TECHNISCHE FAKULTÄT DER CHRISTIAN-ALBRECHTS-UNIVERSITÄT ZU KIEL

DIGITALE SIGNALVERARBEITUNG UND SYSTEMTHEORIE

Signals and Systems II

Exam WS 2023/2024

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	01.03.2024
Name:	____________________
Matriculation Number:	____________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ____________________

Marking

Problem	1	2	3
Points	/33	/33	/34

Total number of points: ____________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ____________ Signature: ____________________

Signals and Systems II

Exam WS 2023/2024

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	OS75, Hörsaal 1 und 2
Date:	01.03.2024
Begin:	09:00 h
Reading Time:	10 Minutes
Working Time:	90 Minutes

Hinweise

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Task 1 (33 Points)

Teil 1 *This part of the task can be solved independently of parts 2 and 3.*

Let the probability distribution $F_k(k)$ with the unknown variables $\alpha, \beta, \gamma \in \mathbb{R}$ be given by:

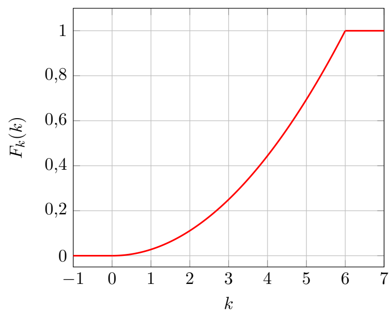
$$F_k(k) = \begin{cases} \frac{3}{2}\alpha, & \text{for } k < 0 \\ (\frac{2}{\beta}k)^2, & \text{for } 0 \leq k \leq 6 \\ 4\gamma, & \text{else} \end{cases}$$

(a) Determine the unkowns α, β, γ. (3 P)
$\alpha = 0, \ \beta = \pm 12, \ \gamma = \frac{1}{4}$

(b) Determine the corresponding probability density $f_k(k)$ with the results from *(a)*. (3 P)
Es gilt $f_k(k) = \frac{\mathrm{d}}{\mathrm{d}k} F_k(k)$.

$$f_k(k) = \begin{cases} \frac{k}{18}, & \text{für } 0 \leq k \leq 6 \\ 0, & \text{sonst} \end{cases}$$

(c) Sketch the probability distribution $F_k(k)$ for the area $-1 \leq k \leq 7$ by using your results from *(a)*. (2 P)



Teil 2 *This part of the task can be solved independently of parts 1 and 3.*

(d) Let be a linear, real-valued, approximately ideal filter that is excited with uniformly distributed noise. Furthermore, figures A-H (figures on the last page of task 1), which describe the signal and system and are shown either measured or idealized, are given. Assign the following labels 1-8 to these figures and explain your answers in detail (12 P)

1. Frequency response $H(j\omega)$

2. Impulse response $h_0(t)$

3. Probability density function (PDF) of the input noise signal $p_v(V)$

4. Autocorrelation function of the input noise signal $s_{vv}(\tau)$

5. Step response $h_{-1}(t)$

6. Autocovariance function of the input noise signal $\psi_{vv}(\tau)$

7. Probability density function (PDF) of the output signal $p_y(Y)$

8. Autocorrelation function of the output signal $s_{yy}(\tau)$.

1. Frequency response $H(j\omega)$: ideal filter, i.e. square wave as frequency response; real-valued filter, i.e. frequency response is symmetrical to the y-axis $\rightarrow$ E.

2. Impulse response $h_0(t)$: Inverse Fourier transform of the square wave function in E results in an si function $\rightarrow$ D.

3. Probability density function (PDF) of the input noise signal $p_v(V)$: A, C and H should be excluded because these functions are even and are therefore reserved for the autocorrelations and autocovariance, therefore the PDFs can only be F and G. Using the theorem of mathematics, for an infinite or sufficiently long impulse response, the bell curve for the distribution density of the output signal results, which means that the VDF of the input is $\rightarrow$ G.

4. Autocorrelation function of the input signal $s_{vv}(\tau)$: The input signal - as can be seen in G - has a mean value. The autocorrelation function of the input noise signal is therefore $\rightarrow$ C.

5. Step response $h_{-1}(t)$: Relationship between step and impulse response:
$h_{-1}(t) = \int\limits_{-\infty}^{t} h_0(\tau)d\tau \rightarrow$ B.

6. Autocovariance function of the input noise signal $\psi_{vv}(\tau)$: The autocovariance function corresponds to the autocorrelation function minus the mean squared $\rightarrow$ H.

