$$

$$F_b(b) = \begin{cases} 0, & \text{for } b < 1 \\ \frac{1}{8}(b-1)^3, & \text{for } 1 \leq b < 3 \\ 1, & \text{for } b \geq 3 \end{cases} \qquad F_d(d) = \begin{cases} d^2, & \text{for } 0 \leq d < 2 \\ 1, & \text{for } d \geq 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) For which of the above functions can you rule out that they are distribution functions of continuous random variables? Justify your answer! Also identify the functions that are distribution functions. (4 P)
Hint: One or more distribution functions may be present.
$F_a(a)$ and $F_d(d)$ are not monotonically increasing and can therefore not be distribution functions. $F_c(c)$ takes negative values and can therefore not be a distribution function. $F_b(b)$ is a distribution function.

(b) Determine the probability density corresponding to the distribution function $F_b(b)$. State the relationship between the distribution function and the probability density. (4 P)

It holds that $f_b(b) = \frac{d}{db} F_b(b)$.

$$f_b(b) = \begin{cases} \frac{3}{8}(b-1)^2, & \text{for } 1 \leq b < 3, \\ 0, & \text{otherwise.} \end{cases}$$

(c) Determine the first moment, the second moment, and the second central moment of the distribution function $F_b(b)$. (7 P)

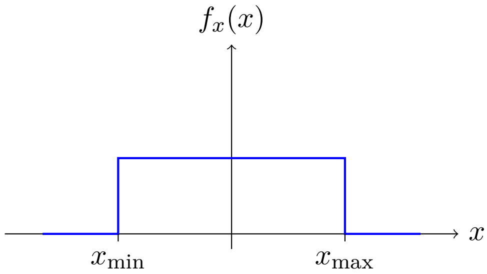
$$
\begin{aligned}
m_b^{(1)} = m_b &= \int_{-\infty}^{\infty} b f_b(b) db \\
&= \int_1^3 \frac{3}{8} b(b-1)^2 db \\
&\text{Substitute: } u = b - 1 \rightarrow \frac{du}{db} = 1 \\
&= \frac{3}{8} \int_{u(1)=0}^{2} (u+1)u^2 du \\
&= \frac{3}{8} \int_0^2 u^3 + u^2 du \\
&= \frac{3}{8} \left[\frac{1}{4} u^4 + \frac{1}{3} u^3 \right]_0^2 \\
&= \frac{5}{2}
\end{aligned}
$$

$$
\begin{aligned}
m_b^{(2)} &= \int_{-\infty}^{\infty} b^2 f_b(b) db \\
&= \int_1^3 \frac{3}{8} b^2 (b-1)^2 db \\
&\text{Expand} \\
&= \frac{3}{8} \left(\int_1^3 b^4 - 2b^3 + b^2 \right) db \\
&= \frac{3}{8} \left[\frac{1}{5} b^5 - \frac{1}{2} b^4 + \frac{1}{3} b^3 \right]_1^3 \\
&= \frac{32}{5}
\end{aligned}
$$

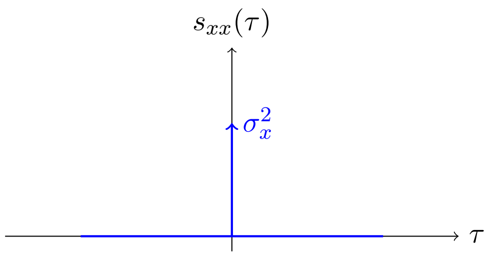
$$
\sigma_b^2 = m_b^{(2)} - m_b^2 = \frac{3}{20}
$$

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

The probability density is given as $f(x) = \begin{cases} \frac{1}{x_{\max}-x_{\min}}, & \text{for } x_{\min} \leq x \leq x_{\max} \\ 0, & \text{otherwise} \end{cases}$
for uniformly distributed white continuous noise with $|x_{\max}| = |x_{\min}|$.



