Signals and Systems II

Exam WS 2024

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	28.03.2025
Name:	______________________
Matriculation Number:	______________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________

Marking

Problem	1	2	3
Points	/33	/33,5	/33,5

Total number of points: ______________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ______________ Signature: ______________________

Signals and Systems II

Exam WS 2024

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	CAP3, lecture hall 2 and 3
Date:	28.03.2025
Begin:	09:00 h
Reading Time:	10 minutes
Working Time:	90 minutes

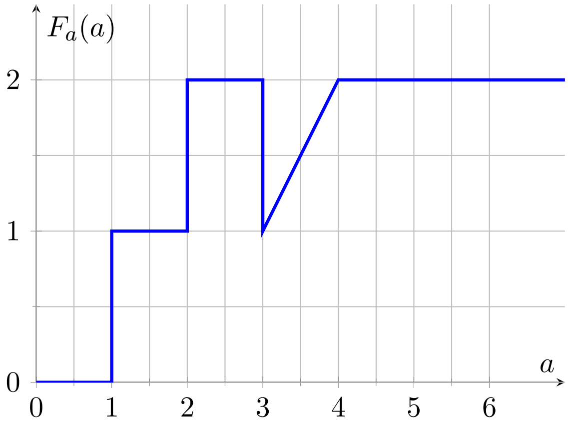
Notes

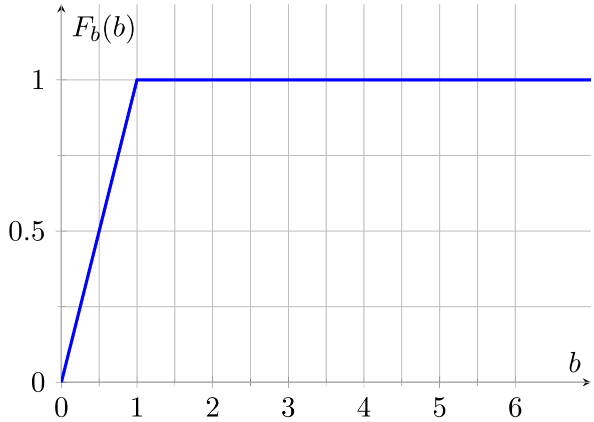
- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

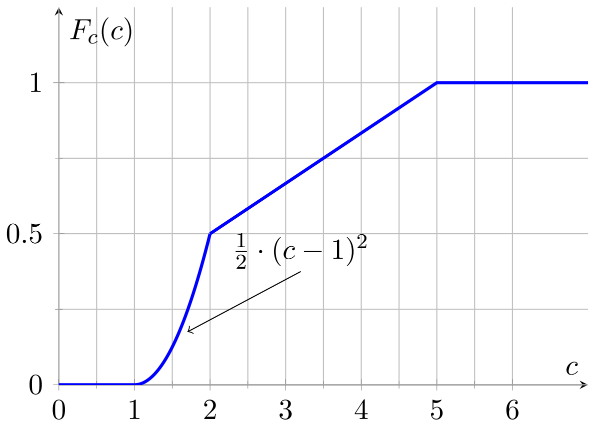
Task 1 (33 points)

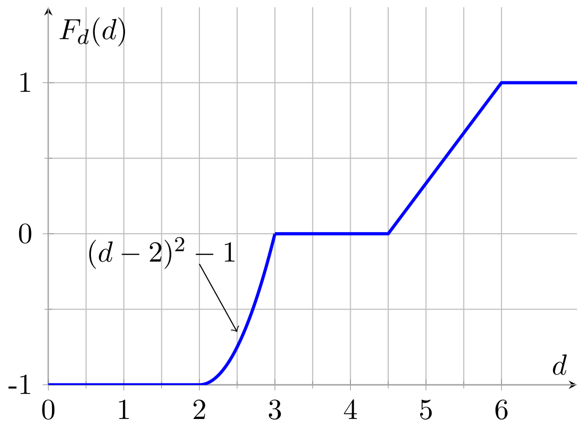
Part 1 *This part of the task can be solved independently of parts 2 and 3.*

The following four functions are given in graphical form:









(a) For which of the above functions can you exclude that they are distribution functions of continuous random variables? Give reasons for your answer! Also identify the functions that are distribution functions. (4 P)
$F_a(a)$ has discontinuities and is not monotonically increasing. (1 P) $F_d(d)$ has negative values. (1 P) $F_c(c)$ and $F_b(b)$ are distribution functions. (2 P)

(b) Determine the probability densities for the functions that you identified as distribution functions in (*a*). (4 P)
It applies $f_c(c) = \frac{\mathrm{d}}{\mathrm{d}c} F_c(c)$. and $f_b(b) = \frac{\mathrm{d}}{\mathrm{d}b} F_b(b)$.

$$f_c(c) = \begin{cases} c-1, & \text{for } 1 \le c < 2, \text{ (1 P)} \\ \frac{1}{6}, & \text{for } 2 \le c < 5, \text{ (1 P)} \\ 0, & \text{else. (0.5 P)} \end{cases}$$

$$f_b(b) = \begin{cases} 1, & \text{for } 0 \leq b < 1, \text{ (1 P)} \\ 0, & \text{else. (0.5 P)} \end{cases}$$

(c) Determine for the functions that you identified as distribution functions in (a), the 2nd statistical moment as well as the probabilities that the values of the functions lie between 4 and 5. (6 P)

$$\begin{aligned} m_c^{(2)} &= \int_{-\infty}^{\infty} c^2 f_c(c) dc \\ &= \int_1^2 c^2(c-1)dc + \frac{1}{6}\int_2^5 c^2 dc \text{ (1 P)} \\ &= [\frac{1}{4}c^4 - \frac{1}{3}c^3]_1^2 + \frac{1}{18}[c^3]_2^5 \text{ (1 P)} \\ &\approx 7{,}92 \text{ (1 P)} \end{aligned}$$