7. Probability density function (PDF) of the output noise signal $p_y(Y)$: see point 3 $\rightarrow$ F.

8. Autocorrelation function of the output signal $s_{yy}(\tau)$: The autocorrelation function of the output signal results from the (double) convolution of the function C with the function D $\rightarrow$ A.

The input noise signal is now discretized so that the discrete stochastic process $v(k)$ with the probability density function $p_v(V)$ results.

(e) Is $v(k)$ a stationary process? Explain your answer. (1 P)
$v(k)$ is a stationary process since $p_v(V)$ is independent of k.

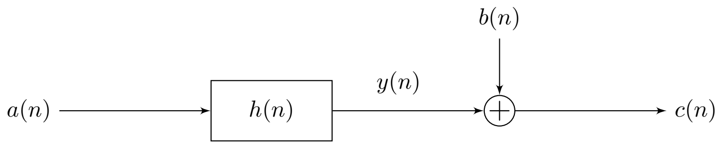
(f) What must be true for for ergodicity to exist? Provide the definition. (1 P)
A stationary random process is called ergodic if the time averages of any sample function (realization) agree with the corresponding group averages with probability one.

Teil 3 *This part of the task can be solved independently of parts 1 and 2.*

The real-valued and ergodic random process $a(n)$ is given.

(g) Is the random process stationary? Explain! (1 P)
Yes, because ergodicity requires stationarity.

The random process $a(n)$ should now be transmitted via the following linear time-invariant system:



In the following subtasks it can be assumed that $b(n)$ and $y(n)$ are orthogonal to each other.

(h) Determine the autocorrelation sequence $s_{cc}(\kappa)$ depending on the correlation sequences of $b(n)$ and $a(n)$, as well as the impulse response $h(n)$. (2 P)

$$\begin{aligned} s_{cc}(\kappa) &= E\{c(n)c(n+\kappa)\} \\ &= E\{(y(n)+b(n))(y(n+\kappa)b(n+\kappa))\} \\ &= E\{y(n)y(n+\kappa)\} + E\{y(n)b(n+\kappa)\} + E\{b(n)y(n+\kappa)\} + E\{b(n)b(n+\kappa)\} \\ &= s_{yy}(\kappa) + s_{yb}(\kappa) + s_{by}(\kappa) + s_{bb}(\kappa) \end{aligned}$$

Since $b(n)$ and $y(n)$ are orthogonal to each other, the result of the cross-correlation functions is zero and we get:

$$s_{cc}(\kappa) = s_{aa}(\kappa) * h(\kappa) * h(-\kappa) + s_{bb}(\kappa).$$

(i) Let the expected value of the output signal $y(n)$ be $\mu_y = 0$, while that of the input signal $\mu_a = 4$. Can a statement be made about the transmission behavior of the filter based on this information? If so, what behavior do you expect? Justify your answers. (3 P)

$$\mu_y = \mu_a H_{cc}(e^{j\Omega})|_{\Omega=0} = 0$$

In order to fulfill the specified information, the depths must be attenuated frequencies are present. No statement can be made regarding the high frequencies. That's why this can be a high-pass or band-pass filter.

The following applies to the impulse response of the system under consideration:

$$h(n) = \frac{1}{4} \cdot \gamma_0(n).$$

(j) Determine the cross-correlation sequence $s_{ay}(\kappa)$ between the input $a(n)$ and the system output $y(n)$. (3 P)

$$\begin{aligned} s_{ay}(\kappa) &= \sum_{\kappa=-\infty}^{\infty} h_0(\kappa) s_{aa}(n-\kappa) \\ &= \sum_{\kappa=-\infty}^{\infty} \frac{1}{4} \gamma_0(n) s_{aa}(n-\kappa) \\ &= \frac{1}{4} \cdot s_{aa}(\kappa) \end{aligned}$$

(k) Are the output $y(n)$ and the input $a(n)$ uncorrelated? Justify your answer mathematically. (2 P)

