Signals and Systems II

Exam WS 25/26

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Date:	27.03.2026
Name:	______________________
Matriculation Number:	______________________

Declaration of the candidate before the start of the examination

I hereby confirm that I am registered for, authorized to sit and eligible to take this examination.
I understand that the date for inspecting the examination will be announced by the EE&IT Examination Office, as soon as my provisional examination result has been published in the QIS portal. After the inspection date, I am able to request my final grade in the QIS portal. I am able to appeal against this examination procedure until the end of the period for academic appeals for the second examination period at the CAU. After this, my grade becomes final.

Signature: ______________________

Marking

Problem	1	2	3
Points	/32	/33.5	/34.5

Total number of points: ____________ /100

Inspection/Return

I hereby confirm that I have acknowledged the marking of this examination and that I agree with the marking noted on this cover sheet.

☐ The examination papers will remain with me. Any later objection to the marking or grading is no longer possible.

Kiel, dated ____________ Signature: ______________________

Signals and Systems II

Exam WS 25/26

Examiner:	Prof. Dr.-Ing. Gerhard Schmidt
Room:	OS40, Norbert-Gansel-HS + R. 201
Date:	27.03.2026
Begin:	09:00 h
Reading Time:	10 minutes
Working Time:	90 minutes

Notes

- Lay out your student or personal ID for inspection.
- Label **each** paper with your **name** and **matriculation number**. Please use a **new sheet of paper** for **each task**. Additional paper is available on request.
- Do **not** use **pencil or red pen**.
- All aids – except for those which allow the communication with another person – are allowed. Prohibited aids are to be kept out of reach and should be turned off.
- The direct communication with any person who is not part of the exam supervision team is prohibited.
- For full credit, your solution is required to be comprehensible and well-reasoned. All sketches of functions require proper labeling of the axes. Please understand that the shown point distribution is only preliminary!
- In case you should feel negatively impacted by your surroundings during the exam, you must notify an exam supervisor immediately.
- The imminent ending of the exam will be announced 5 minutes and 1 minute prior to the scheduled ending time. Once the **end of the exam** has been announced, you **must stop writing** immediately.
- At the end of the exam, put together all solution sheets and hand them to an exam supervisor together with the exam tasks and the **signed cover sheet**.
- Before all exams have been collected, you are prohibited from talking or leaving your seat. Any form of communication at this point in time will still be regarded as an **attempt of deception**.
- During the **reading time, working on the exam tasks is prohibited**. Consequently, all writing tools and other allowed aids should be set aside. Any violation of this rule will be considered as an **attempt of deception**.

Task 1 (32 points)

Part 1 *This part of the exercise can be solved independently of Part 2.*

(a) What conditions must a cumulative distribution function $F_x(x)$ satisfy? (3 P)
A cumulative distribution function must be monotonically increasing , non-negative , and bounded between $\lim_{x\to-\infty} F_x(x) = 0$ and $\lim_{x\to+\infty} F_x(x) = 1$.

(b) State the relationship between a probability density function $f_x(x)$ and a cumulative distribution function $F_x(x)$. (1 P)

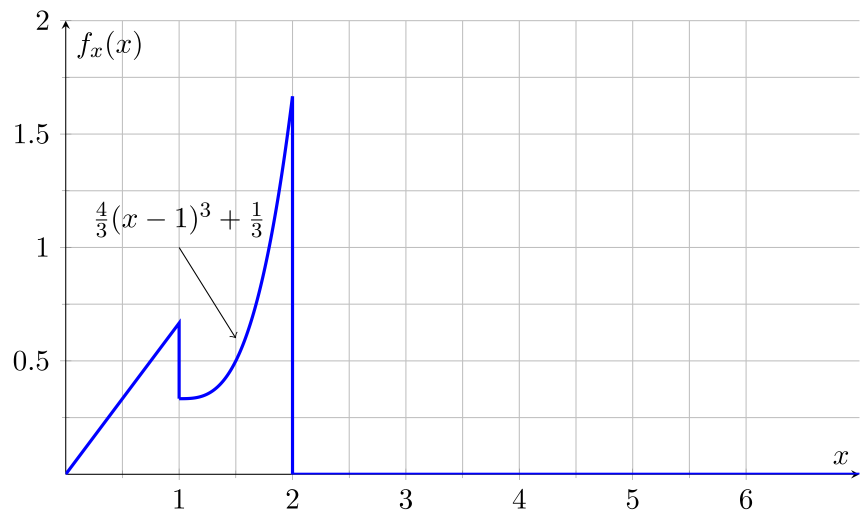
$$f_x(x) = \frac{d}{dx}F_x(x)$$

or

$$F_x(x) = \int_{u=-\infty}^{x} f_x(v)dv$$

(c) Explain the difference between a deterministic signal and a stochastic signal. (2 P)
A deterministic signal can be described analytically or is otherwise uniquely defined, whereas a stochastic signal cannot. It contains random components, which can be characterized by probability distributions or density functions and correlation functions.