(d) Specify the mean value m_x and the variance σ_x^2 of the signal $x(t)$. Also sketch its autocorrelation function $s_{xx}(\tau)$ with all necessary details. (6 P)
$m_x = \frac{x_{\max}+x_{\min}}{2} = 0$
$\sigma_x^2 = \frac{1}{12}(x_{\max} - x_{\min})^2$
The autocorrelation function can be sketched according to: $s_{xx}(\tau) = m_x^2 + \sigma_x^2 \cdot \gamma_0(\tau)$



(e) When are two processes orthogonal? What condition must the signal $y(t)$ satisfy so that the cross-correlation $s_{x,y}(t_1, t_2)$ is orthogonal? (2 P)
Two signals are orthogonal if the following condition for the cross-correlation holds:

$$s_{x,y}(t_1, t_2) = 0$$

Since $x(t)$ already has zero mean, it is sufficient if the signals $x(t)$ and $y(t)$ are uncorrelated.

(f) What does stationarity mean? (1 P)
Stationarity means that the statistical properties of the signal do not change over time.

(g) What does ergodicity mean? What condition must hold for a signal to be ergodic? (2 P)
Ergodicity means that the ensemble averages equal the time averages. Ergodicity requires that a signal is stationary.

(h) Explain the difference between a deterministic signal and a stochastic signal. (2 P)
A deterministic signal is fully predictable and can be described exactly mathematically. A stochastic signal is not fully predictable; it contains random components.

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

The random variables x and y are given with the corresponding joint probability density $f_{x,y}(x, y)$:

$$f_{x,y}(x, y) = \begin{cases} 12x^2y & \text{for } 0 \leq x \leq 1,\ \frac{1}{3} \leq y \leq \frac{2}{3}, \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the corresponding marginal densities $f_x(x)$ and $f_y(y)$. (3 P)

$$f_x(x) = \int_{\frac{1}{3}}^{\frac{2}{3}} f_{x,y}(x, y)\, dy = \begin{cases} 2x^2, & \text{for } 0 \leq x \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

$$f_y(y) = \int_{0}^{1} f_{x,y}(x, y)\, dx = \begin{cases} 4y, & \text{for } \frac{1}{3} \leq y \leq \frac{2}{3} \\ 0, & \text{otherwise} \end{cases}$$

Task 2 (34 points)

Part 1 *This part of the task can be solved independently of part 2.*

Given the following difference equation of a linear time-invariant system with input $v(n)$ and output $y(n)$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = v(n) + 2v(n-1) \ .$$

(a) Is the given system causal? Explain! (1 P)

Since only current and past values are used, the system is causal.

(b) What is the transfer function $H(z)$ of the system? (4 P)

$$H(z) = \frac{Y(z)}{V(z)} = \frac{1 + 2z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

(c) What is the impulse response of the system? (10 P)

Use of the transfer function from the previous part of the task:

$$H(z) = \frac{Y(z)}{V(z)} = \frac{1 + 2z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Multiply with z^2:

$$H(z) = \frac{Y(z)}{V(z)} = \frac{z^2 + 2z}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

Applying partial fraction decomposition:
Determining the zeros of the denominator:

$$z^2 - \frac{3}{4}z + \frac{1}{8} = 0$$

Use of the pq formula:

$$z_0 = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

with $p = -\frac{3}{4}$ and $q = \frac{1}{8}$

$$z_0 = \frac{3}{8} \pm \sqrt{\left(-\frac{3}{8}\right)^2 - \frac{1}{8}}$$

$z_{0,1} = \frac{1}{2}$, $z_{0,2} = \frac{1}{4}$.
Representation in partial fractions:

$$H(z) = \frac{z^2 + 2z}{(z - \frac{1}{2})(z - \frac{1}{4})}$$

Partial fraction decomposition:

$$H(z) = B_0 + \sum_{\mu=1}^{k} \frac{B_\mu z}{z - z_{\infty,\mu}}$$

$$B_0 = H(0) = \frac{\alpha_0}{\beta_0}$$

$$B_0 = H(0) = 0$$

$$B_\mu = \lim_{z \to z_{\infty,\mu}} \left[\frac{H(z)}{z} (z - z_{\infty,\mu}) \right]$$