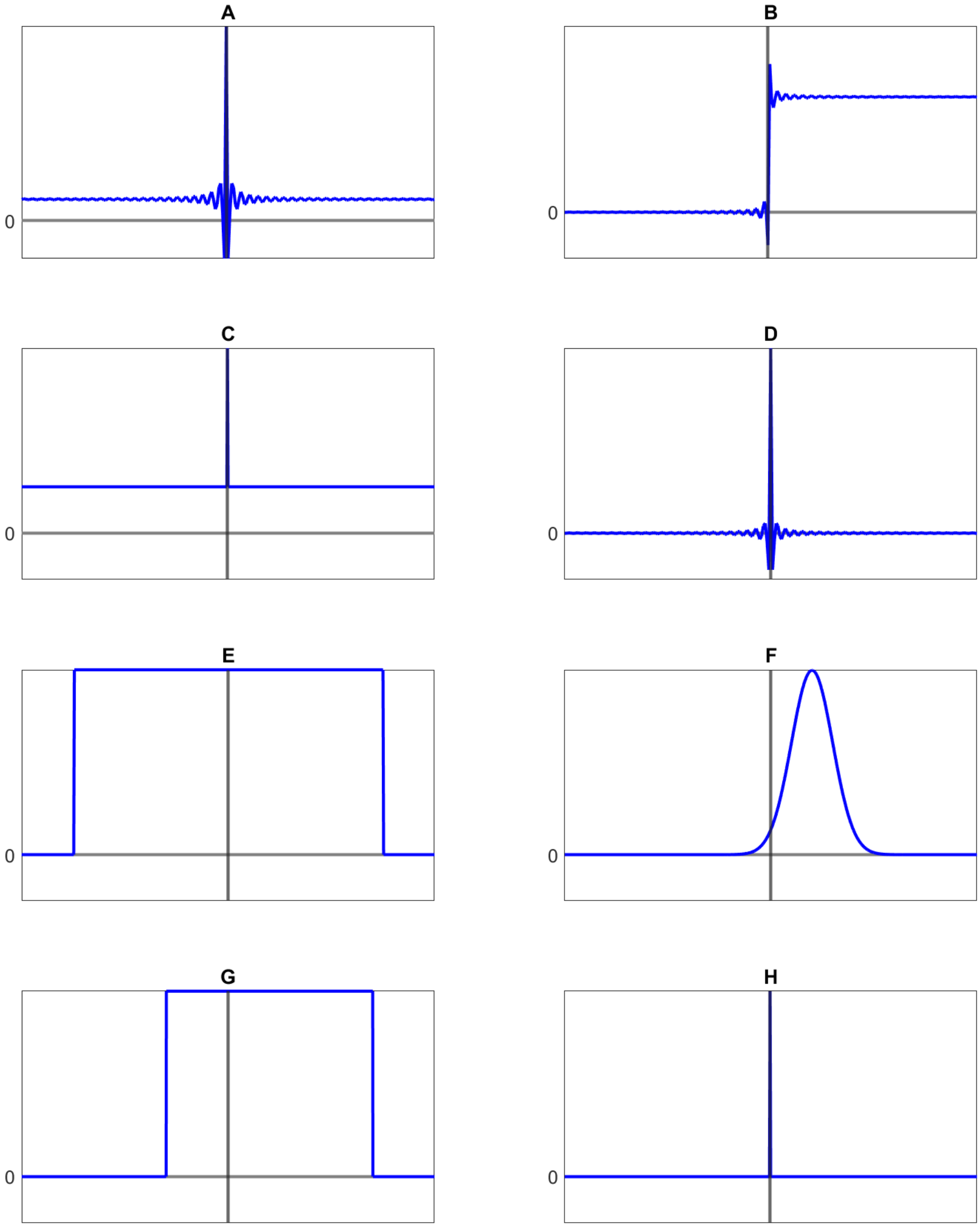
For uncorrelatedness, the cross-covariance sequence must apply

$$\psi_{ay}(\kappa) = s_{ay}(\kappa) - \mu_a \mu_y = 0$$

is. However, this applies here

$$\psi_{ay}(\kappa) = \frac{1}{4} \cdot s_{aa}(\kappa),$$

from which it follows that the two sequences are correlated.