The following probability density function $f_x(x)$ is given:



Additionally, it holds that:

$$f_x(x) = \begin{cases} 0 & \text{for } x \le 0 \\ 0 & \text{for } x \ge 2 \end{cases}$$

(d) Determine the cumulative distribution function $F_x(x)$ corresponding to the proba- (4 P)

bility density function $f_x(x)$ for the range $0 \leq x \leq 6$. **Hint**: $f_x(1) = \frac{2}{3}$

$$F_x(x) = \begin{cases} \frac{1}{3}x^2 & \text{for } 0 \leq x < 1 \\ \frac{1}{3}(x-1)^4 + \frac{1}{3}x & \text{for } 1 \leq x < 2 \\ 1 & \text{for } x > 2 \end{cases}$$

(e) Calculate the first moment, the second moment, and the second central moment of the probability density function $f_x(x)$. (7 P)

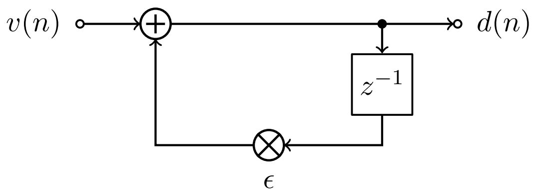
$$\begin{aligned} m_x^{(1)} = m_x &= \int_{-\infty}^{\infty} x f_x(x) dx \\ &= \int_0^1 \frac{2}{3}x^2 dx + \int_1^2 x\left(\frac{4}{3}(x-1)^3 + \frac{1}{3}\right) dx \\ &= \frac{2}{9} + \frac{11}{10} \\ &= \frac{119}{90} \end{aligned}$$

$$\begin{aligned} m_x^{(2)} &= \int_{-\infty}^{\infty} x^2 f_x(x) dx \\ &= \int_0^1 \frac{2}{3}x^3 dx + \int_1^2 x^2\left(\frac{4}{3}(x-1)^3 + \frac{1}{3}\right) dx \\ &= \frac{1}{6} + \frac{28}{15} \\ &= \frac{61}{30} \end{aligned}$$

$$\sigma_x^2 = m_x^{(2)} - m_x^2 \approx 0.285$$

Part 2 *This part of the exercise can be solved independently of Part 1.*

The following system is given with a real impulse response $h_0(n)$ and a real constant $|\epsilon| < 1$:



In the following, the system is excited with zero-mean white noise of power σ_v^2.
Hint: *Use the statistical definitions of the quantities requested.*

(f) The system $d(n) = v(n) + 0.5 \cdot d(n-1)$ is given. The input signal $v(n)$ is zero-mean white noise with variance $\sigma_v^2 = 1$.

Give the autocorrelation function of the input signal $v(n)$. (1 P)

$$s_{vv}(\kappa) = 1 \cdot \gamma_0(\kappa)$$

(g) Calculate the power $m_d^{(2)}$ of the process $d(n)$. (5 P)
Hint: *Use the property that $v(n)$ and $d(n-1)$ are uncorrelated, i.e.,* $E\{v(n)d(n-1)\} = E\{v(n)\} \cdot E\{d(n-1)\}$.

Approach using the quadratic expectation:

$$\begin{aligned} m_d^{(2)} &= E\Big\{d^2(n)\Big\} = E\Big\{[v(n) + 0.5d(n-1)]^2\Big\} \\ &= E\Big\{v^2(n)\Big\} + 2 \cdot 0.5 \cdot \underbrace{E\Big\{v(n)d(n-1)\Big\}}_{=0} + 0.5^2 \cdot E\Big\{d^2(n-1)\Big\} \\ &= 1 + 0 + 0.25 \cdot m_d^{(2)} \end{aligned}$$

Rearranging for $m_d^{(2)}$:

$$0.75 \cdot m_d^{(2)} = 1 \quad \Rightarrow \quad m_d^{(2)} = \frac{1}{0.75} = \frac{4}{3}$$