Calculation of B_1 with $z_{\infty,1} = \frac{1}{2}$

$$\begin{aligned} B_1 &= \lim_{z \to \frac{1}{2}} \left[\frac{H(z)}{z} (z - \frac{1}{2}) \right] \\ &= \lim_{z \to \frac{1}{2}} \left[\frac{z+2}{(z - \frac{1}{2})(z - \frac{1}{4})} (z - \frac{1}{2}) \right] \\ &= \lim_{z \to \frac{1}{2}} \left[\frac{z+2}{(z - \frac{1}{4})} \right] \\ &= \frac{\frac{10}{4}}{\frac{1}{4}} = 10 \end{aligned}$$

Calculation of B_2 with $z_{\infty,2} = \frac{1}{4}$

$$\begin{aligned} B_1 &= \lim_{z \to \frac{1}{4}} \left[\frac{H(z)}{z} (z - \frac{1}{4}) \right] \\ &= \lim_{z \to \frac{1}{4}} \left[\frac{z+2}{(z - \frac{1}{2})(z - \frac{1}{4})} (z - \frac{1}{4}) \right] \\ &= \lim_{z \to \frac{1}{4}} \left[\frac{z+2}{(z - \frac{1}{2})} \right] \\ &= \frac{\frac{9}{4}}{-\frac{1}{4}} = -9 \end{aligned}$$

This results in the representation of the transfer function in partial fractions

$$H(z) = 0 + \frac{10z}{z - \frac{1}{2}} - \frac{9z}{z - \frac{1}{4}}$$

Using

$$h_0(n) = B_0 \gamma_0(n) + \sum_{\mu=1}^{k} B_\mu z_{\infty,\mu}^n \gamma_{-1}(n)$$

is obtained by inserting the coefficients

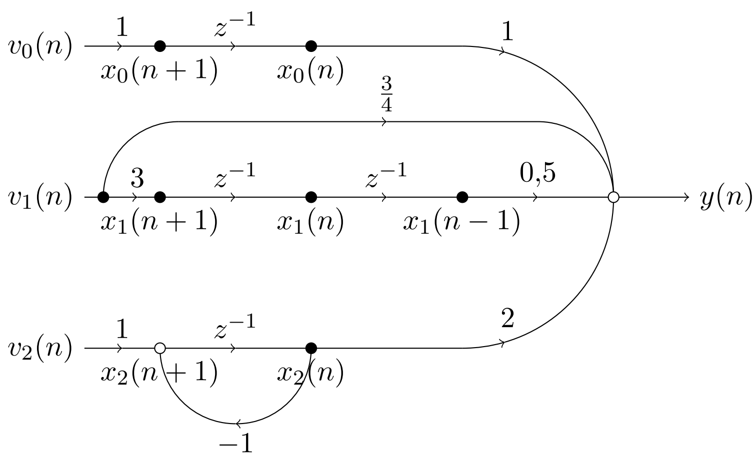
$$h_0(n) = 10 \cdot \left(\frac{1}{2}\right)^n \gamma_{-1}(n) - 9 \cdot \left(\frac{1}{4}\right)^n \gamma_{-1}(n)$$

Part 2 *This part of the task can be solved independently of Part 1.*

The state space model is described by the following state equations:

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\boldsymbol{x}(n) + \boldsymbol{B}\boldsymbol{v}(n),$$
$$\boldsymbol{y}(n) = \boldsymbol{C}\boldsymbol{x}(n) + \boldsymbol{D}\boldsymbol{v}(n).$$

The following system is given in the form of the signal flow graph:



(d) State the number of inputs, outputs, and states and determine the dimensions of the dynamics matrix $\boldsymbol{A}$, the input matrix $\boldsymbol{B}$, the output matrix $\boldsymbol{C}$, and the dimensions of the pass-through matrix $\boldsymbol{D}$. Define the state vector $\boldsymbol{x}(n)$ for this purpose. (4,5 P)

$$\boldsymbol{x}(n) = \begin{bmatrix} x_0(n) \\ x_1(n-1) \\ x_1(n) \\ x_2(n) \end{bmatrix}$$

$$\begin{aligned} \text{Inputs} &:= 3 \\ \text{Outputs} &:= 1 \\ \text{States} &:= 4 \\ \boldsymbol{A} &:= [4 \times 4] \\ \boldsymbol{B} &:= [4 \times 3] \\ \boldsymbol{C} &:= [1 \times 4] \\ \boldsymbol{D} &:= [1 \times 3] \end{aligned}$$

(e) Determine the dynamic matrix $\boldsymbol{A}$, the input matrix $\boldsymbol{B}$, the output matrix $\boldsymbol{C}$, and the pass-through matrix $\boldsymbol{D}$. (4,5 P)

The dynamics matrix $\boldsymbol{A}$ represents the influence of past states on current states.

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The input matrix $\boldsymbol{B}$ describes the influence of the inputs on the states.

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The output matrix $\boldsymbol{C}$ describes the influence of the states on the output.

$$\boldsymbol{C} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 2 \end{bmatrix}$$

The pass-through matrix $\boldsymbol{D}$ describes the direct pass-through of the inputs to the output.

$$\boldsymbol{D} = \begin{bmatrix} 0 & \frac{3}{4} & 0 \end{bmatrix}$$

(f) Determine the transfer function $H(z)$. (10 P)

Note: If you cannot calculate the inverse $(z\boldsymbol{I} - \boldsymbol{A})^{-1}$, you can continue with the following result:

$$(z\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} \frac{1}{z^2} & 0 & 0 & 2 \\ 0 & \frac{1}{z} & \frac{1}{z} & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 1 & 0 & 0 & \frac{1}{2z-1} \end{bmatrix}$$

Solving the task using the above result as a guide leads to a maximum achievable score of (6 P) instead of the original (10 P) for this part of the task!

Calculating the transfer function $\boldsymbol{H}(z)$

$$\boldsymbol{H}(z) = \boldsymbol{C}\,(z\boldsymbol{I} - \boldsymbol{A})^{-1}\,\boldsymbol{B} + \boldsymbol{D}\ .$$

Intermediate step 1:

$$z\boldsymbol{I} - \boldsymbol{A} = \begin{bmatrix} z & 0 & 0 & 0 \\ 0 & z & -1 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & z+1 \end{bmatrix}$$

Intermediate step 2:

$$(z\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} z & 0 & 0 & 0 \\ 0 & z & -1 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & z+1 \end{bmatrix}^{-1}$$

Invert by transforming to the identity matrix

$$\left[\begin{array}{cccc|cccc} z & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & z & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & z+1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Zeile 1: Division by z
Zeile 3: Division by z
Zeile 4: Division by $z+1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{z} & 0 & 0 & 0 \\ 0 & z & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{z+1} \end{array}\right]$$

Addition line 2 + line 3

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{z} & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 & 1 & \frac{1}{z} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{z+1} \end{array}\right]$$

Division line 2 by z

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{z} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{z} & \frac{1}{z^2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{z+1} \end{array}\right]$$

Resulting in:

$$(z\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix} \frac{1}{z} & 0 & 0 & 0 \\ 0 & \frac{1}{z} & \frac{1}{z^2} & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{z+1} \end{bmatrix}$$

Intermediate step 3:

$$\boldsymbol{C}\,(z\boldsymbol{I}-\boldsymbol{A})^{-1} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{z} & 0 & 0 & 0 \\ 0 & \frac{1}{z} & \frac{1}{z^2} & 0 \\ 0 & 0 & \frac{1}{z} & 0 \\ 0 & 0 & 0 & \frac{1}{z+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{z} & \frac{1}{2z} & \frac{1}{2z^2} & \frac{2}{z+1} \end{bmatrix}$$