A
0
B
0
C
0
D
0
E
0
F
0
G
0
H
0

Task 2 (33 Points)

Teil 1 *This part of the task can be solved independently of parts 2 and 3.*

A system is given which is described by the following equations:

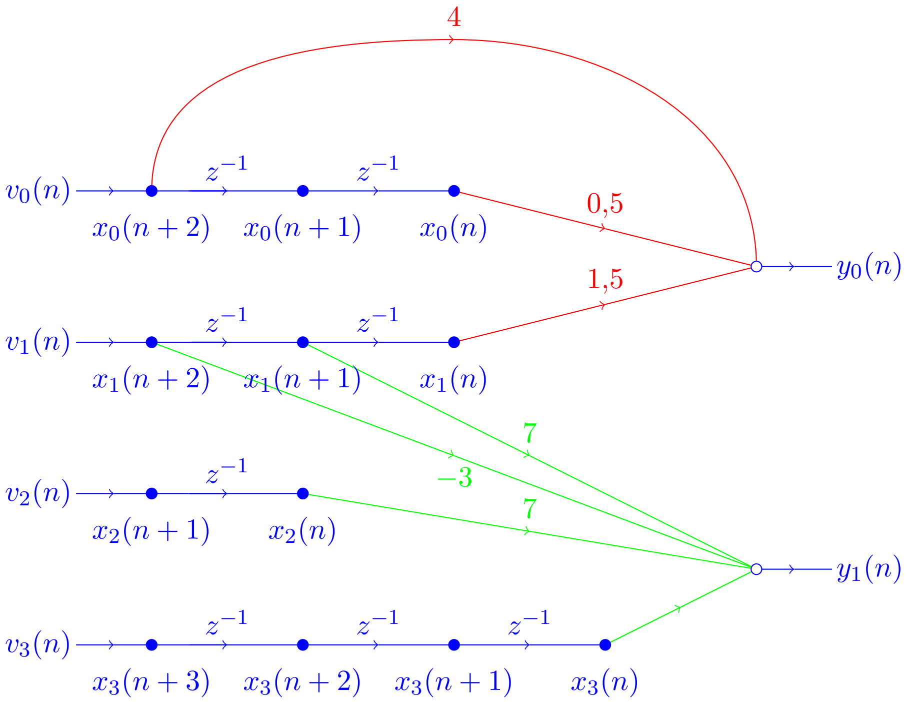
$$2y_0(n) = 3v_1(n-2) + 8v_0(n) + v_0(n-2) \quad ,$$
$$y_1(n) = 7v_1(n-1) - 3v_1(n) + v_3(n-3) + 7v_2(n-1) \quad .$$

(a) Specify the number L of inputs, N of states and R of outputs of the system. (1.5 P)
Inputs L: 4
States N: 8
Outputs R: 2

(b) Call the dimension of the matrices **A**, **B**, **C** and **D**. (2 P)
$\mathbf{A}: 8 \times 8\ [N \times N]$, $\mathbf{B}: 8 \times 4\ [N \times L]$, $\mathbf{C}: 2 \times 8\ [R \times N]$, $\mathbf{D}: 2 \times 4\ [R \times L]$

(c) Like , the names of the individual matrices are **A**, **B**, **C** and **D** and which ones Are they significant for the system? (2 P)
A: Feedback (system behavior without external influences), describes the intrinsic behavior, is called the system matrix)
B: Coupling of the input signal (control of the system, is called input matrix)
C: Decoupling the system (observation of the system, is called the output matrix)
D: Direct connection from input to output (pass-through of the system, is called pass-through matrix)

(d) Draw the signal flow graph for the given system. (5 P)



Teil 2 *This part of the task can be solved independently of parts 1 and 3.*

The system is parameterized with the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0{,}5 \\ 3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 4 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 1 \end{bmatrix} \quad .$$

In addition, the state space is described by the following equations:

$$\begin{aligned} \boldsymbol{x}(n+1) &= \boldsymbol{A}\boldsymbol{x}(n) + \boldsymbol{B}\boldsymbol{v}(n) \quad , \\ \boldsymbol{y}(n) &= \boldsymbol{C}\boldsymbol{x}(n) + \boldsymbol{D}\boldsymbol{v}(n) \quad . \end{aligned}$$

(e) Determine the transfer function $H(z)$. (Simplify as much as possible.) (4 P)

$$\begin{aligned} \mathbf{H}(z) &= \mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \\ H(z) &= \frac{1}{(z-1)(z-2)}(0{,}5z - 1 - 24 + 12z) + 1 \\ &= \frac{1}{(z-1)(z-2)}(12{,}5z - 25) + \frac{(z-1)(z-2)}{(z-1)(z-2)} \\ &= \frac{1}{(z-1)(z-2)}(12{,}5(z-2) + (z-1)(z-2)) \\ &= \frac{12{,}5 + z - 1}{z-1} \\ &= \frac{11{,}5 + z}{z-1} \end{aligned}$$