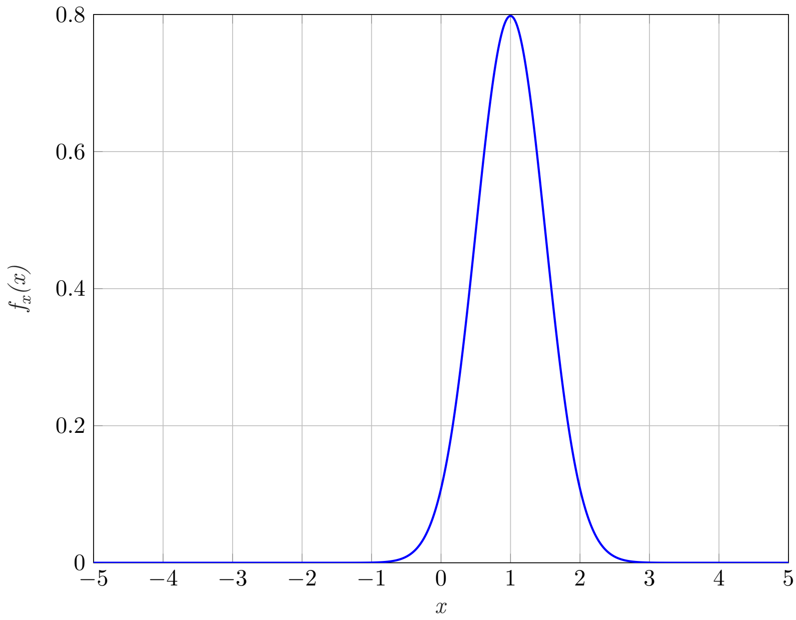
$$\begin{aligned} P(4 \leq c \leq 5) &= \int_4^5 \frac{1}{6} dc \\ &\approx 0{,}167 \text{ (0.5 P)} \end{aligned}$$

$$\begin{aligned} m_b^{(2)} &= \int_{-\infty}^{\infty} b^2 f_b(b) db \\ &= \int_0^1 b^2 db \text{(1 P)} \\ &\approx 0{,}33 \text{ (1 P)} \end{aligned}$$

$$\begin{aligned} P(4 \leq b \leq 5) &= \int_4^5 0 db \\ &= 0 \text{ (0.5 P)} \end{aligned}$$

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

Given is the probability density $f_x(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-m_x)^2}{2\sigma_x^2}}$ of normally distributed white noise:

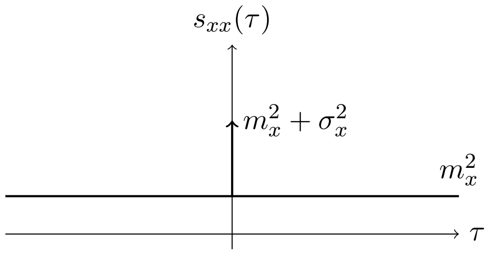


(d) Calculate the total power of the signal $x(t)$ and sketch its autocorrelation function $s_{xx}(\tau)$ with all the necessary details. (5 P)

The total power of the signal can be calculated by $m_x^{(2)} = \sigma_x^2 + m_x^2$.
The mean value $m_x = 1$ can be read from the graph. (1 P)
The variance can be determined by converting the probability density of the normal distribution to σ_x.

$$\begin{aligned} f_x(1) &= \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(1-m_x)^2}{2\sigma_x^2}} \\ &= \frac{1}{\sqrt{2\pi}\sigma_x} = 0{,}8 \end{aligned}$$

Conversion results in $\sigma_x \approx 0{,}5$ (1 P). The total power of the signal is $m_x^{(2)} = 1\ 25$ (1 P)
The autocorrelation function can be sketched according to: $s_{xx}(\tau) = m_x^2 + \sigma_x^2 \cdot \gamma_0(\tau)$ (2 P)



Given is the signal

$$y(t) = \frac{1}{2} + \sin(\omega t).$$

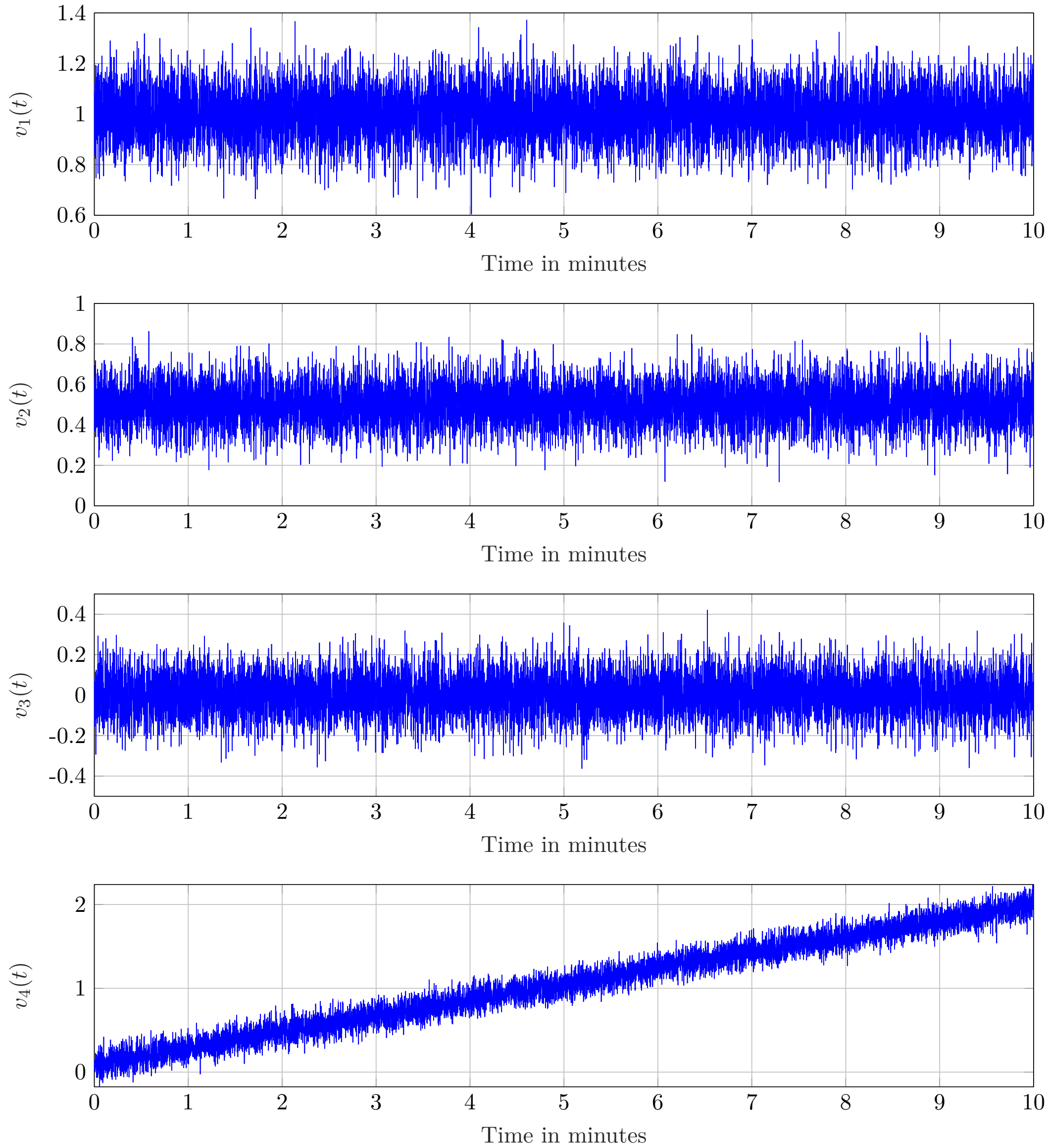
(e) Are the signals $x(t)$ and $y(t)$ orthogonal over a multiple of the period duration T? Give reasons! (2 P)