(h) Calculate the autocorrelation function $s_{dd}(\kappa)$ of $d(n)$ only at $\kappa = 1$. (3 P)

$$\begin{aligned} s_{dd}(1) &= E\Big\{d(n)d(n-1)\Big\} = E\Big\{[v(n) + 0.5d(n-1)]d(n-1)\Big\} \\ &= \underbrace{E\Big\{v(n)d(n-1)\Big\}}_{0} + 0.5 \cdot \underbrace{E\Big\{d^2(n-1)\Big\}}_{m_d^{(2)}} \\ &= 0.5 \cdot \frac{4}{3} = \frac{2}{3} \end{aligned}$$

Now, let the autocorrelation function be given as $s_{dd}(\kappa) = m_d^{(2)} \cdot \epsilon^{\kappa}$ for $\kappa \in \mathbb{R}$.

(i) Specify the function for all $\kappa \in \mathbb{Z}$ based on the form $s_{dd}(\kappa) = m_d^{(2)} \cdot \epsilon^{\kappa}$. (2 P)

$$s_{dd}(\kappa) = m_d^{(2)} \cdot \epsilon^{|\kappa|}$$

The autocorrelation function decays exponentially (maximum at $\kappa = 0$).
The system acts as a low-pass filter, turning the white noise into correlated noise.

(j) What effect does the system have on the originally white noise? (2 P)
The autocorrelation function decays exponentially (maximum at $\kappa = 0$).
The system acts as a low-pass filter and turns white noise into correlated noise.

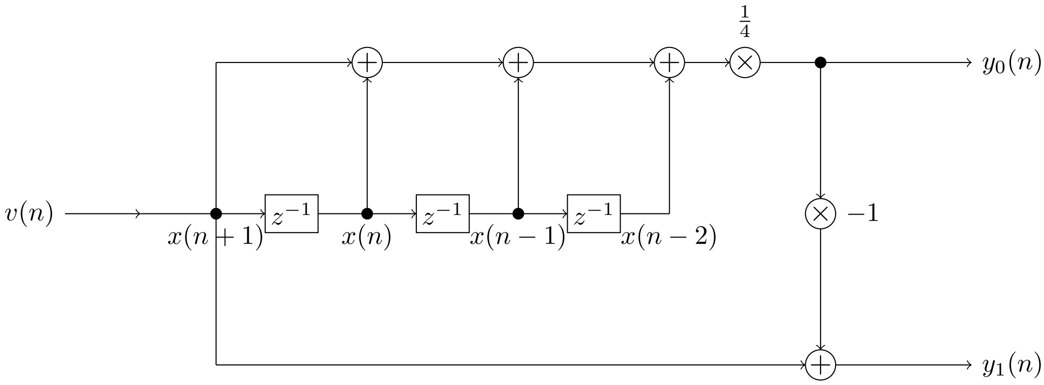
(k) Is the system stable? Justify briefly based on the factor $\epsilon = 0.5$. (2 P)
Yes, the system is stable because the feedback factor (pole in the z-domain) has magnitude less than 1 ($|0.5| < 1$).

Task 2 (33.5 points)

Part 1 *This part of the task can be solved independently of part 2.*

A discrete system should calculate the mean of the last four (current, as well as the previous three) measurements of $v(n)$ and output it as output $y_0(n)$. The output $y_1(n)$ should be the input $v(n)$ adjusted for the mean $y_0(n)$.

(a) Draw the signal flow graph belonging to the system described above. (6 P)



(b) Determine the number of inputs L, outputs M, and states N of the system. (1,5 P)

$$L = 1$$
$$M = 2$$
$$N = 3$$

(c) Is the given system causal? Explain. (1 P)
Since only current and past values are used, the system is causal.

(d) Determine matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$! (4 P)

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\boldsymbol{x}(n) + \boldsymbol{b}v(n)$$
$$\boldsymbol{y}(n) = \boldsymbol{C}\boldsymbol{x}(n) + \boldsymbol{d}v(n)$$
$$\boldsymbol{x}(n) = \begin{bmatrix} x(n) \\ x(n-1) \\ x(n-2) \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ -0.25 & -0.25 & -0.25 \end{bmatrix} \text{ and } \boldsymbol{d} = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$$