(f) Determine the difference equation. (4 P)

$$H(z) = \frac{Y(z)}{V(z)} = \frac{2z + 23}{2z - 2}$$

Divide by z:

$$\frac{Y(z)}{V(z)} = \frac{2 + 23z^{-1}}{2 - 2z^{-1}}$$

$$Y(z)(2 - 2z^{-1}) = V(z)(2 + 23z^{-1})$$

$$Y(z) = Y(z)z^{-1} + V(z)(1 + 11{,}5z^{-1})$$

Time domain solution:

$$y(n) = y(n-1) + v(n) + 11{,}5v(n-1)$$

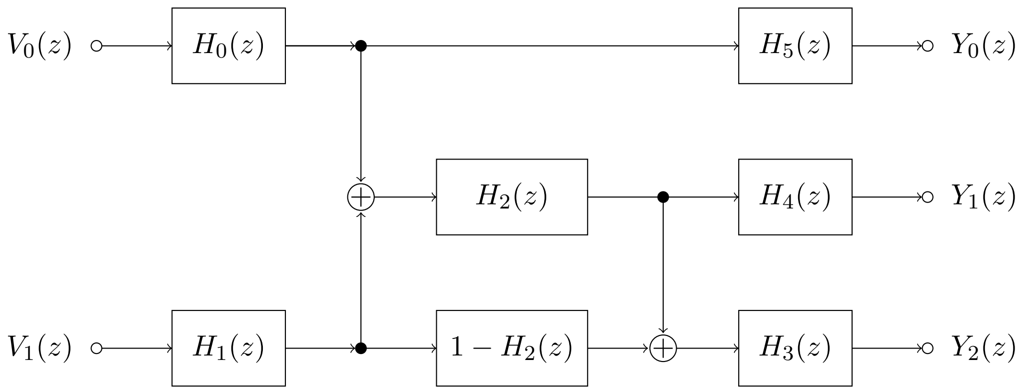
(g) Does have direct access to the system? Give reasons for your answer. (1 P)
The system has direct access because $D = 1$.

(h) Determine the impulse response $h_0(n)$. (4 P)

$$\mathcal{Z}^{-1}\left\{H_0(z)\right\} = h_0(n)$$
$$H_0(z) = \frac{2z+23}{2z-2}$$
$$\frac{2z+23}{2z-2} = \frac{2z-2+2+23}{2z-2}$$
$$\frac{2z-2+2+23}{2z-2} = 1 + \frac{12{,}5}{z-1}$$
$$\mathcal{Z}^{-1}\left\{1 + 12{,}5\frac{1}{z-1}\right\} = \mathcal{Z}^{-1}\left\{1\right\} + 12{,}5\mathcal{Z}^{-1}\left\{\frac{1}{z-1}\right\}$$
$$\mathcal{Z}^{-1}\left\{1\right\} = \gamma_0(n)$$
$$\mathcal{Z}^{-1}\left\{\frac{1}{z-1}\right\} = \gamma_{-1}(n-1)$$
$$\mathcal{Z}^{-1}\left\{1 + 12{,}5\frac{1}{z-1}\right\} = \gamma_0(n) + 12{,}5\gamma_{-1}(n-1)$$

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

For this part of the task, a system is given by the following block diagram.



(i) Determine the transfer matrix $\mathbf{H}_{ges}(z)$ with $\mathbf{Y}(z) = \mathbf{H}_{ges}(z) \cdot \mathbf{V}(z)$. (5,5 P)

$$\mathbf{H}_{ges}(z) = \begin{bmatrix} H_0(z)H_5(z) & 0 \\ H_4(z)H_2(z)H_0(z) & H_4(z)H_2(z)H_1(z) \\ H_3(z)H_2(z)H_0(z) & H_3(z)H_1(z) \end{bmatrix}$$

(j) Give a definition of an all-pass filter. (2 P)
An all-pass filter, also just called all-pass, is a filter that ideally has a constant magnitude frequency response for all frequencies.

The partial transfer functions of $\mathbf{H}_{ges}(z)$ are given by:

$$
\begin{aligned}
H_0(z) &= \frac{z^3+1}{z^3-az^2} \quad , \\
H_1(z) &= 1 \quad , \\
H_2(z) &= \frac{z}{(z-2)(z+2)} \quad , \\
H_3(z) &= \frac{-2+z^{-1}+3z^2}{z^3-2z^2} \quad , \\
H_4(z) &= \frac{z^2-4}{z^3-2z^2} \quad \text{und,} \\
H_5(z) &= \frac{za(z+a)}{(z-a)^3} \quad .
\end{aligned}
$$

(k) Which condition must be fulfilled for a transfer function $H(z)$ in order for it to be causal? is therefore the entire system $\mathbf{H}_{ges}(z)$ causal? (Explain!) (1)
The numerator degree must be less than or equal to the denominator degree for causality.
Yes, because the denominator degree is greater than or equal to the numerator degree for all $H_i(z) \in [0, 1, 2, 3, 4, 5]$.