Two signals are orthogonal if the following condition applies to the cross-correlation

$$s_{x,y}(t_1, t_2) = m_x(t_1) \cdot m_y(t_2) = 0 \text{ (1 P)}$$

Since neither of the two signals is mean-free, the signals are not orthogonal. (1 P)

Four series of measurements of the signal $v(t)$ are now given:



(f) What does stationarity mean? Is the signal $v(t)$ stationary? Give reasons! (2 P)
Stationarity means that the statistical properties of the signal do not change over time. (1 P) The measurement $v_4(t)$ has a time-dependent mean value, which means that $v(t)$ is not stationary. (1 P)

(g) What does ergodicity mean? Is the signal $v(t)$ ergodic? Give reasons! (2 P)
Ergodicity means that the average values of the coulter match the time averages. (1 P) Ergodicity requires stationarity. Therefore $v(t)$ is not ergodic. (1 P)

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

The real output signal $y(n)$ of an LTI system is generally described by the following equation, where $v(n)$ is the real input signal of the system:

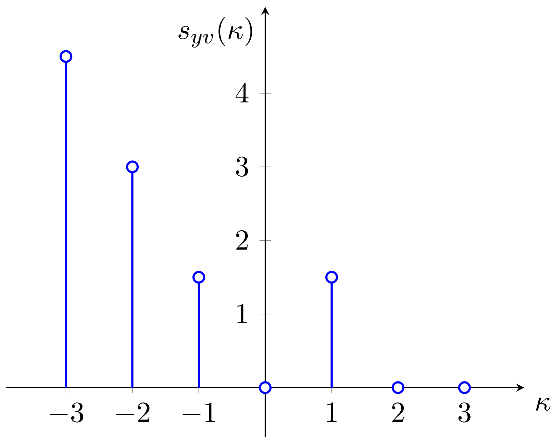
$$y(n) = \sum_{i=-\infty}^{\infty} h_0(i)\, v(n-i), \quad h_0(i) \in \mathbb{R}.$$

(h) Specify a relationship for the correlation functions $s_{yv}(\kappa)$ and $s_{vv}(\kappa)$ of the signals with stationary excitation. Use the summation form. Also state the relationship with the convolution operator. (2 P)

$$s_{yv}(\kappa) = \sum_{i=-\infty}^{\infty} h_0(-i)\, s_{vv}(\kappa - i) = h_0(-\kappa) * s_{vv}(\kappa) \text{ (2 P)}$$

Now the system is excited with white noise $v(n)$ ($m_v = 0$, $\sigma_v^2 = \frac{3}{2}$). In addition, $h_0(i) = |i|$ applies in the interval $-2 < i < 4$ and $h_0(i) = 0$ otherwise.

(i) Sketch $s_{yv}(\kappa)$ from $-3 < \kappa < 3$ with all the necessary details. (5 P)



(1 P) For each correctly drawn value

(j) What relationship can be recognized between $s_{yv}(\kappa)$ and $h_0(\kappa)$? (1 P)

$$s_{yv}(\kappa) = \sigma_v^2 h_0(-\kappa) \text{ (1 P)}$$

Task 2 (33,5 points)

Part 1 *This part of the task can be solved independently of part 2.*

Given a system which is described by the state space description

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\boldsymbol{x}(n) + \boldsymbol{B}\boldsymbol{v}(n), \text{ as well as}$$
$$\boldsymbol{y}(n) = \boldsymbol{C}\boldsymbol{x}(n) + \boldsymbol{D}\boldsymbol{v}(n).$$

The matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are parameterized as follows:

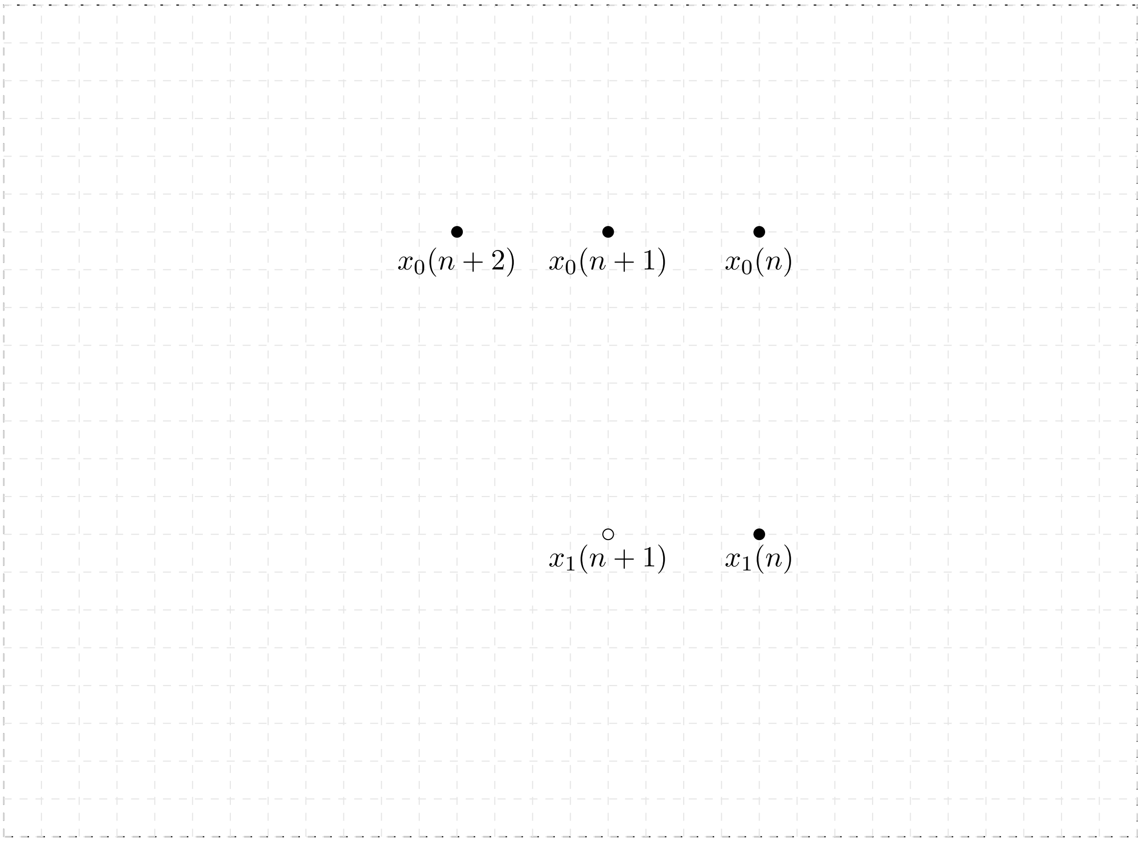
$$\boldsymbol{A} := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 2 & 2 \end{bmatrix}, \boldsymbol{B} := \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix}, \boldsymbol{C} := \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \text{and } \boldsymbol{D} := \begin{bmatrix} 0 & 2 \end{bmatrix}$$

(a) Specify the number L of inputs, N of states and R of outputs of the system. (1,5 P)
Inputs L: 2 (0.5 points)
States N: 3 (0.5 points)
Outputs R: 1 (0.5 points)

(b) What are the names of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ and what significance do they have for the system? (2 P)
$\boldsymbol{A}$: The dynamic matrix describes the influence of the previous states on the current state. (0.5 points)
$\boldsymbol{B}$: The input matrix describes the influence of the input variables on the states. (0.5 points)
$\boldsymbol{C}$: The output matrix describes the influence of the states on the output variables. (0.5 points)
$\boldsymbol{D}$: The passthrough matrix describes the direct influence of the inputs on the output. (0.5 points)

(c) Draw the corresponding signal flow graph. Use the following beginning as a guide: (8 P)

$$\begin{bmatrix} x_0(n+1) \\ x_0(n+2) \\ x_1(n+1) \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x_0(n) \\ x_0(n+1) \\ x_1(n) \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} v_0(n) \\ v_1(n) \end{bmatrix}$$



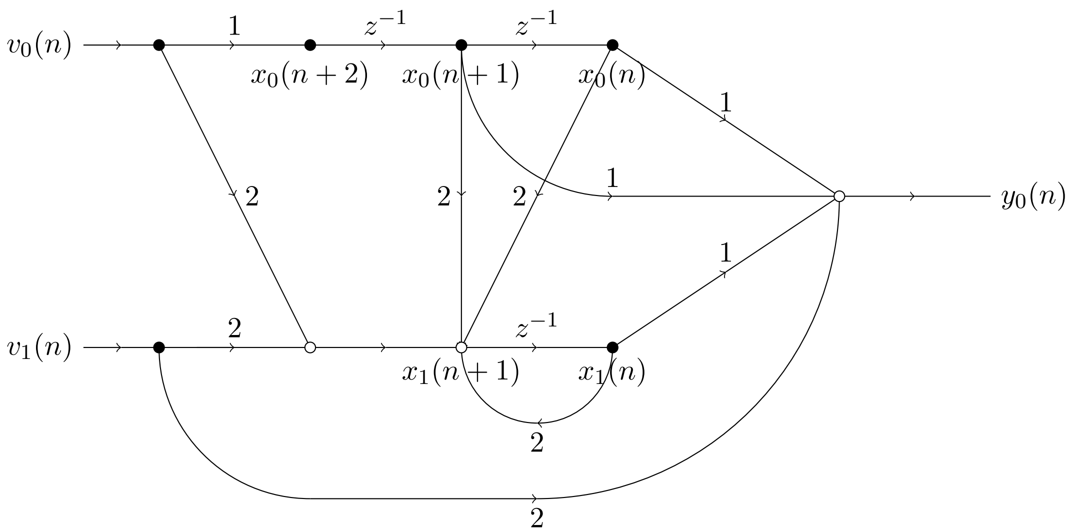
State equation:

$$\begin{bmatrix} x_0(n+1) \\ x_0(n+2) \\ x_1(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_0(n) \\ x_0(n+1) \\ x_1(n) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} v_0(n) \\ v_1(n) \end{bmatrix}$$

$$x_0(n+1) = x_0(n+1)$$
$$x_0(n+2) = v_0(n)$$
$$x_1(n+1) = 2x_0(n) + 2x_0(n+1) + 2x_1(n) + 2v_0(n) + 2v_1(n)$$

Output equation:

$$y_0(n) = x_0(n) + x_0(n+1) + x_1(n) + 2v_1(n)$$



Correct labeling of inputs, states and outputs (1.5 point)
Correct implementation of the dynamic matrix $\boldsymbol{A}$ (2 points)
Correct implementation of the input matrix $\boldsymbol{B}$ (1.5 points)
Correct implementation of the output matrix $\boldsymbol{C}$ (1.5 points)
Correct implementation of the passthrough matrix $\boldsymbol{D}$ (0.5 points)
Completeness of nodes and arrows (1 point)