(e) What is the transfer function $H(z)$ of the system? (7 P)

$$\boldsymbol{H}(z) = \boldsymbol{C}\left(z\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{b} + \boldsymbol{d}$$

$$\left(z\boldsymbol{I} - \boldsymbol{A}\right)^{-1} = \begin{bmatrix} \frac{1}{z} & 0 & 0 \\ \frac{1}{z^2} & \frac{1}{z} & 0 \\ \frac{1}{z^3} & \frac{1}{z^2} & \frac{1}{z} \end{bmatrix}$$

$$\boldsymbol{C}\left(z\boldsymbol{I} - \boldsymbol{A}\right)^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{z} & 0 & 0 \\ \frac{1}{z^2} & \frac{1}{z} & 0 \\ \frac{1}{z^3} & \frac{1}{z^2} & \frac{1}{z} \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} & \frac{1}{z} + \frac{1}{z^2} & \frac{1}{z} \\ -\frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} & -\frac{1}{z} - \frac{1}{z^2} & -\frac{1}{z} \end{bmatrix}$$

$$\boldsymbol{C}\left(z\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{b} = \frac{1}{4}\begin{bmatrix} \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} & \frac{1}{z} + \frac{1}{z^2} & \frac{1}{z} \\ -\frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} & -\frac{1}{z} - \frac{1}{z^2} & -\frac{1}{z} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} \\ -\frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} \end{bmatrix}$$

$$\boldsymbol{C}\left(z\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{b} + \boldsymbol{d} = \frac{1}{4}\begin{bmatrix} \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} \\ -\frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix}$$

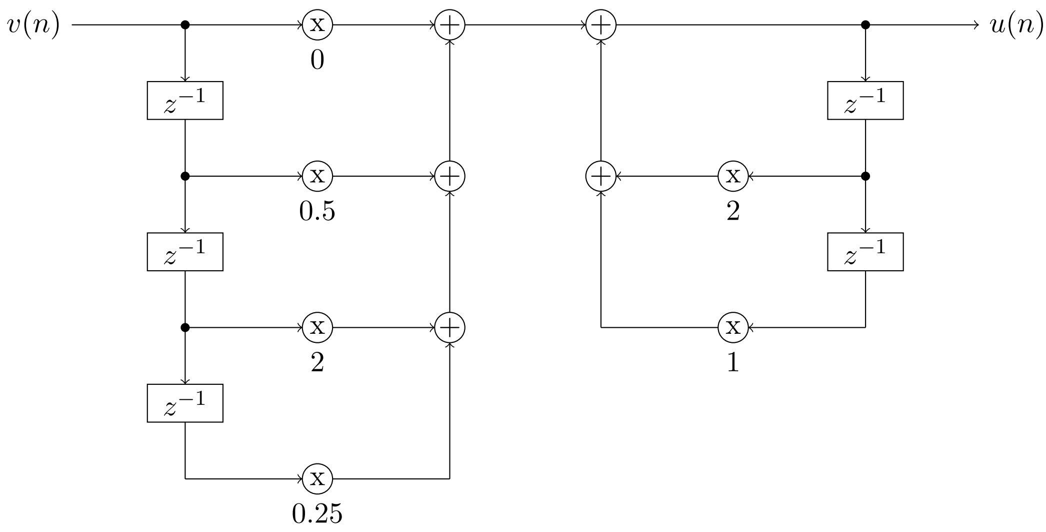
$$= \frac{1}{4}\begin{bmatrix} z^{-1} + z^{-2} + z^{-3} + 1 \\ -z^{-1} - z^{-2} - z^{-3} + 3 \end{bmatrix}$$

(f) Determine the impulse response $\boldsymbol{h}_0(n)$. (2 P)

$$\boldsymbol{h}_0(n) = \begin{bmatrix} \frac{1}{4}\left(\gamma_0(n) + \gamma_0(n-1) + \gamma_0(n-2) + \gamma_0(n-3)\right) \\ \frac{1}{4}\left(3\gamma_0(n) - \gamma_0(n-1) - \gamma_0(n-2) - \gamma_0(n-3)\right) \end{bmatrix}$$