(l) Name the stability criterion for discrete-time systems! (1)
A discrete-time system with a rational z transfer function is asymptotically stable if and only if all poles z_i of $G_z(z)$ lie within the unit circle, i.e.

$$|z_i| < 1 \quad , i = 1, ..., n,$$

where the number of poles corresponds to the system order n. The system is unstable if at least one pole lies outside the unit circle, or if at least one double pole lies on the edge of the circle. The system is boundary stable if single poles appear on the edge of the circle, but all the remaining ones lie within it.

Task 3 (34 Points)

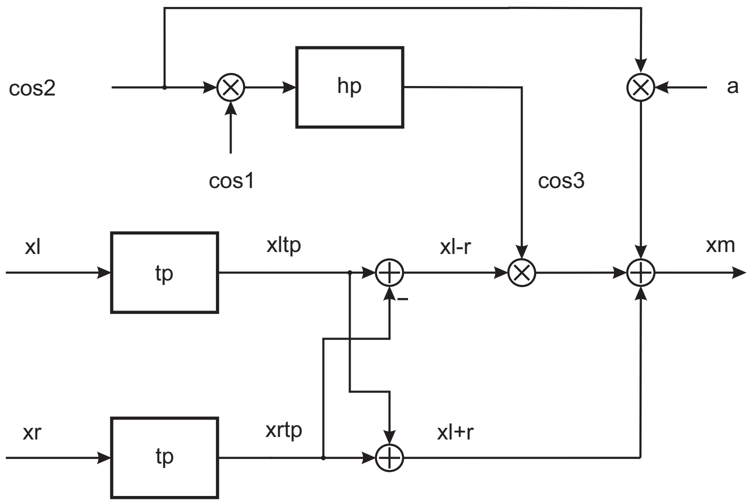
Part 1 *This part of the task can be solved independently of parts 2 and 3.*

(a) What does the FM threshold say and how does it arise? (2 P)
The FM threshold shows up to which SNR the FM transmission works well. It arises from the fact that noise has little influence on the zero crossings up to a certain SNR, especially at sufficiently high carrier frequencies.

(b) Which types of modulation do you know? Name at least three types of modulation. (1 P)
Amplitude, phase and frequency modulation.

Part 2 *This part of the task can be solved independently of Part 1 and Part 3.*

The following block diagram is given to generate a ßtereo baseband signal",



where h_{TP} is an ideal low-pass filter and h_{HP} is an ideal high-pass filter. The cutoff frequencies of the filters are identical at Ω_C and the following applies: $\Omega_C < \Omega_{\mathrm{P}}$.

Below, simplify all of your solutions as much as possible.

(c) Specify the signal $y_{\mathrm{P}}(n)$. (3 P)
The ideal filter h_{HP} removes the constant component resulting from the multiplication.

$$y_{\mathrm{P}}(n) = [-\cos(\Omega_{\mathrm{P}}n)\cdot\cos(\Omega_{\mathrm{P}}n)] * h_{\mathrm{HP}}(n) = -\frac{1}{2}[1+\cos(2\Omega_{\mathrm{P}}n)] * h_{\mathrm{HP}}(n) = -\frac{1}{2}\cos(2\Omega_{\mathrm{P}}n).$$

(d) Give the output signal $x_{\mathrm{m}}(n)$ depending on $\tilde{x}_{\mathrm{L}}(n)$ and $\tilde{x}_{\mathrm{R}}(n)$. (3 P)

$$x_{\mathrm{m}}(n) = \tilde{x}_{\mathrm{L}}(n) + \tilde{x}_{\mathrm{R}}(n) - \frac{1}{2}[\tilde{x}_{\mathrm{L}}(n) - \tilde{x}_{\mathrm{R}}(n)]\cos(2\Omega_{\mathrm{P}}n) + a\cos(\Omega_{\mathrm{P}}n)$$