Intermediate step 4:

$$\boldsymbol{C}\,(z\boldsymbol{I}-\boldsymbol{A})^{-1}\,\boldsymbol{B} = \begin{bmatrix} \frac{1}{z} & \frac{1}{2z} & \frac{1}{2z^2} & \frac{2}{z+1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{z} & \frac{3}{2z^2} & \frac{2}{z+1} \end{bmatrix}$$

Intermediate step 5:

$$\boldsymbol{C}\,(z\boldsymbol{I}-\boldsymbol{A})^{-1}\,\boldsymbol{B}+\boldsymbol{D} = \begin{bmatrix} \frac{1}{z} & \frac{3}{2z^2} & \frac{2}{z+1} \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{4} & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{z} & \frac{3}{2z^2}+\frac{3}{4} & \frac{2}{z+1} \end{bmatrix}$$

Final result:

$$\boldsymbol{H}(z) = \begin{bmatrix} \frac{1}{z} & \frac{6+3z^2}{4z^2} & \frac{2}{z+1} \end{bmatrix}$$

Task 3 (35 points)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

(a) What is meant by the modulation of a signal? Name at least three different types of modulation. (2.5 P)

Signal modulation is the spectral shift of a desired signal based on a carrier signal.

Modulation types: Amplitude modulation (AM), phase modulation (PM), and frequency modulation (FM).

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

Given is the band-limited signal $V(e^{j\Omega})$ shown in Fig. 1 with a maximum frequency of f_v [Hz], which was sampled at a sampling rate of f_S [Hz]. The discrete spectrum was normalized with $\Omega = 2\pi f / f_S$ [rad].

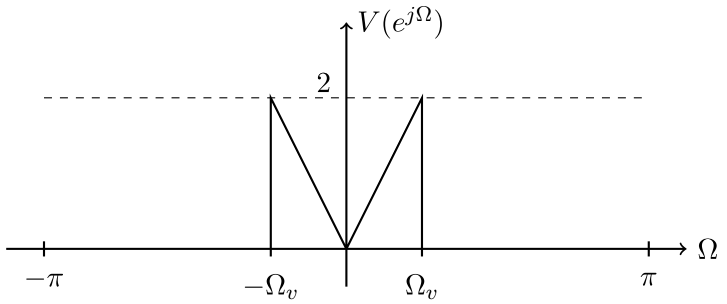


Figure 1: Spectrum of $V(e^{j\Omega})$.

(b) Which condition must the angular frequency Ω_v fulfill so that no aliasing occurs and which frequency range is then sufficient to consider the signal in order to describe it uniquely? (2 P)

The sampling theorem is $f_S \geq 2 \cdot f_{max}$. This results in a Nyquist frequency of $f_{max} = \frac{f_S}{2}$. Normalized, this yields $\Omega_{max} = 2\pi f_{max}/f_S = 2\pi \frac{f_S}{2}/f_S = \pi$. This means that the maximum angular frequency occurring in the signal must be less than or equal to π.

Sufficient answer:
For the angular frequency, $\Omega_v \leq \pi$ must be satisfied.
The relevant frequency range is $\Omega \in [-\pi, \pi]$.

(2 P)

(c) Look at Fig. 1 and determine the expression $V(e^{j\Omega})$, i.e., the DTFT of this signal as a function of the normalized angular frequency Ω.
Tip: Remember the periodicity in the spectrum of the DTFT.

$$V(e^{j\Omega}) = \begin{cases} 2 \cdot |\frac{\Omega - \lambda \cdot 2\pi}{\Omega_v}| & \text{for } \lambda \cdot 2\pi - \Omega_v \leq \Omega \leq \lambda \cdot 2\pi + \Omega_v \text{ with } \lambda \in \mathbb{Z} \\ 0 & \text{else} \end{cases}$$

(d) The signal $v(n) \circ\!\!-\!\!\bullet V(e^{j\Omega})$ is now modulated with the carrier signal $c_1(n) = \cos(\Omega_{\mathrm{T}} n)$. The carrier frequency is $\Omega_v \ll \Omega_{\mathrm{T}}$ and $\Omega_{\mathrm{T}} + \Omega_v \leq \pi$. The resulting spectrum is denoted by $U(e^{j\Omega})$. What type of modulation is this? (1 P)

The modulation used here is double-sideband modulation (DSB modulation for short).