(d) Determine $\boldsymbol{H}(z)$. Use the following result as a guide: (8 P)

$$\operatorname{adj}(z\boldsymbol{I}-\boldsymbol{A}) = \begin{bmatrix} z(z-2) & z-2 & 0 \\ 0 & z(z-2) & 0 \\ 2z & 2z+2 & z^2 \end{bmatrix}.$$

$\boldsymbol{H}(z) = \boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}$ (1 point)

$$\boldsymbol{H}(z) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 2 & 2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 \end{bmatrix}$$

Inversion of $(z\boldsymbol{I}-\boldsymbol{A})$: (4 points)

$$\boldsymbol{M} = z\boldsymbol{I}-\boldsymbol{A} = \begin{bmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} z & -1 & 0 \\ 0 & z & 0 \\ -2 & -2 & z-2 \end{bmatrix}$$

Inversion of a 3×3 matrix

$$\boldsymbol{M}^{-1} = \frac{1}{\det(\boldsymbol{M})} \operatorname{adj}(\boldsymbol{M})$$

$$\det(\boldsymbol{M}) = z^2(z-2)$$

$$\operatorname{adj}(\boldsymbol{M}) = \begin{bmatrix} z(z-2) & z-2 & 0 \\ 0 & z(z-2) & 0 \\ 2z & 2z+2 & z^2 \end{bmatrix}$$

$$\boldsymbol{M}^{-1} = \frac{1}{z^2(z-2)} \begin{bmatrix} z(z-2) & z-2 & 0 \\ 0 & z(z-2) & 0 \\ 2z & 2z+2 & z^2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{z} & \frac{1}{z^2} & 0 \\ 0 & \frac{1}{z} & 0 \\ \frac{2}{z(z-2)} & \frac{2z+2}{z^2(z-2)} & \frac{1}{z-2} \end{bmatrix}$$

(1 point: Calculate determinant)
(2 point: Correct result for the inverse)
Inserting the inverse into the equation:

$$\boldsymbol{H}(z) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{z} & \frac{1}{z^2} & 0 \\ 0 & \frac{1}{z} & 0 \\ \frac{2}{z(z-2)} & \frac{2(z+1)}{z^2(z-2)} & \frac{1}{z-2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 \end{bmatrix}$$

Calculation $\boldsymbol{R}_{\text{intermediate},1} = \boldsymbol{C}(z\boldsymbol{I} - \boldsymbol{A})^{-1}$: (1 point)

$$\boldsymbol{H}(z) = \begin{bmatrix} \frac{1}{z} + \frac{2}{z(z-2)} & \frac{1}{z^2} + \frac{1}{z} + \frac{2(z+1)}{z^2(z-2)} & \frac{1}{z-2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 \end{bmatrix}$$

Calculation $\boldsymbol{R}_{\text{intermediate},2} = \boldsymbol{R}_{\text{intermediate},1}\boldsymbol{B}$: (1 point)

$$\boldsymbol{H}(z) = \begin{bmatrix} \frac{1}{z^2} + \frac{1}{z} + \frac{2(z+1)}{z^2(z-2)} + \frac{2}{z-2} & \frac{2}{z-2} \end{bmatrix} + \begin{bmatrix} 0 & 2 \end{bmatrix}$$

This results in: (1 point)

$$\boldsymbol{H}(z) = \begin{bmatrix} \frac{1}{z^2} + \frac{1}{z} + \frac{2(z+1)}{z^2(z-2)} + \frac{2}{z-2} & \frac{2}{z-2} + 2 \end{bmatrix}$$

Simplify: (1 point)

$$\boldsymbol{H}(z) = \begin{bmatrix} \frac{3(z+\frac{1}{3})}{z(z-2)} & 2\frac{z-1}{z-2} \end{bmatrix}$$

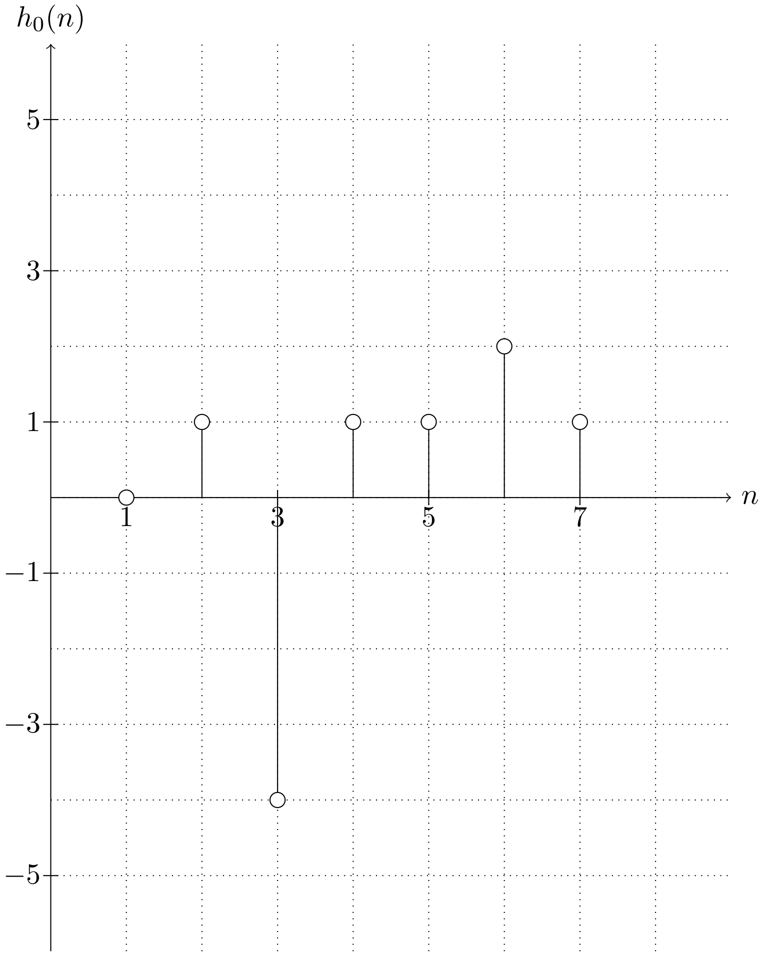
(e) Make a statement about the stability of the system and justify it. (1 P)
Poles of the system lie outside the unit circle (0.5 points), therefore the given system is unstable (0.5 points).