(e) Compute the Fourier transform of $x_\mathrm{m}(n)$. (4 P)

$$
\begin{aligned}
X_\mathrm{m}\left(\mathrm{e}^{j\Omega}\right) = {} & \tilde{X}_\mathrm{L}\left(\mathrm{e}^{j\Omega}\right) + \tilde{X}_\mathrm{R}\left(\mathrm{e}^{j\Omega}\right) \\
& + \frac{1}{2\pi}\left[\tilde{X}_\mathrm{L}\left(\mathrm{e}^{j\Omega}\right) - \tilde{X}_\mathrm{R}\left(\mathrm{e}^{j\Omega}\right)\right] \circledast \left(-\frac{\pi}{2}\sum_{k=-\infty}^{\infty}\left[\gamma_0(\Omega - 2\Omega_\mathrm{P} - 2\pi k) + \gamma_0(\Omega + 2\Omega_\mathrm{P} - 2\pi k)\right]\right) \\
& + a\pi \sum_{k=-\infty}^{\infty}\left[\gamma_0(\Omega - \Omega_\mathrm{P} - 2\pi k) + \gamma_0(\Omega + \Omega_\mathrm{P} - 2\pi k)\right] \\
= {} & \tilde{X}_\mathrm{L}\left(\mathrm{e}^{j\Omega}\right) + \tilde{X}_\mathrm{R}\left(\mathrm{e}^{j\Omega}\right) - \frac{1}{4}\left[\tilde{X}_\mathrm{L}\left(\mathrm{e}^{j\Omega - 2\Omega_\mathrm{P}}\right) + \tilde{X}_\mathrm{L}\left(\mathrm{e}^{j\Omega + 2\Omega_\mathrm{P}}\right)\right] \\
& + \frac{1}{4}\left[\tilde{X}_\mathrm{R}\left(\mathrm{e}^{j\Omega - 2\Omega_\mathrm{P}}\right) + \tilde{X}_\mathrm{R}\left(\mathrm{e}^{j\Omega + 2\Omega_\mathrm{P}}\right)\right] \\
& + a\pi \sum_{k=-\infty}^{\infty}\left[\gamma_0(\Omega - \Omega_\mathrm{P} - 2\pi k) + \gamma_0(\Omega + \Omega_\mathrm{P} - 2\pi k)\right]
\end{aligned}
$$

(f) Give in general the power spectral density $S_{x_{dif}x_{dif}}\left(\mathrm{e}^{j\Omega}\right)$ in dependence from the input variables. Assume that the signals $x_\mathrm{L}(n)$ and $x_\mathrm{R}(n)$ are in the pass band of the low pass $h_\mathrm{TP}(n)$. (4 P)

$$
\begin{aligned}
s_{x_\mathrm{dif}x_\mathrm{dif}}(\kappa) &= \mathrm{E}\left\{x_\mathrm{dif}(n)x_\mathrm{dif}^*(n+\kappa)\right\} \\
&= \mathrm{E}\left\{\left[x_\mathrm{L}(n) - x_\mathrm{R}(n)\right]\left[x_\mathrm{L}(n+\kappa) - x_\mathrm{R}(n+\kappa)\right]^*\right\} \\
&= \mathrm{E}\left\{x_\mathrm{L}(n)x_\mathrm{L}^*(n+\kappa) - x_\mathrm{R}(n)x_\mathrm{L}^*(n+\kappa) - x_\mathrm{L}(n)x_\mathrm{R}^*(n+\kappa) + x_\mathrm{R}(n)x_\mathrm{R}^*(n+\kappa)\right\}
\end{aligned}
$$

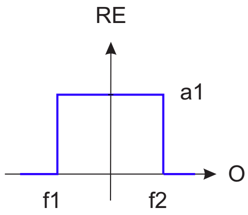
$$\circ\!\!-\!\!\bullet$$

$$
\begin{aligned}
S_{x_\mathrm{dif}x_\mathrm{dif}}\left(\mathrm{e}^{j\Omega}\right) &= S_{x_\mathrm{L}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right) - S_{x_\mathrm{R}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right) - S_{x_\mathrm{L}x_\mathrm{R}}\left(\mathrm{e}^{j\Omega}\right) + S_{x_\mathrm{R}x_\mathrm{R}}\left(\mathrm{e}^{j\Omega}\right) \\
&= S_{x_\mathrm{L}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right) - S_{x_\mathrm{R}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right) - S_{x_\mathrm{R}x_\mathrm{L}}^*\left(\mathrm{e}^{j\Omega}\right) + S_{x_\mathrm{R}x_\mathrm{R}}\left(\mathrm{e}^{j\Omega}\right) \\
&= S_{x_\mathrm{L}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right) - 2\mathrm{Re}\left\{S_{x_\mathrm{R}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right)\right\} + S_{x_\mathrm{R}x_\mathrm{R}}\left(\mathrm{e}^{j\Omega}\right)
\end{aligned}
$$