(e) Calculate the spectrum $U(e^{j\Omega})$ of the modulated signal and sketch it in the range $-\pi < \Omega < \pi$. (5 P)
Tip: Simply sketching the ***correctly labeled*** *axis system will earn you points. Pay particular attention to the type of transformation and the notation of all corresponding quantities.*

$$u(n) = v(n) \cdot c_1(n)$$

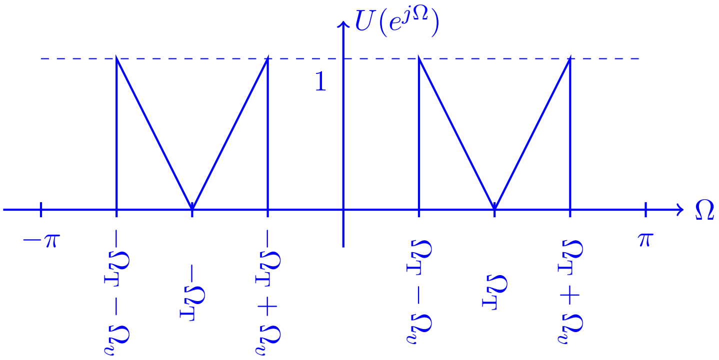
$$\bullet\!\!-\!\!\circ$$

$$U(e^{j\Omega}) = \mathcal{F}\{v(n) \cdot c_1(n)\}$$
$$= \mathcal{F}\{v(n) \cdot \cos(\Omega_{\mathrm{T}} n)\}$$

Applying the correspondence for cosine term and multiplication in the time domain:
$\cos(\Omega_v n) \circ\!\!-\!\!\bullet \pi \sum_{\lambda=-\infty}^{\infty} [\delta_0(\Omega + \Omega_v - 2\pi\lambda) + \delta_0(\Omega - \Omega_v - 2\pi\lambda)]$

$$U(e^{j\Omega}) = \frac{1}{2\pi} V(e^{j\Omega}) \circledast \pi \sum_{\lambda=-\infty}^{\infty} [\delta_0(\Omega + \Omega_T - 2\pi\lambda) + \delta_0(\Omega - \Omega_T - 2\pi\lambda)]$$
$$= \frac{1}{2}\left[V(e^{j(\Omega+\Omega_T)}) + V(e^{j(\Omega-\Omega_T)})\right]$$

0.5 for correct time domain solution, 0.5 for rearrangement, 0.5 for correct cosine correspondence, 0.5 for correct multiplication correspondence, 1 for complete solution at the end, 2 for sketch (0.5 for correct axis system, 0.5 for correct frequencies and amplitude, 0.5 per signal projection)



The signal is now to be transmitted over an ideal channel with unlimited bandwidth i.e., $H(e^{j\Omega}) = 1$. At the same time, another participant transmits the signal shown in Fig. 2, which couples additively into the received signal as an interference signal $b(n)$. The angular frequency of the interference signal is $\Omega_b < \Omega_T - \Omega_v$.