Part 2 *This part of the task can be solved independently of part 1.*

Given is the discrete impulse response $h_0(n)$:

$$h_0(n) = \gamma_{-1}(n-2) - 5\gamma_0(n-3) + \gamma_0(n-6) - \gamma_{-1}(n-8)\ .$$

(f) Draw the impulse response in the interval $0 < n < 8$. Remember to label the axes. (4 P)
Axis labels (0.5 points)
Correct values for $n = 1$ bis 7 je (0.5 points)



(g) Determine the transfer function $H(z)$ and simplify it as much as possible. (5 P)
Each term of $h_0(n)$ (1 point)
Simplify (1 point)

$$\begin{aligned}
H(z) &= z^{-2}\frac{z}{z-1} - 5z^{-3} + z^{-6} - z^{-8}\frac{z}{z-1} \\
&= \frac{z^{-2}}{1-z^{-1}} - 5z^{-3} + z^{-6} - \frac{z^{-8}}{1-z^{-1}} \\
&= \frac{z^{-2}}{1-z^{-1}} - \frac{5z^{-3}(1-z^{-1})}{1-z^{-1}} + \frac{z^{-6}(1-z^{-1})}{1-z^{-1}} - \frac{z^{-8}}{1-z^{-1}} \\
&= \frac{z^{-2} - 5z^{-3}(1-z^{-1}) + z^{-6}(1-z^{-1}) - z^{-8}}{1-z^{-1}} \\
&= \frac{z^{-2} - 5z^{-3} + 5z^{-4} + z^{-6} - z^{-7} - z^{-8}}{1-z^{-1}}
\end{aligned}$$

(h) Determine the difference equation. (3 P)

The relation between input $V(z)$, output $Y(z)$, and transfer function $H(z)$ is:

$$H(z) = \frac{Y(z)}{V(z)}$$

$$\frac{Y(z)}{V(z)} = \frac{z^{-2} - 5z^{-3} + 5z^{-4} + z^{-6} - z^{-7} - z^{-8}}{1 - z^{-1}}$$

Resulting in:

$$Y(z) = (z^{-2} - 5z^{-3} + 5z^{-4} + z^{-6} - z^{-7} - z^{-8})V(z) + z^{-1}Y(z)$$

Transformation into time domain:

$$y(n) = v(n-2) - 5v(n-3) + 5v(n-4) + v(n-6) - v(n-7) - v(n-8) + y(n-1)$$

(i) Does the system have a direct passthrough? Justify your answer. (1 P)

No, this can be seen, for example, in the difference equation, since in this $y(n)$ does not depend on $v(n)$.

Task 3 (33,5 points)

Part 1 *This part of the task can be solved independently of Part 2 and Part 3.*

(a) Name at least two disadvantages of amplitude modulation and briefly explain why it is still used. (3 P)

- Prone to disruption
- Poor frequency efficiency
+ Simple and inexpensive implementation

(b) Calculate the discrete Fourier transform of the double-sideband modulated signal $c_\text{T}(n) = \sin(\Omega_\text{S} n) \cdot \cos(\Omega_\text{T} n)$. (4,5 P)
Tip: Use the trigonometric relationships to simplify the transmitted signal.

Reading trigonometric relationships from formula collection:
$\sin(x)\cos(y) = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$
Transforming the transmission signal:

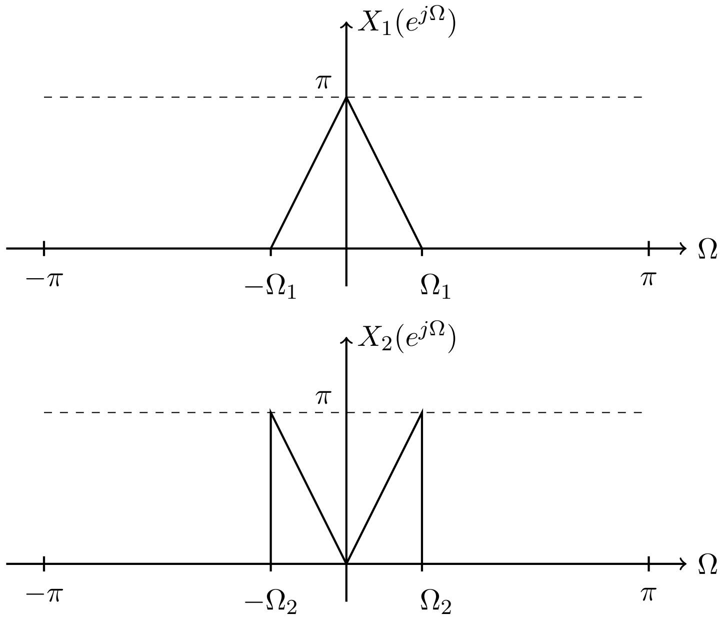
$$c(n) = \frac{1}{2}\left[\sin\Big((\Omega_\text{S} - \Omega_\text{T})n\Big) + \sin\Big((\Omega_\text{S} + \Omega_\text{T})n\Big)\right]$$
$$= \frac{1}{2}\left[\sin\Big((\Omega_\text{T} + \Omega_\text{S})n\Big) - \sin\Big((\Omega_\text{T} - \Omega_\text{S})n\Big)\right]$$