(g) Is the power spectral density $S_{x_{dif}x_{dif}}\left(\mathrm{e}^{j\Omega}\right)$ complex or real? (1 P)

Auto power density spectra are always real. The power spectral density $S_{x_{dif}x_{dif}}\left(\mathrm{e}^{j\Omega}\right)$ contains two complex cross terms $-S_{x_\mathrm{R}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right)$ and $-S_{x_\mathrm{L}x_\mathrm{R}}\left(\mathrm{e}^{j\Omega}\right)$, but these are conjugately complex to each other.

Now two processes $x_\mathrm{L}(n)$ and $x_\mathrm{R}(n)$ with the following auto power spectral density $S_{xx}\left(\mathrm{e}^{j\Omega}\right) = S_{x_\mathrm{R}x_\mathrm{R}}\left(\mathrm{e}^{j\Omega}\right) = S_{x_\mathrm{L}x_\mathrm{L}}\left(\mathrm{e}^{j\Omega}\right)$ are transferred, where $\Omega_1 < \Omega_C < \Omega_\mathrm{P}$.

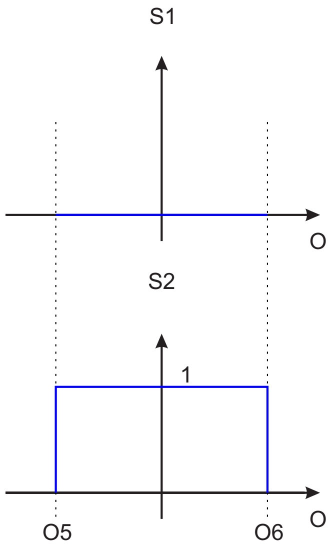


(h) Sketch the auto power spectral density $S_{x_{\mathrm{dif}}x_{\mathrm{dif}}}\left(\mathrm{e}^{j\Omega}\right)$ of the output process $x_{\mathrm{dif}}(n)$ for the following two cases.

(i) The processes are equal, $x_{\mathrm{R}}(n) = x_{\mathrm{L}}(n)$. (3 P)

(ii) The processes $x_{\mathrm{R}}(n)$ and $x_{\mathrm{L}}(n)$ are orthogonal to each other. (4 P)

If the processes are the same, then $x_{\mathrm{dif}}(n)$ is always zero. If the processes are orthogonal, then $x_{\mathrm{dif}}(n)$ is not zero.



Part 3 *This part of the task can be solved independently of Part 1 and Part 2.*

The following phase-modulated signal is given:

$$v(t) = A_T \cos\left[\Phi_T(t)\right] = A_T \cos\left[2\pi f_T t + \eta \cos(2\pi f_N t)\right].$$

(i) Determine the instantaneous angular frequency $\Omega_T(t)$ of the signal $v(t)$. (3 P)

$$\begin{aligned}\Omega_T(t) &= \frac{d\Phi_T(t)}{dt} \\ &= 2\pi f_T - \eta \sin(2\pi f_N t) 2\pi f_N.\end{aligned}$$

(j) Enter the frequency deviation Δf for the signal $v(t)$. Explain the meaning of the frequency deviation in your own words. (3 P)

$$\Delta f = \eta f_N .$$

The frequency deviation Δf is nothing other than the maximum deviation of the instantaneous frequency from the constant carrier frequency f_T.

(k) Enter the minimum and maximum FM bandwidth f_{Bmin} and f_{Bmax} at a frequency swing $\Delta f = 50$ kHz for a useful signal in the range from 50 Hz to 20 kHz. (3 P)

$$\begin{aligned} f_{\text{B}} &= 2\left(\frac{\Delta f}{f_1} + 2\right) f_1 \\ f_{\text{Bmin}} &= 2\left(\frac{50\text{ kHz}}{50\text{ Hz}} + 2\right) 50\text{ Hz} = 100{,}2\text{ kHz} \\ f_{\text{Bmax}} &= 2\left(\frac{50\text{ kHz}}{20\text{ kHz}} + 2\right) 20\text{ kHz} = 180\text{ kHz}. \end{aligned}$$

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