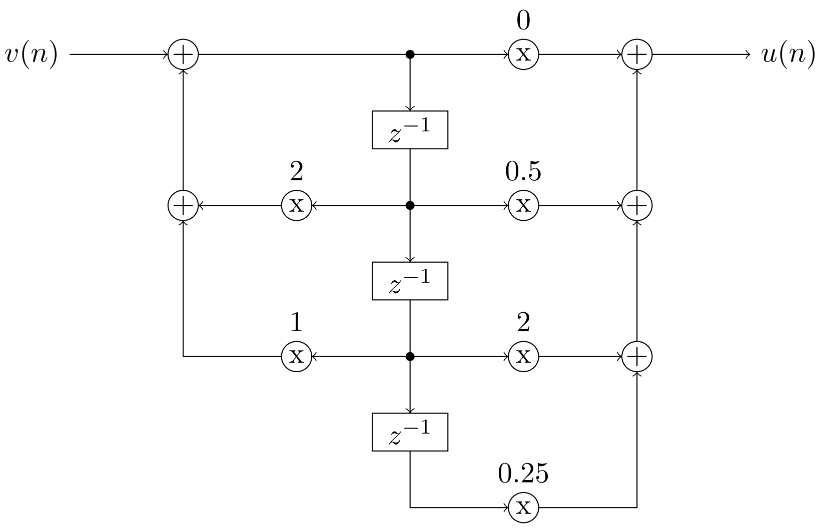
Part 2 *This part of the task can be solved independently of part 1.*

This part of the exercise deals with direct forms 1 and 2. A system in one of the two direct forms is given, see the following figure:



(g) Which of the two direct forms is being referred to here? (1 P)
This is the so-called direct form 1.

(h) Convert the given system into a representation of the remaining direct form. (4 P)



(i) Which of the direct forms is suitable if your system has limited memory? Explain your answer! (2 P)
If the system has limited memory, direct form 2 is best suited because the number of memory elements required is smaller.

(j) Determine the difference equation based on the given system. (5 P)

$$u(n) = 2u(n-1) + u(n-2) + 0.5v(n-1) + 2v(n-2) + 0.25v(n-3)$$

Task 3 (34.5 points)

Part 1 *This part of the task can be solved independently of parts 2 and 3.*

(a) What is meant by modulation, and for what purpose is it used? (2 P)
Modulation refers to the alteration of a property of a carrier signal in response to the useful signal.
It is used to adapt the spectrum of the useful signal to the characteristics of the transmission, storage, or processing medium.

(b) Name two different types of modulation. (2 P)
Examples of modulation types include:

- Amplitude modulation (AM)
- Angle modulation (e.g., frequency or phase modulation)

(c) What is meant by a carrier signal in the context of modulation? (2 P)
A carrier signal is a signal (usually high-frequency) onto which the useful signal is modulated.

Part 2 *This part of the task can be solved independently of parts 1 and 3.*

The company *FrischFunk AG* operates a system that must exchange data with logger stations via radio link on a daily basis. An example signal from such a station is denoted below as $v(n)$ and has the spectrum $V(e^{j\Omega})$ shown below, with frequencies $f_l = 5\,\text{MHz}$ and $f_u = 10\,\text{MHz}$. The sampling frequency is $f_S = 200\,\text{MHz}$.
Note: In all of the following tasks, keep in mind the relationship between frequency and the normalized angular frequency.

(d) Determine the normalized angular frequencies of the DTFT. For clearer notation, you can and should express these as multiples of π. (2.5 P)
General formula: $\Omega = 2\pi \frac{f}{f_S}$
$\Omega_l = 0.05 \cdot \pi$ and $\Omega_u = 0.1 \cdot \pi$

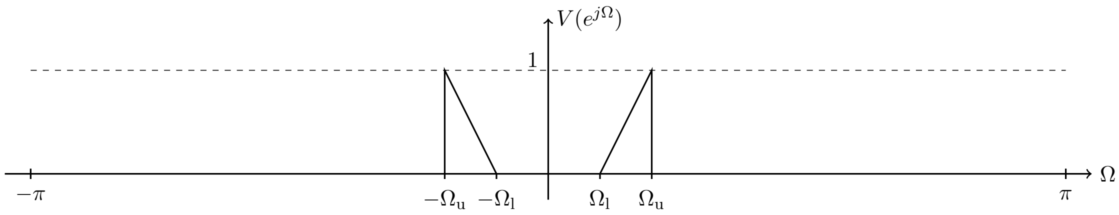


Figure 1: Spectrum $V(e^{j\Omega})$ of the useful signal $v(n)$ from *FrischFunk AG*.

All stations and the central facility are equipped with systems that enable low-pass, high-pass, or band-pass filtering, as well as modulation **before** transmission and **after** reception. The transmitter and receiver allow signals to be transmitted and received in the frequency band between $f_{\text{FF}} \in [80\,\text{MHz}, 100\,\text{MHz}]$. You now want to perform double-sideband modulation without making any further modifications to the signal. The carrier signal is $c_1(n) = \cos(\Omega_{\text{T,FF}} \cdot n)$ and the transmitted signal is $V_{\text{DSB}}(e^{j\Omega})$.