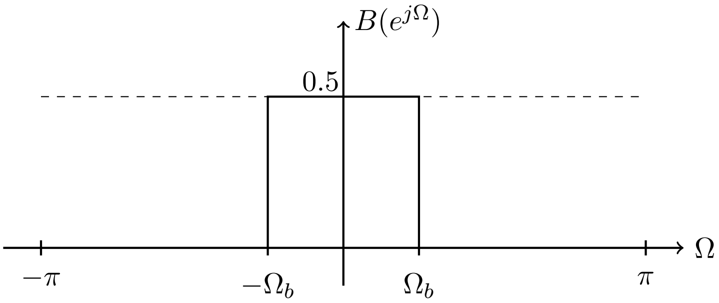


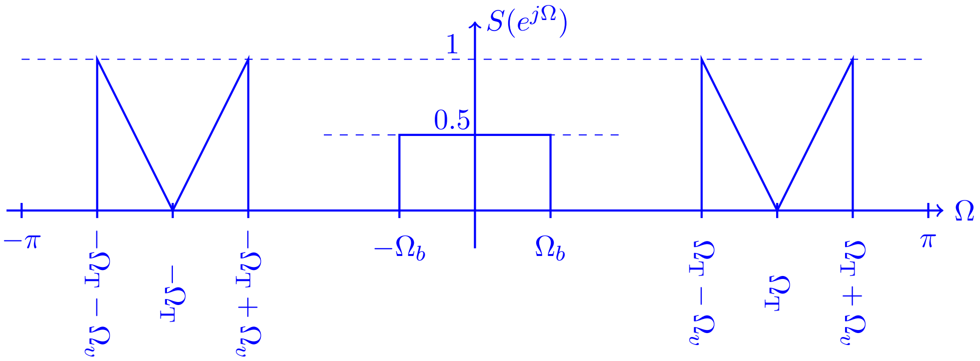
Figure 2: Interference signal from another participant in the channel.

(f) Calculate the spectrum of the disturbed received signal $S(e^{j\Omega}) = \mathcal{F}\{s(n)\}$ and sketch this in the range of $-\pi < \Omega < \pi$. (3.5 P)

$S(e^{j\Omega}) = \mathcal{F}\{s(n)\}$ results in

$$S(e^{j\Omega}) = U(e^{j\Omega}) + B(e^{j\Omega})$$

2.5 for sketch (0.5 for axis system correct, 0.5 for frequencies and amplitude correct, 0.5 per signal projection correct)



The other channel participant is not adhering to the specifications and is now transmitting in such a way that everything below the carrier frequency Ω_T is severely interfered with. The spectrum of the interference signal is no longer based on Fig. 2, but can be considered arbitrary, since it is unknown at the receiving end.

(g) How can you theoretically reconstruct your transmitted signal ideally? Describe your procedure step by step and briefly comment on your steps. If you use filters, they should be defined in the range $-\pi < \Omega < \pi$ – outside this range, periodicity is assumed. Make sure that scaling must be performed in the last step of your processing at the latest, so that the desired signal $V(e^{j\Omega})$ is available again without attenuation. (10 P)
Tip: The modulation type used does not need to be changed. The spectrum of the baseband signal can be represented by $V(e^{j\Omega}) = V_{neg}(e^{j\Omega}) + V_{pos}(e^{j\Omega})$, where $V_{neg}(e^{j\Omega})$ describes all negative frequency components of $V(e^{j\Omega})$ and $V_{pos}(e^{j\Omega})$ describes all positive frequency components of $V(e^{j\Omega})$.

Due to the DSB modulation, the baseband signal is redundant.
1.) With appropriate ideal high-pass filtering, all noise components can be removed before demodulation.

$$H_\mathrm{HP}(e^{j\Omega}) = \begin{cases} 1 & \Omega_\mathrm{T} \leq |\Omega| \leq \pi \\ 0 & \text{else} \end{cases}$$

The resulting signal is:

$$\begin{aligned} S_\mathrm{HP}(e^{j\Omega}) &= S(e^{j\Omega}) \cdot H_\mathrm{HP}(e^{j\Omega}) \\ &= \frac{1}{2}\left[V_\mathrm{neg}(e^{j(\Omega+\Omega_T)}) + V_\mathrm{pos}(e^{j(\Omega-\Omega_T)})\right] \end{aligned}$$