$$\bullet\!\!-\!\!\circ$$

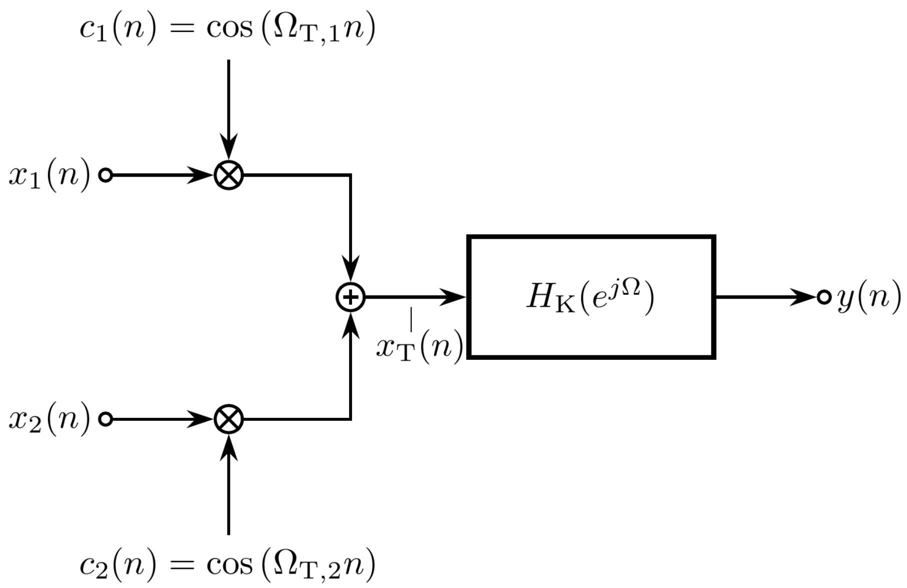
$$C(e^{j\Omega}) = \frac{1}{2}\Bigg[j\pi \sum_{\lambda=-\infty}^{\infty} \Big[\delta_0(\Omega + (\Omega_\text{T} + \Omega_\text{S}) - 2\pi\lambda) - \delta_0(\Omega - (\Omega_\text{T} + \Omega_\text{S}) - 2\pi\lambda)\Big]$$
$$+ j\pi \sum_{\lambda=-\infty}^{\infty} \Big[\delta_0(\Omega + (\Omega_\text{T} - \Omega_\text{S}) - 2\pi\lambda) - \delta_0(\Omega - (\Omega_\text{T} - \Omega_\text{S}) - 2\pi\lambda)\Big]\Bigg]$$

Part 2 *This part of the task can be solved independently of Part 1 and Part 3.*

The two signals $X_1(e^{j\Omega})$ and $X_2(e^{j\Omega})$ with $\Omega_\text{X} = \Omega_1 = \Omega_2$ are now to be transmitted via the same channel $H_\text{K}(e^{j\Omega})$ using double-sideband modulation. For this purpose, the carrier frequency $\Omega_{\text{T},1}$ is available for $X_1(e^{j\Omega})$ and the carrier frequency $\Omega_{\text{T},2}$ is available for $X_2(e^{j\Omega})$, where $\Omega_{\text{T},1} \leq \Omega_{\text{T},2}$.



(c) Draw the transmitter structure and the transmission channel $H_{\mathrm{K}}(e^{j\Omega})$ as a block diagram. The output signal of the transmission channel is denoted by $y(n)$. (4 P)
Tip: Remember to label all signals that appear in the block diagram.



(d) What must apply to the carrier frequencies so that the signals can be transmitted without interference and without frequency gaps? (1 P)

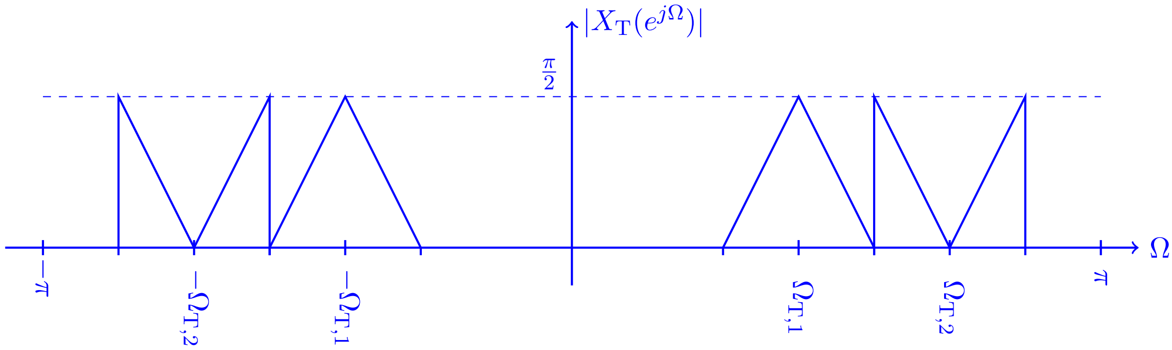
The modulated signals must not overlap in the spectrum (aliasing):
$2 \cdot \Omega_{\mathrm{X}} \leq \Omega_{\mathrm{T},2} - \Omega_{\mathrm{T},1}$

(e) Calculate the Fourier transform $X_{\mathrm{T}}(e^{j\Omega})$ of the transmitted signal and sketch its magnitude $|X_{\mathrm{T}}(e^{j\Omega})|$ in the range $-\pi \leq \Omega \leq \pi$. (5 P)

$$x_{\mathrm{T}}(n) = x_1(n) \cdot \cos(\Omega_{\mathrm{T},1} n) + x_2(n) \cdot \cos(\Omega_{\mathrm{T},2} n)$$

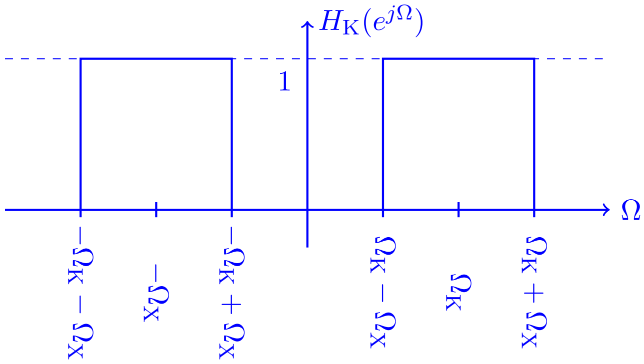
$$\circ\!\!-\!\!\bullet$$

$$X_{\mathrm{T}}(e^{j\Omega}) = \frac{1}{2}[X_1(e^{j(\Omega-\Omega_{\mathrm{T},1})}) + X_1(e^{j(\Omega+\Omega_{\mathrm{T},1})})] + \frac{1}{2}[X_2(e^{j(\Omega-\Omega_{\mathrm{T},2})}) + X_2(e^{j(\Omega+\Omega_{\mathrm{T},2})})]$$



The channel operator has now informed you of the frequency band available for transmission. The channel has a center frequency of Ω_{K} and a bandwidth of $2 \cdot \Omega_{\mathrm{X}}$.