(e) What carrier frequency $f_{\text{T,FF}}$ (in MHz) must you select for DSB modulation in order to successfully transmit the useful signal? (1 P)

$$f_{\text{T,FF}} = 90\,\text{MHz}$$

(f) Calculate the spectrum $V_{\text{DSB}}(e^{j\Omega})$ of the modulated signal $v_{\text{DSB}}(n)$ and sketch it in the range $-\pi < \Omega < \pi$. Note that $|\Omega_{\text{T,FF}} + \Omega_{\text{u}}| < \pi$. (5 P)
*Hint: Simply sketching the **correctly labeled** coordinate system earns you points. Pay particular attention to the type of transformation and the notation of all relevant quantities.*

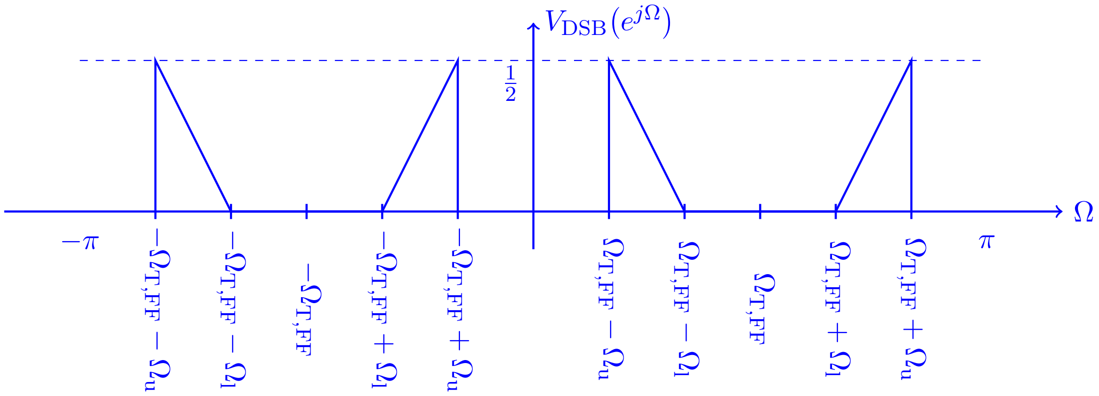
$$v_{\text{DSB}}(n) = v(n) \cdot c_1(n)$$

$$\bullet\!\!-\!\!\circ$$

$$\begin{aligned} V_{\text{DSB}}(e^{j\Omega}) &= \mathcal{F}\{v(n) \cdot c_1(n)\} \\ &= \mathcal{F}\{v(n) \cdot \cos(\Omega_{\text{T,FF}} n)\} \end{aligned}$$

Applying the cosine theorem and a multiplication in the time domain:
$\cos(\Omega_v n) \circ\!\!-\!\!\bullet\ \pi \sum_{\lambda=-\infty}^{\infty} [\delta_0(\Omega + \Omega_v - 2\pi\lambda) + \delta_0(\Omega - \Omega_v - 2\pi\lambda)]$

$$\begin{aligned} V_{\text{DSB}}(e^{j\Omega}) &= \frac{1}{2\pi} V(e^{j\Omega}) \circledast \pi \sum_{\lambda=-\infty}^{\infty} [\delta_0(\Omega + \Omega_{\text{T,FF}} - 2\pi\lambda) + \delta_0(\Omega - \Omega_{\text{T,FF}} - 2\pi\lambda)] \\ &= \frac{1}{2}\left[V(e^{j(\Omega+\Omega_{\text{T,FF}})}) + V(e^{j(\Omega-\Omega_{\text{T,FF}})})\right] \end{aligned}$$