2.) Coherent demodulation with $c_2(n) = \cos(\Omega_\mathrm{T} n)$ yields the reconstructed spectrum:

$$
\begin{aligned}
Y\left(e^{j\Omega}\right) &= \mathcal{F}\left\{s_{\text{HP}}(n)\cdot c_2(n)\right\} \\
&= \underbrace{\frac{1}{2}\left[V_{\text{neg}}\left(e^{j\Omega}\right)+V_{\text{pos}}\left(e^{j\Omega}\right)\right]}_{V\left(e^{j\Omega}\right)} + \frac{1}{4}\left[V_{\text{neg}}\left(e^{j(\Omega+2\Omega_T)}\right)+V_{\text{pos}}\left(e^{j(\Omega-2\Omega_T)}\right)\right] \\
&= \frac{1}{4}V\left(e^{j\Omega}\right) + \frac{1}{4}\left[V_{\text{neg}}\left(e^{j(\Omega+2\Omega_T)}\right)+V_{\text{pos}}\left(e^{j(\Omega-2\Omega_T)}\right)\right]
\end{aligned}
$$

3.) Low-pass filtering to remove noise at twice the carrier frequencies:

$$
H_{\text{TP}}\left(e^{j\Omega}\right) = \begin{cases} 4 & 0 \le |\Omega| \le \Omega_v \\ 0 & \text{else} \end{cases}
$$

The resulting signal is:

$$
\begin{aligned}
Y_{\text{TP}}\left(e^{j\Omega}\right) &= Y\left(e^{j\Omega}\right)\cdot H_{\text{TP}}\left(e^{j\Omega}\right) \\
&= 4\cdot\left[\frac{1}{4}V\left(e^{j\Omega}\right) + \underbrace{\cancel{\frac{1}{4}\left[V_{\text{neg}}\left(e^{j(\Omega+2\Omega_T)}\right)+V_{\text{pos}}\left(e^{j(\Omega-2\Omega_T)}\right)\right]}}_{\text{Eliminated by low-pass}}\right] \\
&= V\left(e^{j\Omega}\right)
\end{aligned}
$$

1 for recognizing the need for HP, 1 for correctly writing down HP, 2 writing down the correct output signal, 1 recognizing coherent demodulation or mentioning ESB demodulation in general, 2 writing down $Y\left(e^{j\Omega}\right)$ correctly, 1 for recognizing the need for LP, 1 for correctly writing down LP, 1 writing down the correct output signal

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

The following phase-modulated signal is given:

$$
c(t) = \hat{c}_{\text{T}}\cos\left[\Phi_{\text{T}}(t)\right] = \hat{c}_{\text{T}}\cos\left[2\pi f_{\text{T}}t + \eta\cos(2\pi f_{\text{N}}t)\right].
$$

Here, f_{N} is the frequency of the wanted signal.

(h) Determine the instantaneous angular frequency $\Omega_{\text{T}}(t)$ of the signal $c(t)$. (3 P)

$$
\begin{aligned}
\Omega_{\text{T}}(t) &= \frac{d\Phi_{\text{T}}(t)}{dt} \\
&= 2\pi f_{\text{T}} - \eta\sin(2\pi f_{\text{N}}t)2\pi f_{\text{N}}
\end{aligned}
$$

(3 P)

(i) Give the frequency deviation Δf for the signal $c(t)$. Explain in your own words the meaning of the frequency deviation.

$$
\Delta f = \eta f_{\text{N}}
$$

The frequency deviation Δf is the maximum deviation of the instantaneous frequency from the constant carrier frequency f_T.

(j) Give the minimum and maximum FM bandwidth f_Bmin and f_Bmax for a frequency deviation $\Delta f = 40$ kHz for a useful signal in the range from 100 Hz to 10 kHz. (3 P)

$$f_\mathrm{B} = 2\left(\frac{\Delta f}{f_1} + 2\right) f_1$$

$$f_\mathrm{Bmin} = 2\left(\frac{40\ \mathrm{kHz}}{100\ \mathrm{Hz}} + 2\right) 100\ \mathrm{Hz} = 80{,}4\ \mathrm{kHz}$$

$$f_\mathrm{Bmax} = 2\left(\frac{40\ \mathrm{kHz}}{10\ \mathrm{kHz}} + 2\right) 10\ \mathrm{kHz} = 120\ \mathrm{kHz}$$

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