(f) Plot the resulting frequency response of the transmission channel assuming that the channel is otherwise ideal and does not distort the signal. (2 P)



(g) Briefly explain whether the current transmitter structure would be able to transmit the signal $x_{\mathrm{T}}(n)$ undistorted and, if not, what adjustments you would need to make to ensure interference-free transmission. (4 P)

With a conventional double-sideband modulation, a bandwidth of $4 \cdot \Omega_{\mathrm{X}}$ would be required for the two signals $x_1(n)$ and $x_2(n)$. Without further adaptation of the transmission signal, the bandwidth of the channel would therefore be too low. The required bandwidth can be reduced using single-sideband modulation. To do this, the carrier frequency for both signals must correspond to the center frequency of the channel: $\Omega_\mathrm{K} = \Omega_{\mathrm{T},1} = \Omega_{\mathrm{T},2}$. In addition, the already modulated signal $x_{\mathrm{T},1} = x_1(n) \cdot \cos(\Omega_{\mathrm{T},1})$ would have to be filtered with a high-pass filter and $x_{\mathrm{T},2} = x_2(n) \cdot \cos(\Omega_{\mathrm{T},2})$ with a low-pass filter (or vice versa).

Part 3 *This part of the task can be solved independently of Part 1 and Part 2.*

The continuous signal $v(t)$ was modulated using FM modulation and transmitted over one channel.

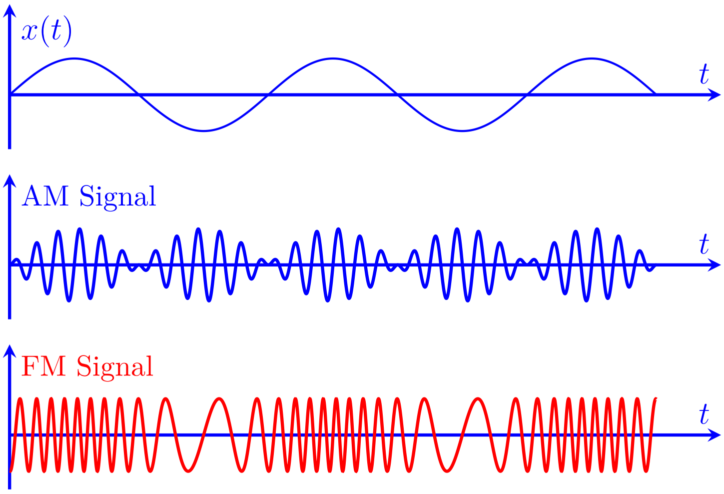
$$\phi_\mathrm{T}(t) = \begin{cases} k_\mathrm{FM} 2\pi 2\,\mathrm{V} \cdot t, & 0\,\mathrm{s} \leq t < 2\,\mathrm{s} \\ k_\mathrm{FM} 2\pi [6\,\mathrm{Vs} - 1\,\mathrm{V} \cdot t], & 2\,\mathrm{s} \leq t < 3\,\mathrm{s} \\ k_\mathrm{FM} 2\pi [9\,\mathrm{Vs} - 2\,\mathrm{V} \cdot t], & 3\,\mathrm{s} \leq t < 6\,\mathrm{s} \\ k_\mathrm{FM} 2\pi [3\,\mathrm{Vs}], & 6\,\mathrm{s} \leq t < 7\,\mathrm{s} \\ k_\mathrm{FM} 2\pi [-10\,\mathrm{Vs} + 1\,\mathrm{V} \cdot t], & 7\,\mathrm{s} \leq t < 10\,\mathrm{s} \\ 0 & \text{else} \end{cases}$$

(h) Explain the advantages of angle modulation over amplitude modulation and when FM modulation makes more sense than AM modulation with regard to the SNR in the transmission channel. (2 P)

Angle modulation methods have the potential to achieve a higher signal-to-noise ratio after demodulation and are therefore less susceptible to interference. However, this advantage can only be exploited from the so-called FM threshold.

(i) Sketch schematically the course of an FM and an AM modulated signal. Remember the corresponding labels. (2 P)

An exact representation is not necessary here. It is only important to recognize that with AM modulation the amplitude fluctuates at a constant frequency (which can be represented graphically), and with FM modulation the frequency fluctuates while the amplitude remains constant.



(j) State which signal would be expected at the receiving end for an ideal, interference-free transmission channel and describe the meaning of **all** variables and terms contained therein. (2 P)

$$c_{\mathrm{T}}(t) = \hat{c}_{\mathrm{T}} \cos\big(\underbrace{2\pi f_{\mathrm{T}} t + k 2\pi \int_{\tau=-\infty}^{t} v(\tau) d\tau}_{\Phi_{\mathrm{T}}(t)}\big)$$

carrier: c_{T}

carrier amplitude: $\hat{c}_{\mathrm{T}}$

carrier frequency: ω_{T}

carrier phase: $\phi_{\mathrm{T}}(t)$

carrier angle instantaneous phase: $\Phi_{\mathrm{T}}(t)$

(k) Give the formula for the instantaneous phase $\Phi_{\mathrm{T}}(t)$ and determine the original signal $v(t)$ from the carrier phase $\phi_{\mathrm{T}}(t)$ given above. (4 P)

$$\Phi_{\mathrm{T}}(t) = 2\pi f_{\mathrm{T}} t + k 2\pi \int_{\tau=-\infty}^{t} v(\tau) d\tau$$

$$v(t) = \begin{cases} 2\,\mathrm{V}, & 0\,\mathrm{s} \le t < 2\,\mathrm{s} \\ -1\,\mathrm{V}, & 2\,\mathrm{s} \le t < 3\,\mathrm{s} \\ -2\,\mathrm{V}, & 3\,\mathrm{s} \le t < 6\,\mathrm{s} \\ 0\,\mathrm{V}, & 6\,\mathrm{s} \le t < 7\,\mathrm{s} \\ 1\,\mathrm{V}, & 7\,\mathrm{s} \le t < 10\,\mathrm{s} \\ 0 & \text{else} \end{cases}$$

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