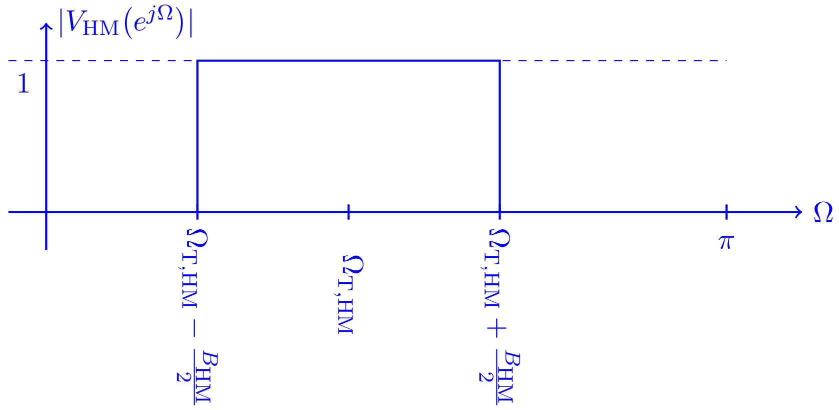
(g) Determine how much bandwidth B_{FF} (in MHz) your useful signal $v(n)$ effectively occupies after DSB modulation, i.e., the sum of the widths of all actually occupied frequency bands. (1 P)

$$B_{\mathrm{FF}} = 2 \cdot (f_{\mathrm{u}} - f_{\mathrm{l}}) = 10\,\mathrm{MHz}$$

The company *HearMe AG* now also wants to transmit data wirelessly. Its system transmits signals using angle modulation. This has a fixed carrier frequency of $f_{\mathrm{T,HM}} = 85\,\mathrm{MHz}$ and is intended to transmit useful signals with a maximum frequency of $f_{\mathrm{S,HM}} = 1\,\mathrm{MHz}$. To prevent the companies from interfering with each other, the transmission architecture of both companies must now be adapted. The *FrischFunk AG* system remains the same as described above. However, additional signal processing techniques can be applied before and after transmission. With *HearMe AG*, the carrier frequency cannot be changed – but the frequency deviation can. A higher frequency deviation has a positive effect on the received signal quality and the SNR. The bandwidth of the *HearMe AG* transmission signal is denoted by B_{HM}.

Note: Assume ideal, interference-free frequency transitions. That is, if the frequency bands of two signals are exactly adjacent, they do not interfere with each other.

(h) Sketch a schematic diagram of the spectrum of the interference signal $V_{\mathrm{HM}}(e^{j\Omega})$ in the positive frequency domain below π. The magnitude at each frequency can be assumed to be equal to one. Be sure to label the axes with all important frequencies. The bandwidth B_{HM} can initially be considered variable. (4 P)



(i) Design a transmission and reception system that enables *HearMe AG* to set its frequency offset to the maximum. Describe your procedure step by step and present all signals used, such as the transmit signal $S_{\mathrm{Tx}}(e^{j\Omega})$, the filters used, the receive signal $S_{\mathrm{Rx}}(e^{j\Omega})$, and the demodulated signal $Y(e^{j\Omega})$, in mathematical form. Next, specify the bandwidth now available to *HearMe AG* and the resulting frequency deviation Δf. (11 P)
Tip: The modulation type used does not need to be changed. The spectrum of the baseband signal can be represented by $V(e^{j\Omega}) = V_{neg}(e^{j\Omega}) + V_{pos}(e^{j\Omega})$*, where* $V_{neg}(e^{j\Omega})$

describes all negative frequency components of $V(e^{j\Omega})$ and $V_{pos}(e^{j\Omega})$ describes all positive frequency components of $V(e^{j\Omega})$. Filters need only be defined in the range $\pi < \Omega < \pi$ and can be assumed to be 2π-periodic outside of this range.

1. To minimize the required bandwidth, the ESB can be used. To do this, the signal $V_{\mathrm{DSB}}(e^{j\Omega})$ must be high-pass filtered before transmission, with a cutoff frequency of $\Omega_{\mathrm{G}} = \Omega_{\mathrm{T,FF}}$:

$$H_{\mathrm{HP}}(e^{j\Omega}) = \begin{cases} 1 & \Omega_{\mathrm{G}} \leq |\Omega| \leq \pi \\ 0 & \text{else} \end{cases}$$

$$S_{\mathrm{Tx}}(e^{j\Omega}) = V_{\mathrm{ESB}}(e^{j\Omega}) = V_{\mathrm{DSB}}(e^{j\Omega}) \cdot H_{\mathrm{HP}}(e^{j\Omega})$$

2. After transmitting over the disrupted channel, the following signal is received:

$$S_{\mathrm{Rx}}(e^{j\Omega}) = S_{\mathrm{Tx}}(e^{j\Omega}) + V_{\mathrm{HM}}(e^{j\Omega})$$

3. To eliminate unwanted signal components, the received signal must be passed through a high-pass filter. This time, however, with an adjusted cutoff frequency $\Omega_{\mathrm{G}} = \Omega_{\mathrm{T,FF}} + \Omega_{\mathrm{l}}$:

$$S_{\mathrm{Rx,HP}}(e^{j\Omega}) = V_{\mathrm{ESB}}(e^{j\Omega}) = S_{\mathrm{Rx}}(e^{j\Omega}) \cdot H_{\mathrm{HP}}(e^{j\Omega})$$

4. Coherent demodulation with $c_2(n) = \cos(\Omega_{\mathrm{T}} n)$ yields the reconstructed spectrum:

$$\begin{aligned} Y(e^{j\Omega}) &= \mathcal{F}\{s_{\mathrm{HP}}(n) \cdot c_2(n)\} \\ &= \frac{1}{2} \underbrace{\left[V_{\mathrm{neg}}(e^{j\Omega}) + V_{\mathrm{pos}}(e^{j\Omega})\right]}_{V(e^{j\Omega})} + \frac{1}{4}\left[V_{\mathrm{neg}}(e^{j(\Omega+2\Omega_{\mathrm{T,FF}})}) + V_{\mathrm{pos}}(e^{j(\Omega-2\Omega_{\mathrm{T,FF}})})\right] \\ &= \frac{1}{4} V(e^{j\Omega}) + \frac{1}{4}\left[V_{\mathrm{neg}}(e^{j(\Omega+2\Omega_{\mathrm{T,FF}})}) + V_{\mathrm{pos}}(e^{j(\Omega-2\Omega_{\mathrm{T,FF}})})\right] \end{aligned}$$

5. Low-pass filtering to remove the spectral components at the double carrier frequencies:

$$H_{\mathrm{TP}}(e^{j\Omega}) = \begin{cases} 4 & 0 \leq |\Omega| \leq \Omega_{\mathrm{u}} \\ 0 & \text{else} \end{cases}$$

The resulting signal is given by:

$$\begin{aligned} Y_{\mathrm{TP}}(e^{j\Omega}) &= Y(e^{j\Omega}) \cdot H_{\mathrm{TP}}(e^{j\Omega}) \\ &= 4 \cdot \left[\frac{1}{4} V(e^{j\Omega}) + \underbrace{\cancel{\frac{1}{4}\left[V_{\mathrm{neg}}(e^{j(\Omega+2\Omega_T)}) + V_{\mathrm{pos}}(e^{j(\Omega-2\Omega_T)})\right]}}_{\text{Eliminated by low-pass}}\right] \\ &= V(e^{j\Omega}) \end{aligned}$$

The resulting signal then lies in the band 95 mHz to 100 mHz. For *HearMe AG*, the band above the carrier frequency of 85 mHz to 95 mHz is thus available. The total bandwidth of HearMe AG is thus $B_{\text{HM}} = 20\,\text{MHz}$. According to Carlson, the bandwidth required for FM can be estimated using the following formula:

$$B_{\text{FM}} = 2 \cdot (\frac{\Delta f}{f_{\text{max}}} + 2) \cdot f_{\text{max}} = 20\,\text{MHz}$$

By rearranging, you get

$$\Delta f = (\frac{B_{\text{FM}}}{2 \cdot f_{\text{max}}} - 2) \cdot f_{\text{max}} = (\frac{20\,\text{MHz}}{2 \cdot 1\,\text{MHz}} - 2) \cdot 1\,\text{MHz} = 8\,\text{MHz}$$

Part 3 *This part of the task can be solved independently of parts 1 and 2.*

There is a low-frequency cosine wave as the useful signal in the audible range with $f_{\text{S}} = 100\,\text{Hz}$, which is to be amplitude- or phase-modulated. Another cosine wave with $f_{\text{C}} = 1$ kHz, also in the audible range, is used as the carrier signal.

(j) The modulated signal is played back through a loudspeaker. Describe what you hear for an amplitude-modulated (AM) and an frequency-modulated (FM) signal waveform, respectively. (3 P)

Amplitude Modulation (AM): A tone with a carrier frequency of approximately 1 kHz is audible, whose volume fluctuates periodically with the frequency of the useful signal (100 Hz).

Frequency Modulation (FM): A tone of approximately 1 kHz is audible, whose pitch varies periodically with the frequency of the useful signal.

(k) What effect does the frequency deviation have on the audible signal in angular modulation? (1 P)

The frequency deviation determines how much the instantaneous frequency deviates from the carrier frequency as a result of amplitude fluctuations in the modulated signal.

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